Computer Algebra Independent Integration Tests

Summer 2023 edition

4-Trig-functions/4.6-Cosecant/128-4.6.11-e-x-^m-a+b-csc-c+d-x^n-^p

Nasser M. Abbasi

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [84]. This is test number [128].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (84)	0.00 (0)
Mathematica	95.24 (80)	4.76 (4)
Fricas	76.19 (64)	23.81 (20)
Maple	61.90 (52)	38.10 (32)
Maxima	60.71 (51)	39.29 (33)
Mupad	55.95 (47)	44.05 (37)
Giac	52.38 (44)	47.62 (40)
Sympy	44.05 (37)	55.95 (47)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

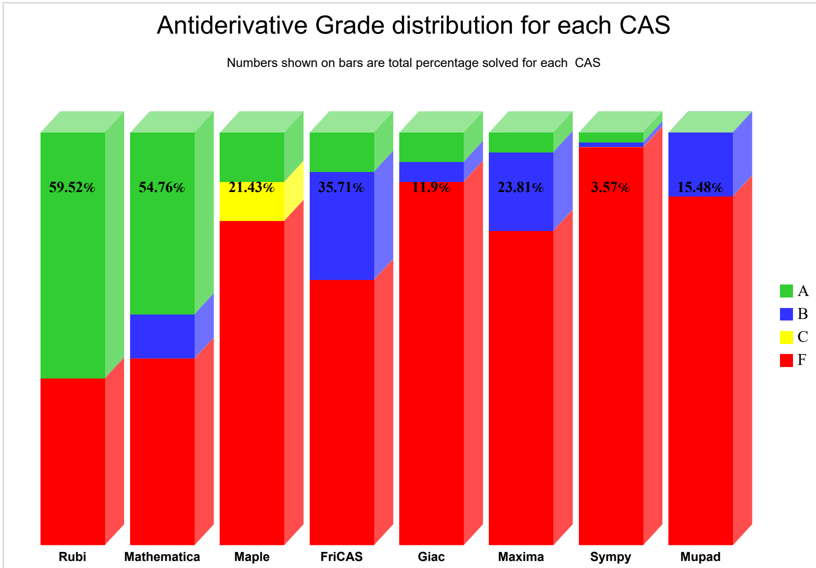
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

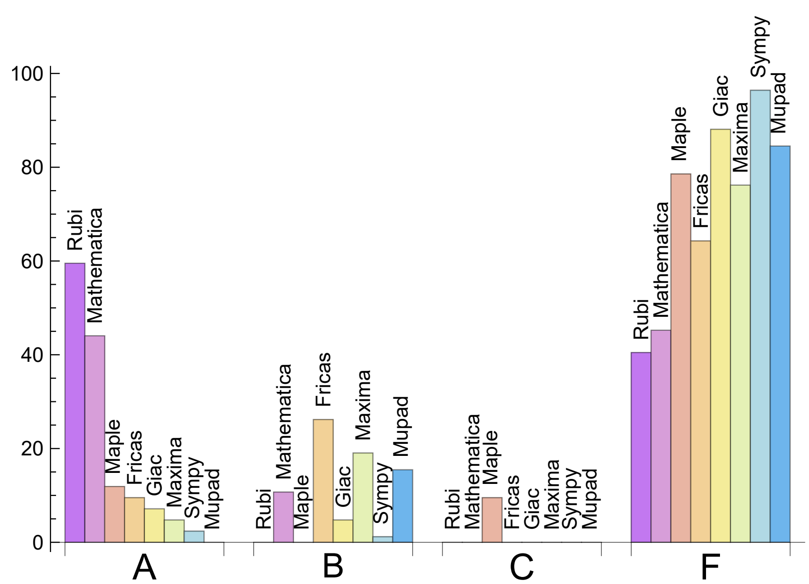
System	% A grade	% B grade	% C grade	% F grade
Rubi	59.524	0.000	0.000	40.476
Mathematica	44.048	10.714	0.000	45.238
Maple	11.905	0.000	9.524	78.571
Fricas	9.524	26.190	0.000	64.286
Giac	7.143	4.762	0.000	88.095
Maxima	4.762	19.048	0.000	76.190
Sympy	2.381	1.190	0.000	96.429
Mupad	0.000	15.476	0.000	84.524

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Fricas	20	100.00	0.00	0.00
Maxima	33	45.45	15.15	39.39
Maple	32	100.00	0.00	0.00
Mupad	37	0.00	100.00	0.00
Giac	40	100.00	0.00	0.00
Sympy	47	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.28
Giac	0.49
Rubi	0.56
Maple	0.67
Maxima	1.41
Sympy	4.19
Mathematica	15.38
Mupad	17.59

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	21.24	1.01	17.00	0.94
Giac	38.32	1.18	20.00	1.11
Mupad	170.23	2.46	22.00	1.22
Maple	174.12	1.44	18.00	1.00
Rubi	308.85	1.00	73.00	1.00
Fricas	392.12	2.26	45.00	2.11
Mathematica	417.25	1.39	73.50	1.11
Maxima	1090.16	33.15	310.00	6.75

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

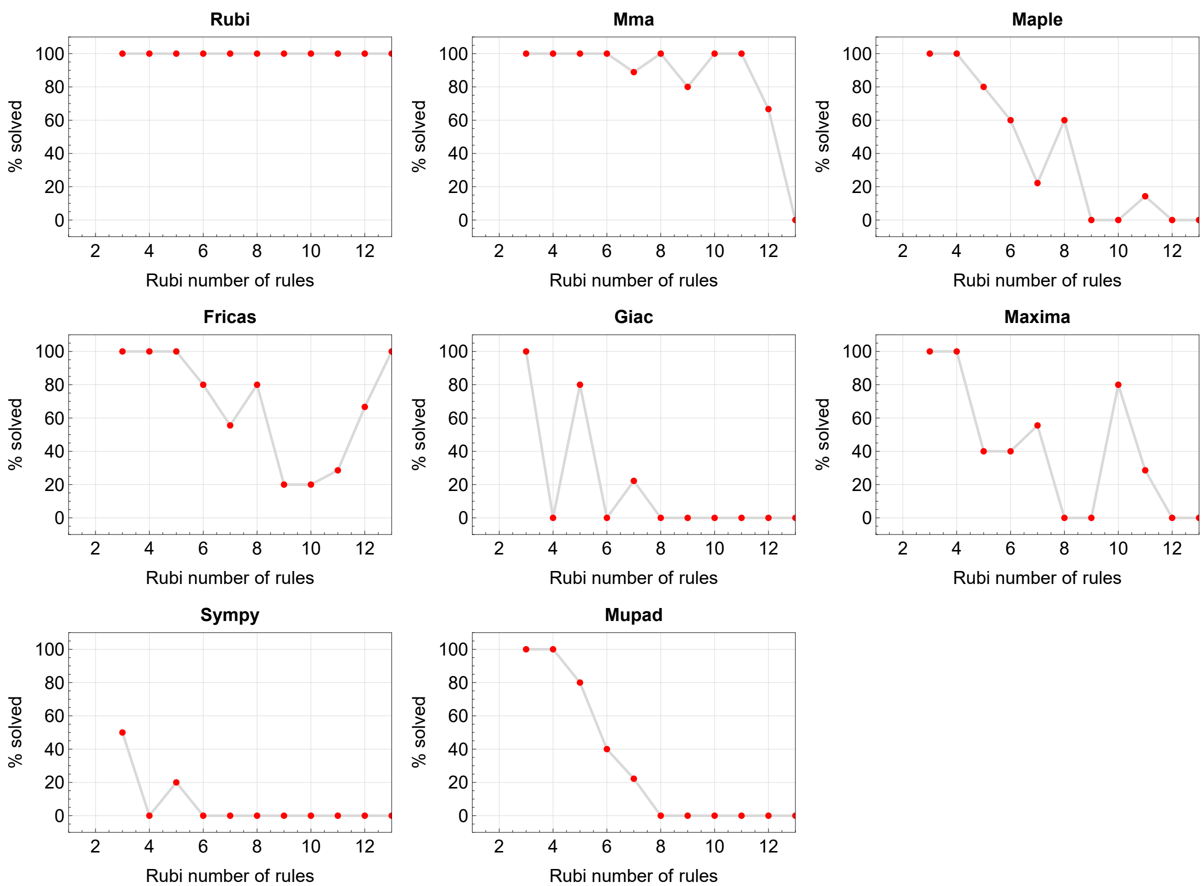


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

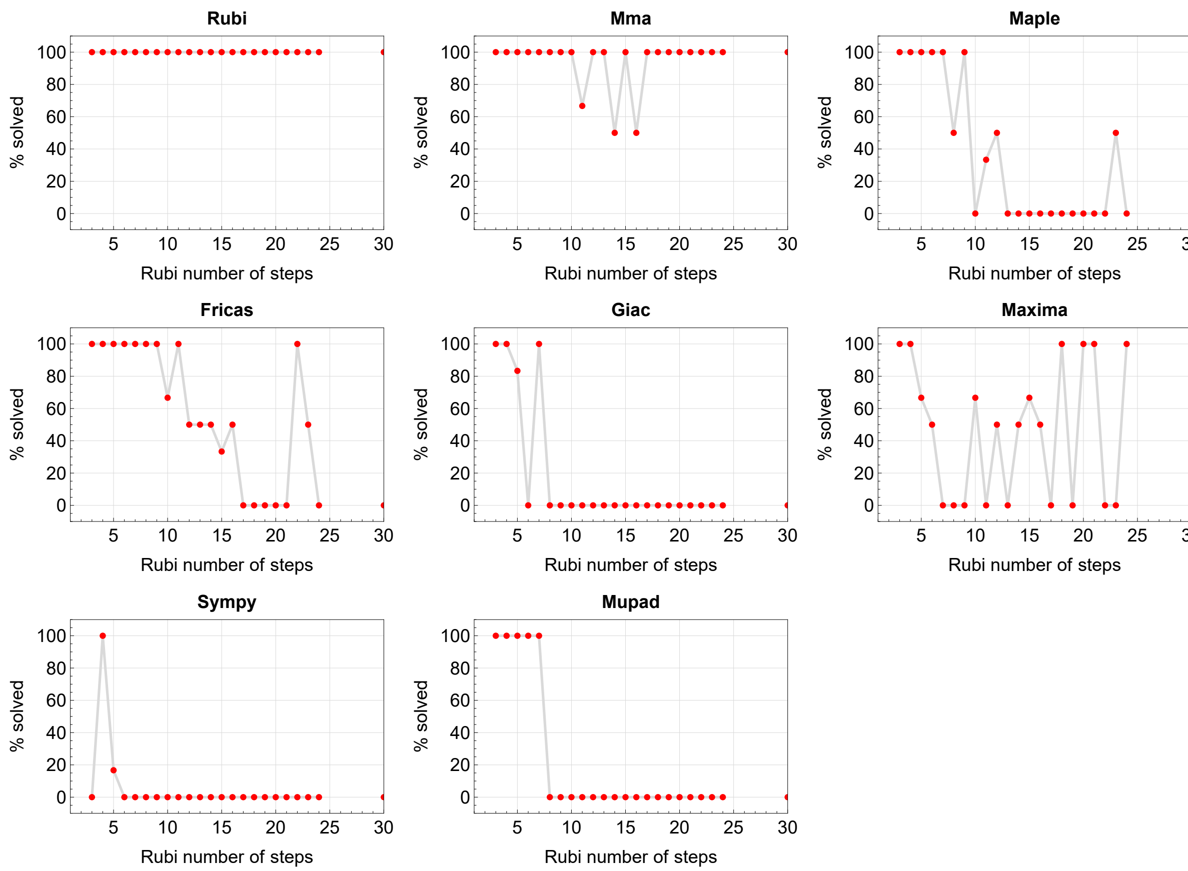


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

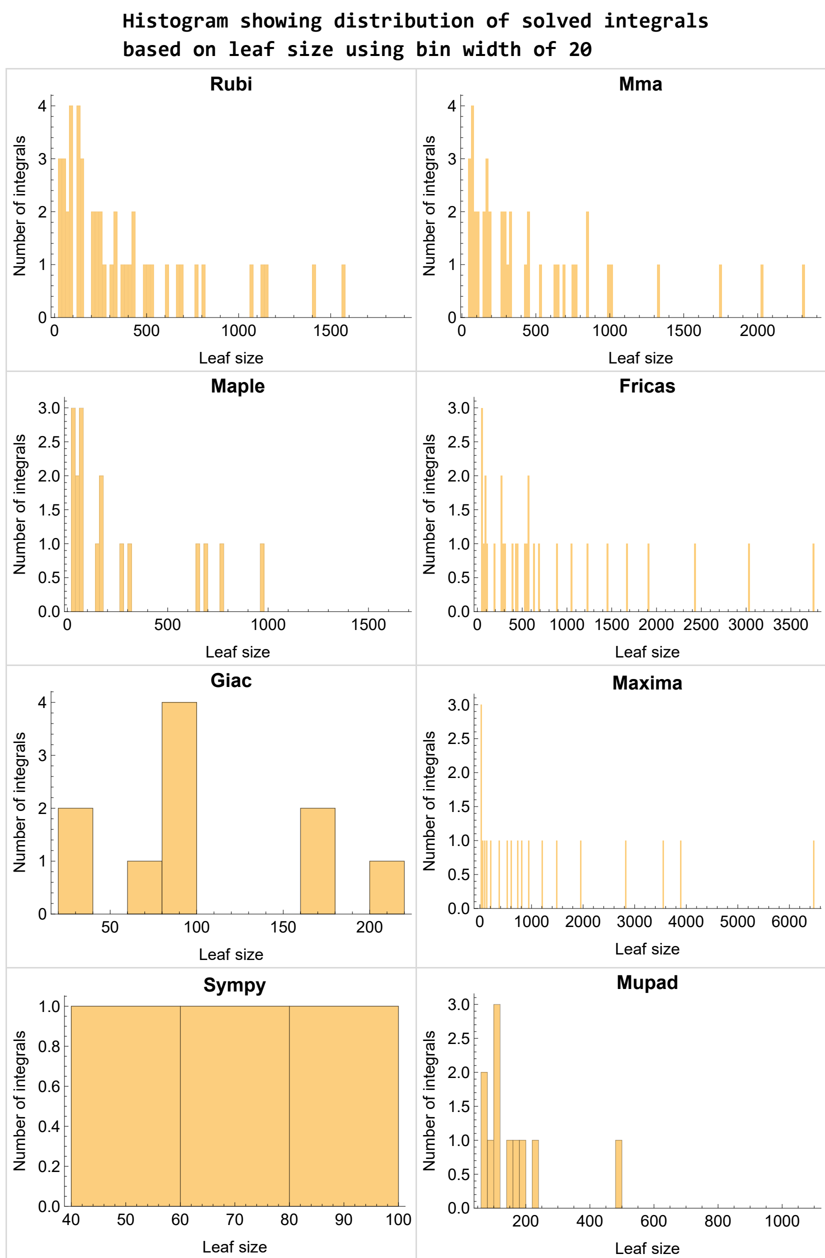


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

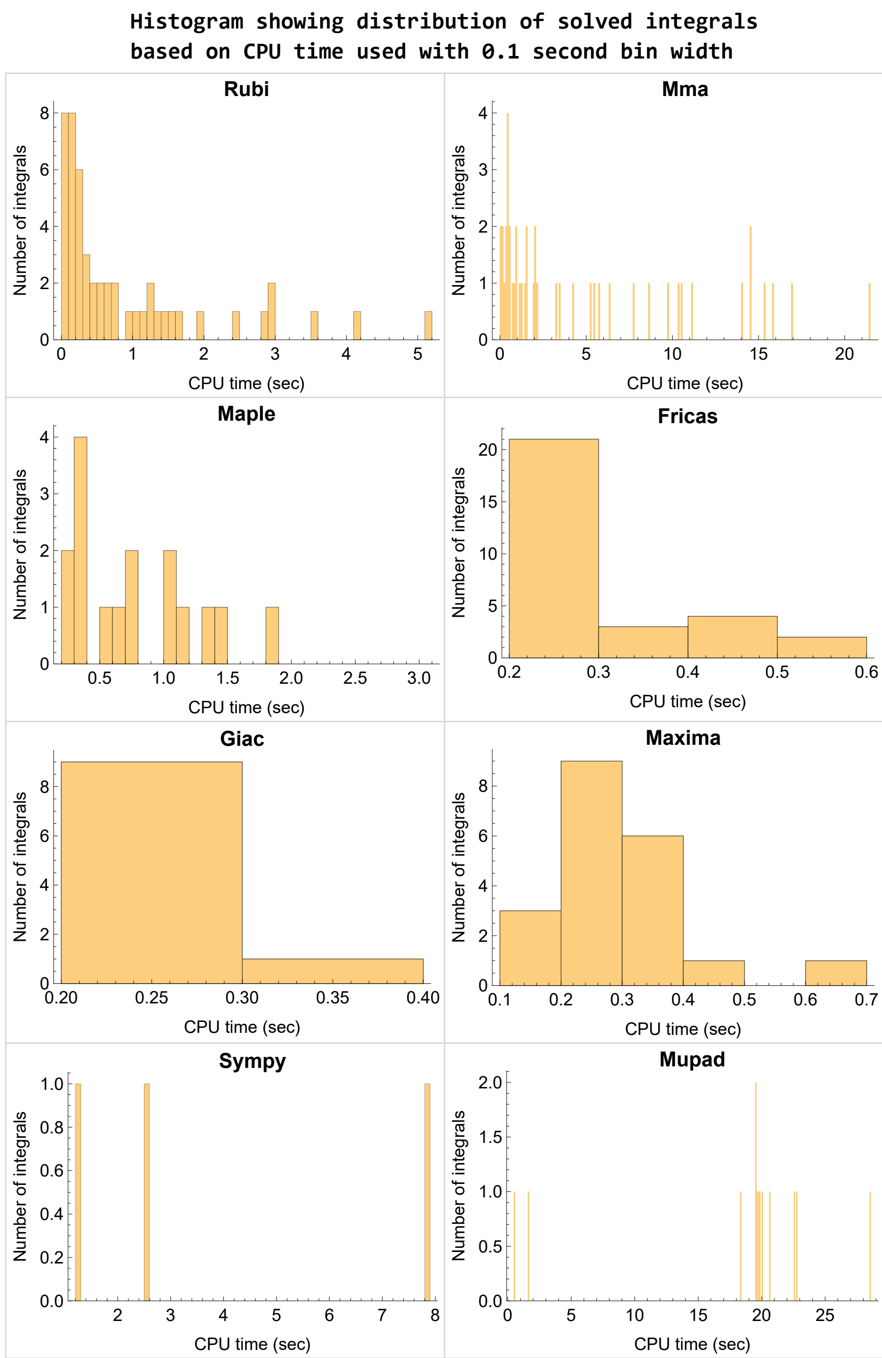


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

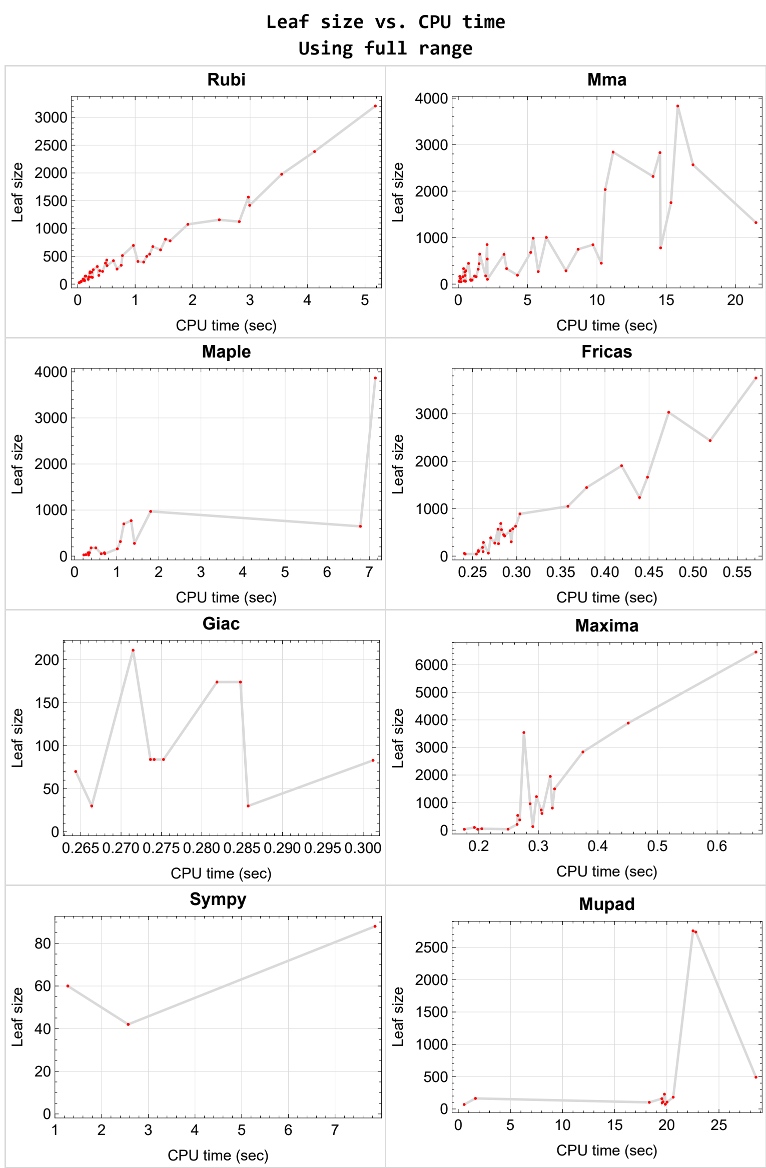


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{2, 4, 6, 7, 9, 11, 13, 14, 17, 19, 21, 22, 24, 26, 28, 29, 30, 34, 35, 39, 40, 44, 45, 49, 50, 54, 55, 59, 60, 65, 66, 70, 71, 72}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {18, 25, 46, 47, 48, 67, 80, 83}

Maple {73, 74, 76, 77, 79, 80, 82, 83}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

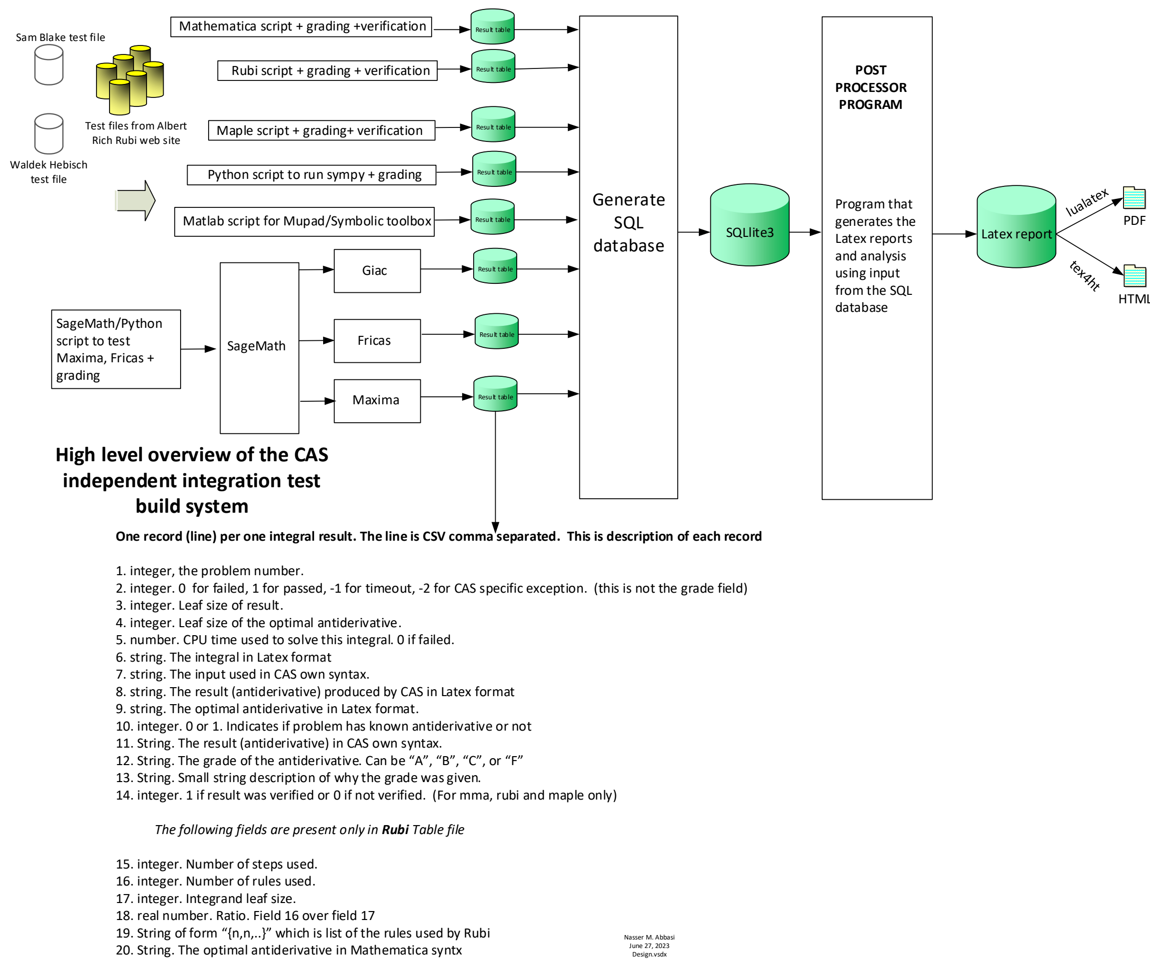
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 3, 5, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 53, 56, 57, 58, 61, 62, 63, 64, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 3, 12, 15, 16, 20, 23, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 53, 56, 58, 62, 63, 64, 67, 68, 69, 73, 74, 76, 77, 79, 82 }

B grade { 5, 8, 10, 18, 25, 57, 61, 80, 83 }

C grade { }

F normal fail { 75, 78, 81, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 5, 12, 15, 20, 27, 53, 58, 61, 64, 69 }

B grade { }

C grade { 73, 74, 76, 77, 79, 80, 82, 83 }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 62, 63, 67, 68, 75, 78, 81, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 5, 20, 53, 64, 73, 76, 79, 82 }

B grade { 1, 3, 8, 10, 12, 15, 16, 18, 23, 25, 27, 58, 61, 69, 74, 75, 77, 78, 80, 81, 83, 84 }

C grade { }

F normal fail { 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 62, 63, 67, 68 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 5, 53, 58, 61 }

B grade { 8, 10, 12, 15, 31, 32, 33, 36, 37, 38, 51, 52, 56, 57, 73, 76 }

C grade { }

F normal fail { 1, 3, 16, 18, 23, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84 }

F(-1) timedout fail { 20, 27, 60, 70, 71 }

F(-2) exception fail { 25, 41, 42, 43, 46, 47, 48, 62, 63, 64, 67, 68, 69 }

Giac

A grade { 5, 20, 27, 53, 64, 69 }

B grade { 12, 15, 58, 61 }

C grade { }

F normal fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 62, 63, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 5, 12, 15, 20, 27, 53, 58, 61, 64, 69, 73, 76, 79 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 3, 8, 10, 16, 18, 23, 25, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 62, 63, 67, 68, 74, 75, 77, 78, 80, 81, 82, 83, 84 }

F(-2) exception fail { }

Sympy

A grade { 53, 58 }

B grade { 5 }

C grade { }

F normal fail { 1, 3, 8, 10, 12, 15, 16, 18, 20, 23, 25, 27, 31, 32, 33, 36, 37, 38, 41, 42, 43, 46, 47, 48, 51, 52, 56, 57, 61, 62, 63, 64, 67, 68, 69, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	159	0	0	425	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	3.29	0.00	0.00	0.00
time (sec)	N/A	0.196	0.332	0.000	0.000	0.286	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	106	21	15	18	20
N.S.	1	1.00	1.12	1.00	6.62	1.31	0.94	1.12	1.25
time (sec)	N/A	0.019	4.137	0.109	0.249	0.258	2.093	0.359	17.162

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	84	118	0	0	288	0	0	0
N.S.	1	1.00	1.40	0.00	0.00	3.43	0.00	0.00	0.00
time (sec)	N/A	0.099	0.163	0.000	0.000	0.262	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	106	21	15	18	20
N.S.	1	1.00	1.12	1.00	6.62	1.31	0.94	1.12	1.25
time (sec)	N/A	0.020	3.273	0.111	0.262	0.242	1.827	0.301	17.513

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	57	26	31	44	42	30	69
N.S.	1	1.00	2.19	1.00	1.19	1.69	1.62	1.15	2.65
time (sec)	N/A	0.029	0.086	0.217	0.176	0.242	2.569	0.266	0.563

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	108	18	14	18	20
N.S.	1	1.00	1.12	1.00	6.75	1.12	0.88	1.12	1.25
time (sec)	N/A	0.021	3.008	0.110	0.252	0.266	0.618	0.269	18.061

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	126	18	15	18	20
N.S.	1	1.00	1.12	1.00	7.88	1.12	0.94	1.12	1.25
time (sec)	N/A	0.021	2.983	0.111	0.245	0.236	0.413	0.398	18.250

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	228	228	639	0	800	687	0	0	0
N.S.	1	1.00	2.80	0.00	3.51	3.01	0.00	0.00	0.00
time (sec)	N/A	0.433	3.300	0.000	0.323	0.282	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	310	42	17	20	22
N.S.	1	1.00	1.11	1.00	17.22	2.33	0.94	1.11	1.22
time (sec)	N/A	0.025	22.316	0.198	0.395	0.236	2.492	1.017	17.669

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	125	268	0	604	451	0	0	0
N.S.	1	1.00	2.14	0.00	4.83	3.61	0.00	0.00	0.00
time (sec)	N/A	0.192	5.766	0.000	0.306	0.285	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	300	42	17	20	22
N.S.	1	1.00	1.11	1.00	16.67	2.33	0.94	1.11	1.22
time (sec)	N/A	0.026	35.070	0.188	0.372	0.249	2.142	0.941	18.882

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	86	51	98	94	0	84	102
N.S.	1	1.00	1.91	1.13	2.18	2.09	0.00	1.87	2.27
time (sec)	N/A	0.059	0.998	0.632	0.193	0.257	0.000	0.274	18.321

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	360	36	15	20	22
N.S.	1	1.00	1.11	1.00	20.00	2.00	0.83	1.11	1.22
time (sec)	N/A	0.026	86.008	0.205	0.370	0.237	1.946	0.319	17.557

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	366	36	17	20	22
N.S.	1	1.00	1.11	1.00	20.33	2.00	0.94	1.11	1.22
time (sec)	N/A	0.026	49.148	0.197	0.403	0.246	0.607	1.139	18.132

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	167	73	3543	183	0	211	491
N.S.	1	1.00	1.86	0.81	39.37	2.03	0.00	2.34	5.46
time (sec)	N/A	0.092	0.108	0.706	0.276	0.261	0.000	0.271	28.546

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	396	396	319	0	0	1445	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	1.148	1.434	0.000	0.000	0.379	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	253	20	15	20	22
N.S.	1	1.00	1.11	1.00	14.06	1.11	0.83	1.11	1.22
time (sec)	N/A	0.032	2.407	0.144	0.399	0.227	0.426	0.311	18.154

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	B	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	271	271	987	0	0	1050	0	0	0
N.S.	1	1.00	3.64	0.00	0.00	3.87	0.00	0.00	0.00
time (sec)	N/A	0.685	5.402	0.000	0.000	0.358	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	253	20	15	20	22
N.S.	1	1.00	1.11	1.00	14.06	1.11	0.83	1.11	1.22
time (sec)	N/A	0.032	4.136	0.119	0.396	0.234	0.346	0.313	18.412

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	66	73	0	261	0	84	163
N.S.	1	1.00	1.05	1.16	0.00	4.14	0.00	1.33	2.59
time (sec)	N/A	0.120	0.492	0.322	0.000	0.279	0.000	0.274	1.646

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	250	19	15	20	22
N.S.	1	1.00	1.11	1.00	13.89	1.06	0.83	1.11	1.22
time (sec)	N/A	0.032	3.330	0.118	0.376	0.246	0.709	0.288	20.828

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	126	18	15	18	20
N.S.	1	1.00	1.12	1.00	7.88	1.12	0.94	1.12	1.25
time (sec)	N/A	0.017	0.195	0.010	0.279	0.237	0.379	0.399	0.002

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1124	1124	2033	0	0	3032	0	0	0
N.S.	1	1.00	1.81	0.00	0.00	2.70	0.00	0.00	0.00
time (sec)	N/A	2.812	10.597	0.000	0.000	0.472	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1286	38	17	20	22
N.S.	1	1.00	1.11	1.00	71.44	2.11	0.94	1.11	1.22
time (sec)	N/A	0.027	12.347	0.164	0.925	0.253	1.118	0.383	21.228

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	616	616	2566	0	0	1906	0	0	0
N.S.	1	1.00	4.17	0.00	0.00	3.09	0.00	0.00	0.00
time (sec)	N/A	1.442	16.929	0.000	0.000	0.419	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	1265	38	17	20	22
N.S.	1	1.00	1.11	1.00	70.28	2.11	0.94	1.11	1.22
time (sec)	N/A	0.027	10.240	0.147	0.808	0.257	0.956	0.395	26.319

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	158	179	0	536	0	174	2755
N.S.	1	1.00	1.32	1.49	0.00	4.47	0.00	1.45	22.96
time (sec)	N/A	0.256	1.288	0.503	0.000	0.293	0.000	0.282	22.512

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	4629	38	17	20	22
N.S.	1	1.00	1.11	1.00	257.17	2.11	0.94	1.11	1.22
time (sec)	N/A	0.026	18.418	0.147	5.455	0.252	1.140	0.733	18.556

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	4560	44	19	20	22
N.S.	1	1.00	1.11	1.00	253.33	2.44	1.06	1.11	1.22
time (sec)	N/A	0.026	14.000	0.145	5.412	0.257	1.027	0.388	17.974

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	3530	44	19	20	22
N.S.	1	1.00	1.11	1.00	196.11	2.44	1.06	1.11	1.22
time (sec)	N/A	0.026	15.641	0.149	5.369	0.257	1.092	1.020	17.759

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	432	432	445	0	1498	0	0	0	0
N.S.	1	1.00	1.03	0.00	3.47	0.00	0.00	0.00	0.00
time (sec)	N/A	0.506	0.738	0.000	0.327	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	316	316	333	0	956	0	0	0	0
N.S.	1	1.00	1.05	0.00	3.03	0.00	0.00	0.00	0.00
time (sec)	N/A	0.341	0.382	0.000	0.286	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	200	260	0	534	0	0	0	0
N.S.	1	1.00	1.30	0.00	2.67	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.474	0.000	0.265	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	105	18	15	18	20
N.S.	1	1.00	1.11	0.89	5.83	1.00	0.83	1.00	1.11
time (sec)	N/A	0.020	11.763	0.329	0.510	0.247	1.649	0.315	18.570

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	111	18	17	18	20
N.S.	1	1.00	1.11	0.89	6.17	1.00	0.94	1.00	1.11
time (sec)	N/A	0.019	16.932	0.322	0.557	0.260	1.575	0.347	18.431

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	695	695	1323	0	6462	0	0	0	0
N.S.	1	1.00	1.90	0.00	9.30	0.00	0.00	0.00	0.00
time (sec)	N/A	0.969	21.468	0.000	0.665	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	513	513	779	0	3885	0	0	0	0
N.S.	1	1.00	1.52	0.00	7.57	0.00	0.00	0.00	0.00
time (sec)	N/A	0.778	14.591	0.000	0.451	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	333	333	449	0	1950	0	0	0	0
N.S.	1	1.00	1.35	0.00	5.86	0.00	0.00	0.00	0.00
time (sec)	N/A	0.511	10.315	0.000	0.320	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	386	36	17	20	22
N.S.	1	1.00	1.10	0.90	19.30	1.80	0.85	1.00	1.10
time (sec)	N/A	0.026	137.819	0.428	0.834	0.262	8.330	0.486	18.794

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	388	36	19	20	22
N.S.	1	1.00	1.10	0.90	19.40	1.80	0.95	1.00	1.10
time (sec)	N/A	0.026	85.216	0.445	1.007	0.253	2.444	0.599	18.801

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1075	1075	850	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.917	2.077	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	807	807	644	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.527	1.548	0.000	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	539	539	438	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.252	1.503	0.000	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	242	19	17	20	22
N.S.	1	1.00	1.10	0.90	12.10	0.95	0.85	1.00	1.10
time (sec)	N/A	0.028	4.542	0.373	0.783	0.248	2.060	0.296	18.210

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	111	18	17	18	20
N.S.	1	1.00	1.11	0.89	6.17	1.00	0.94	1.00	1.11
time (sec)	N/A	0.019	0.211	0.003	0.585	0.256	1.579	0.358	0.002

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	3205	3205	3831	0	0	0	0	0	0
N.S.	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.180	15.831	0.000	0.000	0.000	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	2385	2385	2829	0	0	0	0	0	0
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.122	14.549	0.000	0.000	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1565	1565	1752	0	0	0	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.970	15.341	0.000	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	4411	38	19	20	22
N.S.	1	1.00	1.10	0.90	220.55	1.90	0.95	1.00	1.10
time (sec)	N/A	0.026	61.389	0.402	13.439	0.269	3.883	0.672	18.146

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	4411	44	20	20	22
N.S.	1	1.00	1.10	0.90	220.55	2.20	1.00	1.00	1.10
time (sec)	N/A	0.025	59.362	0.439	18.932	0.256	8.921	0.976	17.921

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	258	258	286	0	730	0	0	0	0
N.S.	1	1.00	1.11	0.00	2.83	0.00	0.00	0.00	0.00
time (sec)	N/A	0.270	0.548	0.000	0.305	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	191	0	370	0	0	0	0
N.S.	1	1.00	1.33	0.00	2.57	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	4.266	0.000	0.269	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	52	32	31	43	60	30	73
N.S.	1	1.00	2.00	1.23	1.19	1.65	2.31	1.15	2.81
time (sec)	N/A	0.027	0.205	0.268	0.198	0.254	1.277	0.286	19.859

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	113	25	19	18	20
N.S.	1	1.00	1.10	0.80	5.65	1.25	0.95	0.90	1.00
time (sec)	N/A	0.017	25.613	0.314	0.622	0.230	1.014	0.298	18.454

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	16	115	25	19	18	20
N.S.	1	1.00	1.10	0.80	5.75	1.25	0.95	0.90	1.00
time (sec)	N/A	0.017	28.586	0.313	0.615	0.244	6.176	0.342	18.240

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	421	421	749	0	2836	0	0	0	0
N.S.	1	1.00	1.78	0.00	6.74	0.00	0.00	0.00	0.00
time (sec)	N/A	0.621	8.645	0.000	0.375	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	241	241	681	0	1217	0	0	0	0
N.S.	1	1.00	2.83	0.00	5.05	0.00	0.00	0.00	0.00
time (sec)	N/A	0.383	5.224	0.000	0.297	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	93	51	52	94	88	83	111
N.S.	1	1.00	1.98	1.09	1.11	2.00	1.87	1.77	2.36
time (sec)	N/A	0.060	0.892	0.716	0.205	0.262	7.856	0.301	19.635

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	794	46	20	20	22
N.S.	1	1.00	1.09	0.82	36.09	2.09	0.91	0.91	1.00
time (sec)	N/A	0.025	76.763	0.459	1.148	0.252	1.980	0.368	18.061

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	46	20	20	22
N.S.	1	1.00	1.09	0.82	0.00	2.09	0.91	0.91	1.00
time (sec)	N/A	0.025	84.563	0.467	0.000	0.247	7.154	0.394	18.008

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	57	24	34	56	0	70	94
N.S.	1	1.00	2.38	1.00	1.42	2.33	0.00	2.92	3.92
time (sec)	N/A	0.026	0.056	0.331	0.249	0.241	0.000	0.264	19.549

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	675	675	539	0	0	0	0	0	0
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.307	2.090	0.000	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	407	407	333	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.048	3.485	0.000	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	68	73	0	275	0	84	159
N.S.	1	1.00	1.03	1.11	0.00	4.17	0.00	1.27	2.41
time (sec)	N/A	0.107	0.427	0.351	0.000	0.275	0.000	0.275	19.521

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	244	27	20	20	22
N.S.	1	1.00	1.09	0.82	11.09	1.23	0.91	0.91	1.00
time (sec)	N/A	0.031	5.100	0.360	0.956	0.258	2.115	0.340	17.956

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	245	27	20	20	22
N.S.	1	1.00	1.09	0.82	11.14	1.23	0.91	0.91	1.00
time (sec)	N/A	0.028	5.021	0.361	1.363	0.233	8.638	0.398	17.725

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	1977	1977	2316	0	0	0	0	0	0
N.S.	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.550	14.048	0.000	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1157	1157	846	0	0	0	0	0	0
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.464	9.712	0.000	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	172	179	0	576	0	174	2737
N.S.	1	1.00	1.38	1.43	0.00	4.61	0.00	1.39	21.90
time (sec)	N/A	0.236	1.173	0.391	0.000	0.296	0.000	0.285	22.781

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	48	22	20	22
N.S.	1	1.00	1.09	0.82	0.00	2.18	1.00	0.91	1.00
time (sec)	N/A	0.027	50.492	0.425	0.000	0.254	5.431	1.023	17.893

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	18	0	48	22	20	22
N.S.	1	1.00	1.09	0.82	0.00	2.18	1.00	0.91	1.00
time (sec)	N/A	0.025	57.919	0.414	0.000	0.247	39.125	1.767	18.033

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	19	22	24
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.95	1.10	1.20
time (sec)	N/A	0.056	4.425	0.375	2.349	0.246	21.909	0.657	17.580

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	45	45	61	158	128	62	0	0	106
N.S.	1	1.00	1.36	3.51	2.84	1.38	0.00	0.00	2.36
time (sec)	N/A	0.058	0.513	1.014	0.291	0.268	0.000	0.000	20.021

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	141	185	699	0	384	0	0	0
N.S.	1	1.00	1.31	4.96	0.00	2.72	0.00	0.00	0.00
time (sec)	N/A	0.130	0.487	1.168	0.000	0.271	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	221	221	0	0	0	557	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.215	0.000	0.000	0.000	0.283	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	80	80	102	275	207	116	0	0	182
N.S.	1	1.00	1.28	3.44	2.59	1.45	0.00	0.00	2.28
time (sec)	N/A	0.109	2.102	1.420	0.264	0.256	0.000	0.000	20.614

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	214	214	286	970	0	568	0	0	0
N.S.	1	1.00	1.34	4.53	0.00	2.65	0.00	0.00	0.00
time (sec)	N/A	0.244	7.769	1.805	0.000	0.279	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	377	377	0	0	0	890	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	2.36	0.00	0.00	0.00
time (sec)	N/A	0.483	0.000	0.000	0.000	0.304	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	85	85	79	315	0	301	0	0	229
N.S.	1	1.00	0.93	3.71	0.00	3.54	0.00	0.00	2.69
time (sec)	N/A	0.183	0.917	1.086	0.000	0.294	0.000	0.000	19.771

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	338	338	1003	769	0	1235	0	0	0
N.S.	1	1.00	2.97	2.28	0.00	3.65	0.00	0.00	0.00
time (sec)	N/A	0.758	6.352	1.342	0.000	0.439	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	499	499	0	0	0	1663	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	3.33	0.00	0.00	0.00
time (sec)	N/A	1.201	0.000	0.000	0.000	0.448	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	156	156	176	646	0	630	0	0	0
N.S.	1	1.00	1.13	4.14	0.00	4.04	0.00	0.00	0.00
time (sec)	N/A	0.367	1.977	6.787	0.000	0.299	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	778	778	2839	3867	0	2435	0	0	0
N.S.	1	1.00	3.65	4.97	0.00	3.13	0.00	0.00	0.00
time (sec)	N/A	1.608	11.168	7.143	0.000	0.519	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	B	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1417	1417	0	0	0	3755	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	2.65	0.00	0.00	0.00
time (sec)	N/A	2.993	0.000	0.000	0.000	0.571	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [23] had the largest ratio of [.66669999999999959]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	6	1.00	16	0.375
2	N/A	0	0	1.00	16	0.000
3	A	8	5	1.00	16	0.312
4	N/A	0	0	1.00	16	0.000
5	A	4	3	1.00	14	0.214
6	N/A	0	0	1.00	16	0.000
7	N/A	0	0	1.00	16	0.000
8	A	15	11	1.00	18	0.611
9	N/A	0	0	1.00	18	0.000
10	A	10	7	1.00	18	0.389
11	N/A	0	0	1.00	18	0.000
12	A	5	5	1.00	16	0.312
13	N/A	0	0	1.00	18	0.000
14	N/A	0	0	1.00	18	0.000
15	A	5	3	1.00	12	0.250
16	A	13	8	1.00	18	0.444
17	N/A	0	0	1.00	18	0.000
18	A	11	7	1.00	18	0.389
19	N/A	0	0	1.00	18	0.000
20	A	5	5	1.00	16	0.312
21	N/A	0	0	1.00	18	0.000
22	N/A	0	0	1.00	16	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	31	12	1.00	18	0.667
24	N/A	0	0	1.00	18	0.000
25	A	22	10	1.00	18	0.556
26	N/A	0	0	1.00	18	0.000
27	A	7	7	1.00	16	0.438
28	N/A	0	0	1.00	18	0.000
29	N/A	0	0	1.00	18	0.000
30	N/A	0	0	1.00	18	0.000
31	A	20	7	1.00	18	0.389
32	A	16	7	1.00	18	0.389
33	A	12	7	1.00	16	0.438
34	N/A	0	0	1.00	18	0.000
35	N/A	0	0	1.00	18	0.000
36	A	30	10	1.00	20	0.500
37	A	24	10	1.00	20	0.500
38	A	18	10	1.00	18	0.556
39	N/A	0	0	1.00	20	0.000
40	N/A	0	0	1.00	20	0.000
41	A	23	9	1.00	20	0.450
42	A	19	9	1.00	20	0.450
43	A	15	9	1.00	18	0.500
44	N/A	0	0	1.00	20	0.000
45	N/A	0	0	1.00	18	0.000
46	A	61	11	1.00	20	0.550
47	A	49	11	1.00	20	0.550
48	A	37	11	1.00	18	0.611
49	N/A	0	0	1.00	20	0.000
50	N/A	0	0	1.00	20	0.000
51	A	14	7	1.00	20	0.350
52	A	10	6	1.00	20	0.300
53	A	4	3	1.00	20	0.150
54	N/A	0	0	1.00	20	0.000
55	N/A	0	0	1.00	20	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	21	10	1.00	22	0.454
57	A	15	11	1.00	22	0.500
58	A	5	5	1.00	22	0.227
59	N/A	0	0	1.00	22	0.000
60	N/A	0	0	1.00	22	0.000
61	A	3	3	1.00	14	0.214
62	A	17	9	1.00	22	0.409
63	A	13	8	1.00	22	0.364
64	A	5	5	1.00	22	0.227
65	N/A	0	0	1.00	22	0.000
66	N/A	0	0	1.00	22	0.000
67	A	43	11	1.00	22	0.500
68	A	31	12	1.00	22	0.546
69	A	7	7	1.00	22	0.318
70	N/A	0	0	1.00	22	0.000
71	N/A	0	0	1.00	22	0.000
72	N/A	0	0	1.00	20	0.000
73	A	5	4	1.00	20	0.200
74	A	9	6	1.00	22	0.273
75	A	11	7	1.00	22	0.318
76	A	6	6	1.00	22	0.273
77	A	11	8	1.00	24	0.333
78	A	16	12	1.00	24	0.500
79	A	6	6	1.00	22	0.273
80	A	12	8	1.00	24	0.333
81	A	14	9	1.00	24	0.375
82	A	8	8	1.00	22	0.364
83	A	23	11	1.00	24	0.458
84	A	32	13	1.00	24	0.542

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^5(a+b\csc(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 129

$$\int x^5(a+b\csc(c+dx^2))\,dx=\frac{ax^6}{6}-\frac{bx^4\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d}+\frac{ibx^2\operatorname{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2}-\frac{ibx^2\operatorname{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2}-\frac{b\operatorname{PolyLog}\left(3,-e^{i(c+dx^2)}\right)}{d^3}+\frac{b\operatorname{PolyLog}\left(3,e^{i(c+dx^2)}\right)}{d^3}$$

```
[Out] 1/6*a*x^6-b*x^4*arctanh(exp(I*(d*x^2+c)))/d+I*b*x^2*polylog(2,-exp(I*(d*x^2
+c)))/d^2-I*b*x^2*polylog(2,exp(I*(d*x^2+c)))/d^2-b*polylog(3,-exp(I*(d*x^2
+c)))/d^3+b*polylog(3,exp(I*(d*x^2+c)))/d^3
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {14, 4290, 4268, 2611, 2320, 6724}

$$\int x^5(a+b\csc(c+dx^2))\,dx=\frac{ax^6}{6}-\frac{bx^4\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{b\operatorname{PolyLog}\left(3,-e^{i(dx^2+c)}\right)}{d^3}+\frac{b\operatorname{PolyLog}\left(3,e^{i(dx^2+c)}\right)}{d^3}+\frac{ibx^2\operatorname{PolyLog}\left(2,-e^{i(dx^2+c)}\right)}{d^2}-\frac{ibx^2\operatorname{PolyLog}\left(2,e^{i(dx^2+c)}\right)}{d^2}$$

```
[In] Int[x^5*(a + b*Csc[c + d*x^2]),x]
```

```
[Out] (a*x^6)/6 - (b*x^4*ArcTanh[E^(I*(c + d*x^2))])/d + (I*b*x^2*PolyLog[2, -E^(
I*(c + d*x^2))])/d^2 - (I*b*x^2*PolyLog[2, E^(I*(c + d*x^2))])/d^2 - (b*Pol
yLog[3, -E^(I*(c + d*x^2))])/d^3 + (b*PolyLog[3, E^(I*(c + d*x^2))])/d^3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^5+bx^5\csc\left(c+dx^2\right)\right)\,dx\\
&= \frac{ax^6}{6}+b\int x^5\csc\left(c+dx^2\right)\,dx\\
&= \frac{ax^6}{6}+\frac{1}{2}b\text{Subst}\left(\int x^2\csc(c+dx)\,dx,x,x^2\right)\\
&= \frac{ax^6}{6}-\frac{bx^4\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{b\text{Subst}\left(\int x\log\left(1-e^{i(c+dx)}\right)\,dx,x,x^2\right)}{d}\\
&\quad+\frac{b\text{Subst}\left(\int x\log\left(1+e^{i(c+dx)}\right)\,dx,x,x^2\right)}{d}\\
&= \frac{ax^6}{6}-\frac{bx^4\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}+\frac{ibx^2\,\text{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2}\\
&\quad-\frac{ibx^2\,\text{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2}-\frac{(ib)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{i(c+dx)}\right)\,dx,x,x^2\right)}{d^2}\\
&\quad+\frac{(ib)\text{Subst}\left(\int \text{PolyLog}\left(2,e^{i(c+dx)}\right)\,dx,x,x^2\right)}{d^2}\\
&= \frac{ax^6}{6}-\frac{bx^4\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}+\frac{ibx^2\,\text{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2}-\frac{ibx^2\,\text{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2}\\
&\quad-\frac{b\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{i(c+dx^2)}\right)}{d^3}+\frac{b\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{i(c+dx^2)}\right)}{d^3}\\
&= \frac{ax^6}{6}-\frac{bx^4\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}+\frac{ibx^2\,\text{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2}\\
&\quad-\frac{ibx^2\,\text{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2}\\
&\quad-\frac{b\,\text{PolyLog}\left(3,-e^{i(c+dx^2)}\right)}{d^3}+\frac{b\,\text{PolyLog}\left(3,e^{i(c+dx^2)}\right)}{d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.23

$$\int x^5\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{ax^6}{6}$$
$$-\frac{b\left(d^2x^4\operatorname{arctanh}\left(\cos\left(c+dx^2\right)+i\sin\left(c+dx^2\right)\right)-idx^2\operatorname{PolyLog}\left(2,-\cos\left(c+dx^2\right)-i\sin\left(c+dx^2\right)\right)+ic\right.}{}$$

```
[In] Integrate[x^5*(a + b*Csc[c + d*x^2]),x]
```

```
[Out] (a*x^6)/6 - (b*(d^2*x^4*ArcTanh[Cos[c + d*x^2] + I*Sin[c + d*x^2]] - I*d*x^
2*PolyLog[2, -Cos[c + d*x^2] - I*Sin[c + d*x^2]] + I*d*x^2*PolyLog[2, Cos[c
 + d*x^2] + I*Sin[c + d*x^2]] + PolyLog[3, -Cos[c + d*x^2] - I*Sin[c + d*x^
2]] - PolyLog[3, Cos[c + d*x^2] + I*Sin[c + d*x^2]]))/d^3
```

Maple [F]

$$\int x^5\left(a+b\csc\left(d\,x^2+c\right)\right)dx$$

```
[In] int(x^5*(a+b*csc(d*x^2+c)),x)
```

```
[Out] int(x^5*(a+b*csc(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(111) = 222.

Time = 0.29 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.29

$$\int x^5\left(a+b\csc\left(c+dx^2\right)\right)\,dx$$
$$=\frac{2\,ad^3x^6-3\,bd^2x^4\log\left(\cos\left(dx^2+c\right)+i\,\sin\left(dx^2+c\right)+1\right)-3\,bd^2x^4\log\left(\cos\left(dx^2+c\right)-i\,\sin\left(dx^2+c\right)+\right.}{}$$

```
[In] integrate(x^5*(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*a*d^3*x^6 - 3*b*d^2*x^4*log(cos(d*x^2 + c) + I*sin(d*x^2 + c) + 1)
- 3*b*d^2*x^4*log(cos(d*x^2 + c) - I*sin(d*x^2 + c) + 1) - 6*I*b*d*x^2*dilo
g(cos(d*x^2 + c) + I*sin(d*x^2 + c)) + 6*I*b*d*x^2*dilog(cos(d*x^2 + c) - I
*sin(d*x^2 + c)) - 6*I*b*d*x^2*dilog(-cos(d*x^2 + c) + I*sin(d*x^2 + c)) +
6*I*b*d*x^2*dilog(-cos(d*x^2 + c) - I*sin(d*x^2 + c)) + 3*b*c^2*log(-1/2*co
s(d*x^2 + c) + 1/2*I*sin(d*x^2 + c) + 1/2) + 3*b*c^2*log(-1/2*cos(d*x^2 + c
) - 1/2*I*sin(d*x^2 + c) + 1/2) + 3*(b*d^2*x^4 - b*c^2)*log(-cos(d*x^2 + c)
 + I*sin(d*x^2 + c) + 1) + 3*(b*d^2*x^4 - b*c^2)*log(-cos(d*x^2 + c) - I*si
```

```
n(d*x^2 + c) + 1) + 6*b*polylog(3, cos(d*x^2 + c) + I*sin(d*x^2 + c)) + 6*b
*polylog(3, cos(d*x^2 + c) - I*sin(d*x^2 + c)) - 6*b*polylog(3, -cos(d*x^2
+ c) + I*sin(d*x^2 + c)) - 6*b*polylog(3, -cos(d*x^2 + c) - I*sin(d*x^2 + c
)))/d^3
```

Sympy [F]

$$\int x^5 \left(a + b \csc \left(c + dx^2\right)\right) \, dx = \int x^5 \left(a + b \csc \left(c + dx^2\right)\right) \, dx$$

```
[In] integrate(x**5*(a+b*csc(d*x**2+c)),x)
[Out] Integral(x**5*(a + b*csc(c + d*x**2)), x)
```

Maxima [F]

$$\int x^5 \left(a + b \csc \left(c + dx^2\right)\right) \, dx = \int \left(b \csc \left(dx^2 + c\right) + a\right) x^5 \, dx$$

```
[In] integrate(x^5*(a+b*csc(d*x^2+c)),x, algorithm="maxima")
[Out] 1/6*a*x^6 + b*(integrate(x^5*sin(d*x^2 + c)/(cos(d*x^2 + c)^2 + sin(d*x^2 +
 c)^2 + 2*cos(d*x^2 + c) + 1), x) + integrate(x^5*sin(d*x^2 + c)/(cos(d*x^2
 + c)^2 + sin(d*x^2 + c)^2 - 2*cos(d*x^2 + c) + 1), x))
```

Giac [F]

$$\int x^5 \left(a + b \csc \left(c + dx^2\right)\right) \, dx = \int \left(b \csc \left(dx^2 + c\right) + a\right) x^5 \, dx$$

```
[In] integrate(x^5*(a+b*csc(d*x^2+c)),x, algorithm="giac")
[Out] integrate((b*csc(d*x^2 + c) + a)*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\int x^5\left(a+\frac{b}{\sin\left(d\,x^2+c\right)}\right)\,dx$$

```
[In] int(x^5*(a + b/sin(c + d*x^2)),x)
[Out] int(x^5*(a + b/sin(c + d*x^2)), x)
```

3.2 $\int x^4(a+b\csc(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{ax^5}{5}+b\mathrm{Int}\left(x^4\csc\left(c+dx^2\right),x\right)$$

```
[Out] 1/5*a*x^5+b*Unintegrable(x^4*csc(d*x^2+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx$$

```
[In] Int[x^4*(a + b*Csc[c + d*x^2]),x]
```

```
[Out] (a*x^5)/5 + b*Defer[Int][x^4*Csc[c + d*x^2], x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax^4+bx^4\csc\left(c+dx^2\right)\right)\,dx\\&=\frac{ax^5}{5}+b\int x^4\csc\left(c+dx^2\right)\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx$$

```
[In] Integrate[x^4*(a + b*Csc[c + d*x^2]),x]
[Out] Integrate[x^4*(a + b*Csc[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^4\left(a+b\csc\left(d\,x^2+c\right)\right)dx$$

```
[In] int(x^4*(a+b*csc(d*x^2+c)),x)
[Out] int(x^4*(a+b*csc(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\int\left(b\csc\left(dx^2+c\right)+a\right)x^4\,dx$$

```
[In] integrate(x^4*(a+b*csc(d*x^2+c)),x, algorithm="fricas")
[Out] integral(b*x^4*csc(d*x^2 + c) + a*x^4, x)
```

Sympy [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\int x^4\left(a+b\csc\left(c+dx^2\right)\right)\,dx$$

```
[In] integrate(x**4*(a+b*csc(d*x**2+c)),x)
[Out] Integral(x**4*(a + b*csc(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 6.62

$$\int x^4(a+b\csc(c+dx^2))\ dx = \int (b\csc(dx^2+c)+a)x^4\,dx$$

```
[In] integrate(x^4*(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + b*(integrate(x^4*sin(d*x^2 + c)/(cos(d*x^2 + c)^2 + sin(d*x^2 +
 c)^2 + 2*cos(d*x^2 + c) + 1), x) + integrate(x^4*sin(d*x^2 + c)/(cos(d*x^2
 + c)^2 + sin(d*x^2 + c)^2 - 2*cos(d*x^2 + c) + 1), x))
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^4(a+b\csc(c+dx^2))\ dx = \int (b\csc(dx^2+c)+a)x^4\,dx$$

```
[In] integrate(x^4*(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^2 + c) + a)*x^4, x)
```

Mupad [N/A]

Not integrable

Time = 17.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^4(a+b\csc(c+dx^2))\ dx = \int x^4\left(a+\frac{b}{\sin(d\,x^2+c)}\right)\,dx$$

```
[In] int(x^4*(a + b/sin(c + d*x^2)),x)
```

```
[Out] int(x^4*(a + b/sin(c + d*x^2)), x)
```

3.3 $\int x^3(a + b\csc(c + dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 84

$$\int x^3(a+b\csc(c+dx^2))\,dx=\frac{ax^4}{4}-\frac{bx^2\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d}+\frac{ib\operatorname{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{2d^2}-\frac{ib\operatorname{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{2d^2}$$

```
[Out] 1/4*a*x^4-b*x^2*arctanh(exp(I*(d*x^2+c)))/d+1/2*I*b*polylog(2,-exp(I*(d*x^2
+c)))/d^2-1/2*I*b*polylog(2,exp(I*(d*x^2+c)))/d^2
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {14, 4290, 4268, 2317, 2438}

$$\int x^3(a+b\csc(c+dx^2))\,dx=\frac{ax^4}{4}-\frac{bx^2\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d}+\frac{ib\operatorname{PolyLog}\left(2,-e^{i(dx^2+c)}\right)}{2d^2}-\frac{ib\operatorname{PolyLog}\left(2,e^{i(dx^2+c)}\right)}{2d^2}$$

```
[In] Int[x^3*(a + b*Csc[c + d*x^2]),x]
[Out] (a*x^4)/4 - (b*x^2*ArcTanh[E^(I*(c + d*x^2))])/d + ((I/2)*b*PolyLog[2, -E^(
I*(c + d*x^2))])/d^2 - ((I/2)*b*PolyLog[2, E^(I*(c + d*x^2))])/d^2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^3 + bx^3 \csc\left(c + dx^2\right)\right) \, dx \\
&= \frac{ax^4}{4} + b \int x^3 \csc\left(c + dx^2\right) \, dx \\
&= \frac{ax^4}{4} + \frac{1}{2} b \text{Subst}\left(\int x \csc(c + dx) \, dx, x, x^2\right) \\
&= \frac{ax^4}{4} - \frac{bx^2 \text{arctanh}\left(e^{i(c+dx^2)}\right)}{d} - \frac{b \text{Subst}\left(\int \log\left(1 - e^{i(c+dx)}\right) \, dx, x, x^2\right)}{2d} \\
&\quad + \frac{b \text{Subst}\left(\int \log\left(1 + e^{i(c+dx)}\right) \, dx, x, x^2\right)}{2d}
\end{aligned}$$

$$= \frac{ax^4}{4} - \frac{bx^2\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d} + \frac{(ib)\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{2d^2} - \frac{(ib)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{2d^2}$$

$$= \frac{ax^4}{4} - \frac{bx^2\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d} + \frac{ib\,\text{PolyLog}\left(2, -e^{i(c+dx^2)}\right)}{2d^2} - \frac{ib\,\text{PolyLog}\left(2, e^{i(c+dx^2)}\right)}{2d^2}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int x^3\left(a+b\csc\left(c+dx^2\right)\right)\,dx = \frac{ax^4}{4} + \frac{b\left(\left(c+dx^2\right)\left(\log\left(1-e^{i(c+dx^2)}\right)-\log\left(1+e^{i(c+dx^2)}\right)\right)-c\log\left(\tan\left(\frac{1}{2}(c+dx^2)\right)\right)+i\left(\text{PolyLog}\left(2,-e^{i\ldots}\right.\right.\right.}{2d^2}$$

```
[In] Integrate[x^3*(a + b*Csc[c + d*x^2]),x]
```

```
[Out] (a*x^4)/4 + (b*((c + d*x^2)*(Log[1 - E^(I*(c + d*x^2))] - Log[1 + E^(I*(c +
 d*x^2))]) - c*Log[Tan[(c + d*x^2)/2]] + I*(PolyLog[2, -E^(I*(c + d*x^2))]
- PolyLog[2, E^(I*(c + d*x^2))])))/(2*d^2)
```

Maple [F]

$$\int x^3\left(a+b\csc\left(d\,x^2+c\right)\right)dx$$

```
[In] int(x^3*(a+b*csc(d*x^2+c)),x)
```

```
[Out] int(x^3*(a+b*csc(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(66) = 132.

Time = 0.26 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.43

$$\int x^3\left(a+b\csc\left(c+dx^2\right)\right)\,dx = \frac{ad^2x^4 - bdx^2\log\left(\cos\left(dx^2+c\right)+i\sin\left(dx^2+c\right)+1\right) - bdx^2\log\left(\cos\left(dx^2+c\right)-i\sin\left(dx^2+c\right)+1\right) - bc\ldots}{\ldots}$$

```
[In] integrate(x^3*(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^2*x^4 - b*d*x^2*log(cos(d*x^2 + c) + I*sin(d*x^2 + c) + 1) - b*d*x
^2*log(cos(d*x^2 + c) - I*sin(d*x^2 + c) + 1) - b*c*log(-1/2*cos(d*x^2 + c)
 + 1/2*I*sin(d*x^2 + c) + 1/2) - b*c*log(-1/2*cos(d*x^2 + c) - 1/2*I*sin(d*
x^2 + c) + 1/2) - I*b*dilog(cos(d*x^2 + c) + I*sin(d*x^2 + c)) + I*b*dilog(
cos(d*x^2 + c) - I*sin(d*x^2 + c)) - I*b*dilog(-cos(d*x^2 + c) + I*sin(d*x^
2 + c)) + I*b*dilog(-cos(d*x^2 + c) - I*sin(d*x^2 + c)) + (b*d*x^2 + b*c)*l
og(-cos(d*x^2 + c) + I*sin(d*x^2 + c) + 1) + (b*d*x^2 + b*c)*log(-cos(d*x^2
 + c) - I*sin(d*x^2 + c) + 1))/d^2
```

Sympy [F]

$$\int x^3(a+b\csc(c+dx^2))\,dx=\int x^3(a+b\csc(c+dx^2))\,dx$$

```
[In] integrate(x**3*(a+b*csc(d*x**2+c)),x)
```

```
[Out] Integral(x**3*(a + b*csc(c + d*x**2)), x)
```

Maxima [F]

$$\int x^3(a+b\csc(c+dx^2))\,dx=\int (b\csc(dx^2+c)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + b*(integrate(x^3*sin(d*x^2 + c)/(cos(d*x^2 + c)^2 + sin(d*x^2 +
 c)^2 + 2*cos(d*x^2 + c) + 1), x) + integrate(x^3*sin(d*x^2 + c)/(cos(d*x^2
 + c)^2 + sin(d*x^2 + c)^2 - 2*cos(d*x^2 + c) + 1), x))
```

Giac [F]

$$\int x^3(a+b\csc(c+dx^2))\,dx=\int (b\csc(dx^2+c)+a)x^3\,dx$$

```
[In] integrate(x^3*(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^2 + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b\csc\left(c + dx^2\right)\right) \, dx = \int x^3 \left(a + \frac{b}{\sin\left(d\,x^2 + c\right)}\right) \, dx$$

```
[In] int(x^3*(a + b/sin(c + d*x^2)),x)
[Out] int(x^3*(a + b/sin(c + d*x^2)), x)
```

3.4 $\int x^2(a+b\csc(c+dx^2))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int x^2\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{ax^3}{3}+b\mathrm{Int}\left(x^2\csc\left(c+dx^2\right),x\right)$$

```
[Out] 1/3*a*x^3+b*Unintegrable(x^2*csc(d*x^2+c),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\int x^2\left(a+b\csc\left(c+dx^2\right)\right)\,dx$$

```
[In] Int[x^2*(a + b*Csc[c + d*x^2]),x]
[Out] (a*x^3)/3 + b*Defer[Int][x^2*Csc[c + d*x^2], x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(ax^2+bx^2\csc\left(c+dx^2\right)\right)\,dx\\&=\frac{ax^3}{3}+b\int x^2\csc\left(c+dx^2\right)\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 3.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2 \left(a + b \csc \left(c + dx^2\right)\right) \, dx = \int x^2 \left(a + b \csc \left(c + dx^2\right)\right) \, dx$$

```
[In] Integrate[x^2*(a + b*Csc[c + d*x^2]),x]
[Out] Integrate[x^2*(a + b*Csc[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int x^2 \left(a + b \csc \left(d\, x^2 + c\right)\right) dx$$

```
[In] int(x^2*(a+b*csc(d*x^2+c)),x)
[Out] int(x^2*(a+b*csc(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int x^2 \left(a + b \csc \left(c + dx^2\right)\right) \, dx = \int \left(b \csc \left(dx^2 + c\right) + a\right) x^2 \, dx$$

```
[In] integrate(x^2*(a+b*csc(d*x^2+c)),x, algorithm="fricas")
[Out] integral(b*x^2*csc(d*x^2 + c) + a*x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int x^2 \left(a + b \csc \left(c + dx^2\right)\right) \, dx = \int x^2 \left(a + b \csc \left(c + dx^2\right)\right) \, dx$$

```
[In] integrate(x**2*(a+b*csc(d*x**2+c)),x)
[Out] Integral(x**2*(a + b*csc(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 6.62

$$\int x^2(a+b\csc(c+dx^2))\,dx=\int(b\csc(dx^2+c)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + b*(integrate(x^2*sin(d*x^2 + c)/(cos(d*x^2 + c)^2 + sin(d*x^2 +
 c)^2 + 2*cos(d*x^2 + c) + 1), x) + integrate(x^2*sin(d*x^2 + c)/(cos(d*x^2
 + c)^2 + sin(d*x^2 + c)^2 - 2*cos(d*x^2 + c) + 1), x))
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int x^2(a+b\csc(c+dx^2))\,dx=\int(b\csc(dx^2+c)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^2 + c) + a)*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 17.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int x^2(a+b\csc(c+dx^2))\,dx=\int x^2\left(a+\frac{b}{\sin(d\,x^2+c)}\right)\,dx$$

```
[In] int(x^2*(a + b/sin(c + d*x^2)),x)
```

```
[Out] int(x^2*(a + b/sin(c + d*x^2)), x)
```

3.5 $\int x(a+b\csc(c+dx^2))\,dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\operatorname{arctanh}(\cos\left(c+dx^2\right))}{2d}$$

```
[Out] 1/2*a*x^2-1/2*b*arctanh(cos(d*x^2+c))/d
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {14, 4290, 3855}

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx=\frac{ax^2}{2}-\frac{b\operatorname{arctanh}(\cos\left(c+dx^2\right))}{2d}$$

```
[In] Int[x*(a + b*Csc[c + d*x^2]),x]
```

```
[Out] (a*x^2)/2 - (b*ArcTanh[Cos[c + d*x^2]])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ax + bx\csc\left(c + dx^2\right)\right)\,dx\\ &= \frac{ax^2}{2} + b\int x\csc\left(c + dx^2\right)\,dx\\ &= \frac{ax^2}{2} + \frac{1}{2}b\text{Subst}\left(\int \csc(c + dx)\,dx, x, x^2\right)\\ &= \frac{ax^2}{2} - \frac{b\text{arctanh}(\cos\left(c + dx^2\right))}{2d}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int x\left(a + b\csc\left(c + dx^2\right)\right)\,dx = \frac{ax^2}{2} - \frac{b\log\left(\cos\left(\frac{c}{2} + \frac{dx^2}{2}\right)\right)}{2d} + \frac{b\log\left(\sin\left(\frac{c}{2} + \frac{dx^2}{2}\right)\right)}{2d}$$

```
[In] Integrate[x*(a + b*Csc[c + d*x^2]),x]
[Out] (a*x^2)/2 - (b*Log[Cos[c/2 + (d*x^2)/2]])/(2*d) + (b*Log[Sin[c/2 + (d*x^2)/
2]])/(2*d)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

method	result	size
norman	$\frac{a\,x^{2}}{2}+\frac{b\ln\left(\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{2d}$	26
parallelrisch	$\frac{a\,x^{2}d+b\ln\left(\tan\left(\frac{d\,x^{2}}{2}+\frac{c}{2}\right)\right)}{2d}$	27
parts	$\frac{a\,x^{2}}{2}-\frac{b\ln\left(\csc\left(d\,x^{2}+c\right)+\cot\left(d\,x^{2}+c\right)\right)}{2d}$	32
derivativedivides	$\frac{\left(d\,x^{2}+c\right)a-b\ln\left(\csc\left(d\,x^{2}+c\right)+\cot\left(d\,x^{2}+c\right)\right)}{2d}$	37
default	$\frac{\left(d\,x^{2}+c\right)a-b\ln\left(\csc\left(d\,x^{2}+c\right)+\cot\left(d\,x^{2}+c\right)\right)}{2d}$	37
risch	$\frac{a\,x^{2}}{2}+\frac{b\ln\left({\mathrm e}^{i\left(d\,x^{2}+c\right)}-1\right)}{2d}-\frac{b\ln\left({\mathrm e}^{i\left(d\,x^{2}+c\right)}+1\right)}{2d}$	48

```
[In] int(x*(a+b*csc(d*x^2+c)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+1/2*b/d*ln(tan(1/2*d*x^2+1/2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx = \frac{2\,adx^{2}-b\log\left(\frac{1}{2}\,\cos\left(dx^{2}+c\right)+\frac{1}{2}\right)+b\log\left(-\frac{1}{2}\,\cos\left(dx^{2}+c\right)+\frac{1}{2}\right)}{4\,d}$$

```
[In] integrate(x*(a+b*csc(d*x^2+c)),x, algorithm="fricas")
[Out] 1/4*(2*a*d*x^2 - b*log(1/2*cos(d*x^2 + c) + 1/2) + b*log(-1/2*cos(d*x^2 + c
) + 1/2))/d
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 2.57 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)\,dx = \begin{cases} \frac{a\left(c+dx^{2}\right)-b\log\left(\cot\left(c+dx^{2}\right)+\csc\left(c+dx^{2}\right)\right)}{2d} & \text{for}\: d \neq 0 \\ \frac{x^{2}\left(a+b\csc\left(c\right)\right)}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*(a+b*csc(d*x**2+c)),x)
[Out] Piecewise(((a*(c + d*x**2) - b*log(cot(c + d*x**2) + csc(c + d*x**2)))/(2*d
), Ne(d, 0)), (x**2*(a + b*csc(c))/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int x(a+b\csc(c+dx^2))\,dx = \frac{1}{2}ax^2 - \frac{b\log(\cot(dx^2+c)+\csc(dx^2+c))}{2d}$$

```
[In] integrate(x*(a+b*csc(d*x^2+c)),x, algorithm="maxima")
[Out] 1/2*a*x^2 - 1/2*b*log(cot(d*x^2 + c) + csc(d*x^2 + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int x(a+b\csc(c+dx^2))\,dx = \frac{(dx^2+c)a + b\log\left(\left|\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)\right|\right)}{2d}$$

```
[In] integrate(x*(a+b*csc(d*x^2+c)),x, algorithm="giac")
[Out] 1/2*((d*x^2 + c)*a + b*log(abs(tan(1/2*d*x^2 + 1/2*c))))/d
```

Mupad [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.65

$$\int x(a+b\csc(c+dx^2))\,dx = \frac{ax^2}{2} - \frac{b\ln\left(-bx\,2\mathrm{i} - bx\,\mathrm{e}^{dx^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}\right)}{2d} + \frac{b\ln\left(bx\,2\mathrm{i} - bx\,\mathrm{e}^{dx^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}\right)}{2d}$$

```
[In] int(x*(a + b/sin(c + d*x^2)),x)
[Out] (a*x^2)/2 - (b*log(- b*x*2i - b*x*exp(d*x^2*1i)*exp(c*1i)*2i))/(2*d) + (b*l
og(b*x*2i - b*x*exp(d*x^2*1i)*exp(c*1i)*2i))/(2*d)
```

3.6 $\int \frac{a+b\csc(c+dx^2)}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\csc(c+dx^2)}{x}\,dx = a\log(x)+b\text{Int}\left(\frac{\csc(c+dx^2)}{x},x\right)$$

```
[Out] a*ln(x)+b*Unintegrable(csc(d*x^2+c)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc(c+dx^2)}{x}\,dx = \int \frac{a+b\csc(c+dx^2)}{x}\,dx$$

```
[In] Int[(a + b*Csc[c + d*x^2])/x,x]
[Out] a*Log[x] + b*Defer[Int][Csc[c + d*x^2]/x, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{x}+\frac{b\csc(c+dx^2)}{x}\right)dx \\ &= a\log(x)+b\int \frac{\csc(c+dx^2)}{x}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 3.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\csc(c + dx^2)}{x} \, dx = \int \frac{a + b\csc(c + dx^2)}{x} \, dx$$

```
[In] Integrate[(a + b*Csc[c + d*x^2])/x,x]
[Out] Integrate[(a + b*Csc[c + d*x^2])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a + b\csc(d\,x^2 + c)}{x} dx$$

```
[In] int((a+b*csc(d*x^2+c))/x,x)
[Out] int((a+b*csc(d*x^2+c))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a + b\csc(c + dx^2)}{x} \, dx = \int \frac{b\csc(dx^2 + c) + a}{x} \, dx$$

```
[In] integrate((a+b*csc(d*x^2+c))/x,x, algorithm="fricas")
[Out] integral((b*csc(d*x^2 + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{a + b\csc(c + dx^2)}{x} \, dx = \int \frac{a + b\csc(c + dx^2)}{x} \, dx$$

```
[In] integrate((a+b*csc(d*x**2+c))/x,x)
[Out] Integral((a + b*csc(c + d*x**2))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 6.75

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x}\,dx=\int \frac{b\csc\left(dx^2+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))/x,x, algorithm="maxima")
```

```
[Out] b*(integrate(sin(d*x^2 + c)/(x*cos(d*x^2 + c)^2 + x*sin(d*x^2 + c)^2 + 2*x*
cos(d*x^2 + c) + x), x) + integrate(sin(d*x^2 + c)/(x*cos(d*x^2 + c)^2 + x*
sin(d*x^2 + c)^2 - 2*x*cos(d*x^2 + c) + x), x)) + a*log(x)
```

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x}\,dx=\int \frac{b\csc\left(dx^2+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))/x,x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^2 + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 18.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x}\,dx=\int \frac{a+\frac{b}{\sin\left(d\,x^2+c\right)}}{x}\,dx$$

```
[In] int((a + b/sin(c + d*x^2))/x,x)
```

```
[Out] int((a + b/sin(c + d*x^2))/x, x)
```

3.7 $\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = -\frac{a}{x} + b\mathrm{Int}\left(\frac{\csc(c+dx^2)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(csc(d*x^2+c)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = \int \frac{a+b\csc(c+dx^2)}{x^2}\,dx$$

```
[In] Int[(a + b*Csc[c + d*x^2])/x^2,x]
```

```
[Out] -(a/x) + b*Defer[Int][Csc[c + d*x^2]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{x^2} + \frac{b\csc(c+dx^2)}{x^2}\right)dx \\ &= -\frac{a}{x} + b\int \frac{\csc(c+dx^2)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 2.98 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*x^2])/x^2,x]
[Out] Integrate[(a + b*Csc[c + d*x^2])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(d\,x^2+c\right)}{x^2}dx$$

```
[In] int((a+b*csc(d*x^2+c))/x^2,x)
[Out] int((a+b*csc(d*x^2+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=\int \frac{b\csc\left(dx^2+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="fricas")
[Out] integral((b*csc(d*x^2 + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x**2+c))/x**2,x)
[Out] Integral((a + b*csc(c + d*x**2))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 7.88

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = \int \frac{b\csc(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="maxima")
```

```
[Out] b*(integrate(sin(d*x^2 + c)/(x^2*cos(d*x^2 + c)^2 + x^2*sin(d*x^2 + c)^2 +
2*x^2*cos(d*x^2 + c) + x^2), x) + integrate(sin(d*x^2 + c)/(x^2*cos(d*x^2 +
 c)^2 + x^2*sin(d*x^2 + c)^2 - 2*x^2*cos(d*x^2 + c) + x^2), x)) - a/x
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = \int \frac{b\csc(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^2 + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = \int \frac{a+\frac{b}{\sin(d\,x^2+c)}}{x^2}\,dx$$

```
[In] int((a + b/sin(c + d*x^2))/x^2,x)
```

```
[Out] int((a + b/sin(c + d*x^2))/x^2, x)
```

3.8 $\int x^5(a+b\csc(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 228

$$\int x^5\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=-\frac{ib^2x^4}{2d}+\frac{a^2x^6}{6}-\frac{2abx^4\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{b^2x^4\cot\left(c+dx^2\right)}{2d}+\frac{b^2x^2\log\left(1-e^{2i(c+dx^2)}\right)}{d^2}+\frac{2iabx^2\operatorname{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2}-\frac{2iabx^2\operatorname{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2}-\frac{ib^2\operatorname{PolyLog}\left(2,e^{2i(c+dx^2)}\right)}{2d^3}-\frac{2ab\operatorname{PolyLog}\left(3,-e^{i(c+dx^2)}\right)}{d^3}+\frac{2ab\operatorname{PolyLog}\left(3,e^{i(c+dx^2)}\right)}{d^3}$$

```
[Out] -1/2*I*b^2*x^4/d+1/6*a^2*x^6-2*a*b*x^4*arctanh(exp(I*(d*x^2+c)))/d-1/2*b^2*
x^4*cot(d*x^2+c)/d+b^2*x^2*ln(1-exp(2*I*(d*x^2+c)))/d^2+2*I*a*b*x^2*polylog
(2,-exp(I*(d*x^2+c)))/d^2-2*I*a*b*x^2*polylog(2,exp(I*(d*x^2+c)))/d^2-1/2*I
*b^2*polylog(2,exp(2*I*(d*x^2+c)))/d^3-2*a*b*polylog(3,-exp(I*(d*x^2+c)))/d
^3+2*a*b*polylog(3,exp(I*(d*x^2+c)))/d^3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.611, Rules

used = {4290, 4275, 4268, 2611, 2320, 6724, 4269, 3798, 2221, 2317, 2438}

$$\int x^5\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\frac{a^2x^6}{6}-\frac{2abx^4\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{2ab\operatorname{PolyLog}\left(3,-e^{i(dx^2+c)}\right)}{d^3}+\frac{2ab\operatorname{PolyLog}\left(3,e^{i(dx^2+c)}\right)}{d^3}+\frac{2iabx^2\operatorname{PolyLog}\left(2,-e^{i(dx^2+c)}\right)}{d^2}-\frac{2iabx^2\operatorname{PolyLog}\left(2,e^{i(dx^2+c)}\right)}{d^2}-\frac{ib^2\operatorname{PolyLog}\left(2,e^{2i(dx^2+c)}\right)}{2d^3}+\frac{b^2x^2\log\left(1-e^{2i(c+dx^2)}\right)}{d^2}-\frac{b^2x^4\cot\left(c+dx^2\right)}{2d}-\frac{ib^2x^4}{2d}$$

```
[In] Int[x^5*(a + b*Csc[c + d*x^2])^2,x]
```

```
[Out] ((-1/2*I)*b^2*x^4)/d + (a^2*x^6)/6 - (2*a*b*x^4*ArcTanh[E^(I*(c + d*x^2))])
/d - (b^2*x^4*Cot[c + d*x^2])/(2*d) + (b^2*x^2*Log[1 - E^((2*I)*(c + d*x^2)
)])/d^2 + ((2*I)*a*b*x^2*PolyLog[2, -E^(I*(c + d*x^2))])/d^2 - ((2*I)*a*b*x
^2*PolyLog[2, E^(I*(c + d*x^2))])/d^2 - ((I/2)*b^2*PolyLog[2, E^((2*I)*(c +
 d*x^2))])/d^3 - (2*a*b*PolyLog[3, -E^(I*(c + d*x^2))])/d^3 + (2*a*b*PolyLo
g[3, E^(I*(c + d*x^2))])/d^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
```

```
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x^2(a+b\csc(c+dx))^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x^2+2abx^2\csc(c+dx)+b^2x^2\csc^2(c+dx)\right)\,dx,x,x^2\right)\\
&= \frac{a^2x^6}{6}+(ab)\text{Subst}\left(\int x^2\csc(c+dx)\,dx,x,x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x^2\csc^2(c+dx)\,dx,x,x^2\right)\\
&= \frac{a^2x^6}{6}-\frac{2abx^4\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{b^2x^4\cot\left(c+dx^2\right)}{2d}\\
&\quad-\frac{(2ab)\text{Subst}\left(\int x\log\left(1-e^{i(c+dx)}\right)\,dx,x,x^2\right)}{d}\\
&\quad+\frac{(2ab)\text{Subst}\left(\int x\log\left(1+e^{i(c+dx)}\right)\,dx,x,x^2\right)}{d}+\frac{b^2\text{Subst}\left(\int x\cot(c+dx)\,dx,x,x^2\right)}{d}\\
&= -\frac{ib^2x^4}{2d}+\frac{a^2x^6}{6}-\frac{2abx^4\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{b^2x^4\cot\left(c+dx^2\right)}{2d}\\
&\quad+\frac{2iabx^2\,\text{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2}-\frac{2iabx^2\,\text{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2}\\
&\quad-\frac{(2iab)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{i(c+dx)}\right)\,dx,x,x^2\right)}{d^2}\\
&\quad+\frac{(2iab)\text{Subst}\left(\int \text{PolyLog}\left(2,e^{i(c+dx)}\right)\,dx,x,x^2\right)}{d^2}\\
&\quad-\frac{\left(2ib^2\right)\text{Subst}\left(\int \frac{e^{2i(c+dx)}x}{1-e^{2i(c+dx)}}\,dx,x,x^2\right)}{d}
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{ib^2x^4}{2d} + \frac{a^2x^6}{6} - \frac{2abx^4\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d} - \frac{b^2x^4\cot(c+dx^2)}{2d} \\
&\quad + \frac{b^2x^2\log\left(1-e^{2i(c+dx^2)}\right)}{d^2} + \frac{2iabx^2\operatorname{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2} \\
&\quad - \frac{2iabx^2\operatorname{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2} - \frac{(2ab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,-x)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{d^3} \\
&\quad + \frac{(2ab)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(2,x)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{d^3} \\
&\quad - \frac{b^2\operatorname{Subst}\left(\int \log\left(1-e^{2i(c+dx)}\right)\,dx, x, x^2\right)}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ib^2x^4}{2d} + \frac{a^2x^6}{6} - \frac{2abx^4\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d} - \frac{b^2x^4\cot(c+dx^2)}{2d} \\
&\quad + \frac{b^2x^2\log\left(1-e^{2i(c+dx^2)}\right)}{d^2} + \frac{2iabx^2\operatorname{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2} \\
&\quad - \frac{2iabx^2\operatorname{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2} - \frac{2ab\operatorname{PolyLog}\left(3,-e^{i(c+dx^2)}\right)}{d^3} \\
&\quad + \frac{2ab\operatorname{PolyLog}\left(3,e^{i(c+dx^2)}\right)}{d^3} + \frac{(ib^2)\operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{2i(c+dx^2)}\right)}{2d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ib^2x^4}{2d} + \frac{a^2x^6}{6} - \frac{2abx^4\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d} - \frac{b^2x^4\cot(c+dx^2)}{2d} \\
&\quad + \frac{b^2x^2\log\left(1-e^{2i(c+dx^2)}\right)}{d^2} + \frac{2iabx^2\operatorname{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2} \\
&\quad - \frac{2iabx^2\operatorname{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2} - \frac{ib^2\operatorname{PolyLog}\left(2,e^{2i(c+dx^2)}\right)}{2d^3} \\
&\quad - \frac{2ab\operatorname{PolyLog}\left(3,-e^{i(c+dx^2)}\right)}{d^3} + \frac{2ab\operatorname{PolyLog}\left(3,e^{i(c+dx^2)}\right)}{d^3}
\end{aligned}
$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 639 vs. 2(228) = 456.

Time = 3.30 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.80

$$
\int x^5\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx
$$

$$
= \frac{-12ib^2d^2x^4 - 2a^2d^3x^6 + 2a^2d^3e^{2ic}x^6 - 12b^2dx^2\log\left(1-e^{-i(c+dx^2)}\right) + 12b^2de^{2ic}x^2\log\left(1-e^{-i(c+dx^2)}\right) - }{}
$$

```
[In] Integrate[x^5*(a + b*Csc[c + d*x^2])^2,x]
```

```
[Out] ((-12*I)*b^2*d^2*x^4 - 2*a^2*d^3*x^6 + 2*a^2*d^3*E^((2*I)*c)*x^6 - 12*b^2*d
*x^2*Log[1 - E^((-I)*(c + d*x^2))] + 12*b^2*d*E^((2*I)*c)*x^2*Log[1 - E^((-
I)*(c + d*x^2))] - 12*a*b*d^2*x^4*Log[1 - E^((-I)*(c + d*x^2))] + 12*a*b*d^
2*E^((2*I)*c)*x^4*Log[1 - E^((-I)*(c + d*x^2))] - 12*b^2*d*x^2*Log[1 + E^((
-I)*(c + d*x^2))] + 12*b^2*d*E^((2*I)*c)*x^2*Log[1 + E^((-I)*(c + d*x^2))]
+ 12*a*b*d^2*x^4*Log[1 + E^((-I)*(c + d*x^2))] - 12*a*b*d^2*E^((2*I)*c)*x^4
*Log[1 + E^((-I)*(c + d*x^2))] + (12*I)*b*(-1 + E^((2*I)*c))*(b - 2*a*d*x^2
)*PolyLog[2, -E^((-I)*(c + d*x^2))] + (12*I)*b*(-1 + E^((2*I)*c))*(b + 2*a*
d*x^2)*PolyLog[2, E^((-I)*(c + d*x^2))] + 24*a*b*PolyLog[3, -E^((-I)*(c + d
*x^2))] - 24*a*b*E^((2*I)*c)*PolyLog[3, -E^((-I)*(c + d*x^2))] - 24*a*b*Pol
yLog[3, E^((-I)*(c + d*x^2))] + 24*a*b*E^((2*I)*c)*PolyLog[3, E^((-I)*(c +
d*x^2))] - 3*b^2*d^2*x^4*Csc[c/2]*Csc[(c + d*x^2)/2]*Sin[(d*x^2)/2] + 3*b^2
*d^2*E^((2*I)*c)*x^4*Csc[c/2]*Csc[(c + d*x^2)/2]*Sin[(d*x^2)/2] - 3*b^2*d^2
*x^4*Sec[c/2]*Sec[(c + d*x^2)/2]*Sin[(d*x^2)/2] + 3*b^2*d^2*E^((2*I)*c)*x^4
*Sec[c/2]*Sec[(c + d*x^2)/2]*Sin[(d*x^2)/2])/(12*d^3*(-1 + E^((2*I)*c)))
```

Maple [F]

$$\int x^5 \left(a + b\csc\left(d\,x^2 + c\right)\right)^2 dx$$

```
[In] int(x^5*(a+b*csc(d*x^2+c))^2,x)
```

```
[Out] int(x^5*(a+b*csc(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 687 vs. 2(195) = 390.

Time = 0.28 (sec) , antiderivative size = 687, normalized size of antiderivative = 3.01

$$\int x^5 \left(a + b\csc\left(c + dx^2\right)\right)^2 \, dx = \frac{a^2 d^3 x^6 \sin\left(dx^2 + c\right) - 3\,b^2 d^2 x^4 \cos\left(dx^2 + c\right) + 6\,ab\mathrm{polylog}(3, \cos\left(dx^2 + c\right) + i\,\sin\left(dx^2 + c\right))\sin\left(dx^2 + c\right) -}{}$$

```
[In] integrate(x^5*(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(a^2*d^3*x^6*sin(d*x^2 + c) - 3*b^2*d^2*x^4*cos(d*x^2 + c) + 6*a*b*poly
log(3, cos(d*x^2 + c) + I*sin(d*x^2 + c))*sin(d*x^2 + c) + 6*a*b*polylog(3,
 cos(d*x^2 + c) - I*sin(d*x^2 + c))*sin(d*x^2 + c) - 6*a*b*polylog(3, -cos(
d*x^2 + c) + I*sin(d*x^2 + c))*sin(d*x^2 + c) - 6*a*b*polylog(3, -cos(d*x^2
 + c) - I*sin(d*x^2 + c))*sin(d*x^2 + c) - 3*(2*I*a*b*d*x^2 + I*b^2)*dilog(
cos(d*x^2 + c) + I*sin(d*x^2 + c))*sin(d*x^2 + c) - 3*(-2*I*a*b*d*x^2 - I*b
```

```
^2)*dilog(cos(d*x^2 + c) - I*sin(d*x^2 + c))*sin(d*x^2 + c) - 3*(2*I*a*b*d*
x^2 - I*b^2)*dilog(-cos(d*x^2 + c) + I*sin(d*x^2 + c))*sin(d*x^2 + c) - 3*(
-2*I*a*b*d*x^2 + I*b^2)*dilog(-cos(d*x^2 + c) - I*sin(d*x^2 + c))*sin(d*x^2
 + c) - 3*(a*b*d^2*x^4 - b^2*d*x^2)*log(cos(d*x^2 + c) + I*sin(d*x^2 + c) +
 1)*sin(d*x^2 + c) - 3*(a*b*d^2*x^4 - b^2*d*x^2)*log(cos(d*x^2 + c) - I*sin
(d*x^2 + c) + 1)*sin(d*x^2 + c) + 3*(a*b*c^2 - b^2*c)*log(-1/2*cos(d*x^2 +
c) + 1/2*I*sin(d*x^2 + c) + 1/2)*sin(d*x^2 + c) + 3*(a*b*c^2 - b^2*c)*log(-
1/2*cos(d*x^2 + c) - 1/2*I*sin(d*x^2 + c) + 1/2)*sin(d*x^2 + c) + 3*(a*b*d^
2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c)*log(-cos(d*x^2 + c) + I*sin(d*x^2 + c)
 + 1)*sin(d*x^2 + c) + 3*(a*b*d^2*x^4 + b^2*d*x^2 - a*b*c^2 + b^2*c)*log(-c
os(d*x^2 + c) - I*sin(d*x^2 + c) + 1)*sin(d*x^2 + c))/(d^3*sin(d*x^2 + c))
```

Sympy [F]

$$\int x^5 \left(a + b\csc\left(c + dx^2\right)\right)^2\,dx = \int x^5 \left(a + b\csc\left(c + dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**5*(a+b*csc(d*x**2+c))**2,x)
[Out] Integral(x**5*(a + b*csc(c + d*x**2))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 800 vs. 2(195) = 390.

Time = 0.32 (sec) , antiderivative size = 800, normalized size of antiderivative = 3.51

$$\int x^5 \left(a + b\csc\left(c + dx^2\right)\right)^2\,dx = \frac{1}{6}\,a^2x^6 - \frac{2\,b^2d^2x^4\cos\left(2\,dx^2 + 2\,c\right) + 2i\,b^2d^2x^4\sin\left(2\,dx^2 + 2\,c\right) - 2\left(abd^2x^4 - b^2dx^2 - \left(abd^2x^4 - b^2dx^2\right)\cos\left(2\,dx^2\right.\right.}{\cdots}$$

```
[In] integrate(x^5*(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
[Out] 1/6*a^2*x^6 - (2*b^2*d^2*x^4*cos(2*d*x^2 + 2*c) + 2*I*b^2*d^2*x^4*sin(2*d*x
^2 + 2*c) - 2*(a*b*d^2*x^4 - b^2*d*x^2 - (a*b*d^2*x^4 - b^2*d*x^2)*cos(2*d*
x^2 + 2*c) + (-I*a*b*d^2*x^4 + I*b^2*d*x^2)*sin(2*d*x^2 + 2*c))*arctan2(sin
(d*x^2 + c), cos(d*x^2 + c) + 1) - 2*(a*b*d^2*x^4 + b^2*d*x^2 - (a*b*d^2*x^
4 + b^2*d*x^2)*cos(2*d*x^2 + 2*c) + (-I*a*b*d^2*x^4 - I*b^2*d*x^2)*sin(2*d*
x^2 + 2*c))*arctan2(sin(d*x^2 + c), -cos(d*x^2 + c) + 1) + 2*(2*a*b*d*x^2 -
 b^2 - (2*a*b*d*x^2 - b^2)*cos(2*d*x^2 + 2*c) - (2*I*a*b*d*x^2 - I*b^2)*sin
(2*d*x^2 + 2*c))*dilog(-e^(I*d*x^2 + I*c)) - 2*(2*a*b*d*x^2 + b^2 - (2*a*b*
d*x^2 + b^2)*cos(2*d*x^2 + 2*c) + (-2*I*a*b*d*x^2 - I*b^2)*sin(2*d*x^2 + 2*
```

```
c))*dilog(e^(I*d*x^2 + I*c)) + (I*a*b*d^2*x^4 - I*b^2*d*x^2 + (-I*a*b*d^2*x
^4 + I*b^2*d*x^2)*cos(2*d*x^2 + 2*c) + (a*b*d^2*x^4 - b^2*d*x^2)*sin(2*d*x^
2 + 2*c))*log(cos(d*x^2 + c)^2 + sin(d*x^2 + c)^2 + 2*cos(d*x^2 + c) + 1) +
 (-I*a*b*d^2*x^4 - I*b^2*d*x^2 + (I*a*b*d^2*x^4 + I*b^2*d*x^2)*cos(2*d*x^2
+ 2*c) - (a*b*d^2*x^4 + b^2*d*x^2)*sin(2*d*x^2 + 2*c))*log(cos(d*x^2 + c)^2
 + sin(d*x^2 + c)^2 - 2*cos(d*x^2 + c) + 1) - 4*(I*a*b*cos(2*d*x^2 + 2*c) -
 a*b*sin(2*d*x^2 + 2*c) - I*a*b)*polylog(3, -e^(I*d*x^2 + I*c)) - 4*(-I*a*b
*cos(2*d*x^2 + 2*c) + a*b*sin(2*d*x^2 + 2*c) + I*a*b)*polylog(3, e^(I*d*x^2
 + I*c)))/(-2*I*d^3*cos(2*d*x^2 + 2*c) + 2*d^3*sin(2*d*x^2 + 2*c) + 2*I*d^3
)
```

Giac [F]

$$\int x^5 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \int \left(b \csc \left(dx^2 + c\right) + a\right)^2 x^5 \, dx$$

```
[In] integrate(x^5*(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate((b*csc(d*x^2 + c) + a)^2*x^5, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \int x^5 \left(a + \frac{b}{\sin \left(d\,x^2 + c\right)}\right)^2 \, dx$$

```
[In] int(x^5*(a + b/sin(c + d*x^2))^2,x)
[Out] int(x^5*(a + b/sin(c + d*x^2))^2, x)
```

3.9 $\int x^4(a+b\csc(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\mathrm{Int}\left(x^4\left(a+b\csc\left(c+dx^2\right)\right)^2,x\right)$$

```
[Out] Unintegrable(x^4*(a+b*csc(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^4\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int x^4\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$

```
[In] Int[x^4*(a + b*Csc[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^4*(a + b*Csc[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral}=\int x^4\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$

Mathematica [N/A]

Not integrable

Time = 22.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4 (a + b \csc (c + dx^2))^2 \, dx = \int x^4 (a + b \csc (c + dx^2))^2 \, dx$$

```
[In] Integrate[x^4*(a + b*Csc[c + d*x^2])^2,x]
[Out] Integrate[x^4*(a + b*Csc[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^4 \left(a + b \csc \left(d\,x^2 + c\right)\right)^2 dx$$

```
[In] int(x^4*(a+b*csc(d*x^2+c))^2,x)
[Out] int(x^4*(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^4 (a + b \csc (c + dx^2))^2 \, dx = \int (b \csc (dx^2 + c) + a)^2 x^4 \, dx$$

```
[In] integrate(x^4*(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(b^2*x^4*csc(d*x^2 + c)^2 + 2*a*b*x^4*csc(d*x^2 + c) + a^2*x^4, x)
```

Sympy [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^4 (a + b \csc (c + dx^2))^2 \, dx = \int x^4 (a + b \csc (c + dx^2))^2 \, dx$$

```
[In] integrate(x**4*(a+b*csc(d*x**2+c))**2,x)
[Out] Integral(x**4*(a + b*csc(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 310, normalized size of antiderivative = 17.22

$$\int x^4 \left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int \left(b\csc\left(dx^2+c\right)+a\right)^2x^4\,dx$$

```
[In] integrate(x^4*(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*a^2*x^5 - (b^2*x^3*sin(2*d*x^2 + 2*c) - (d*cos(2*d*x^2 + 2*c)^2 + d*sin
(2*d*x^2 + 2*c)^2 - 2*d*cos(2*d*x^2 + 2*c) + d)*integrate(1/2*(4*a*b*d*x^4
- 3*b^2*x^2)*sin(d*x^2 + c)/(d*cos(d*x^2 + c)^2 + d*sin(d*x^2 + c)^2 + 2*d*
cos(d*x^2 + c) + d), x) - (d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2
- 2*d*cos(2*d*x^2 + 2*c) + d)*integrate(1/2*(4*a*b*d*x^4 + 3*b^2*x^2)*sin(d
*x^2 + c)/(d*cos(d*x^2 + c)^2 + d*sin(d*x^2 + c)^2 - 2*d*cos(d*x^2 + c) + d
), x))/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 - 2*d*cos(2*d*x^2 +
 2*c) + d)
```

Giac [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^4 \left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int \left(b\csc\left(dx^2+c\right)+a\right)^2x^4\,dx$$

```
[In] integrate(x^4*(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^2 + c) + a)^2*x^4, x)
```

Mupad [N/A]

Not integrable

Time = 17.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^4 \left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int x^4\,\left(a+\frac{b}{\sin\left(d\,x^2+c\right)}\right)^2\,dx$$

```
[In] int(x^4*(a + b/sin(c + d*x^2))^2,x)
```

```
[Out] int(x^4*(a + b/sin(c + d*x^2))^2, x)
```

3.10 $\int x^3(a+b\csc(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 125

$$\int x^3(a+b\csc(c+dx^2))^2\,dx=\frac{a^2x^4}{4}-\frac{2abx^2\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{b^2x^2\cot(c+dx^2)}{2d}+\frac{b^2\log(\sin(c+dx^2))}{2d^2}+\frac{iab\operatorname{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2}-\frac{iab\operatorname{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2}$$

```
[Out] 1/4*a^2*x^4-2*a*b*x^2*arctanh(exp(I*(d*x^2+c)))/d-1/2*b^2*x^2*cot(d*x^2+c)/
d+1/2*b^2*ln(sin(d*x^2+c))/d^2+I*a*b*polylog(2,-exp(I*(d*x^2+c)))/d^2-I*a*b
*polylog(2,exp(I*(d*x^2+c)))/d^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4290, 4275, 4268, 2317, 2438, 4269, 3556}

$$\int x^3(a+b\csc(c+dx^2))^2\,dx=\frac{a^2x^4}{4}-\frac{2abx^2\operatorname{arctanh}\left(e^{i(c+dx^2)}\right)}{d}+\frac{iab\operatorname{PolyLog}\left(2,-e^{i(dx^2+c)}\right)}{d^2}-\frac{iab\operatorname{PolyLog}\left(2,e^{i(dx^2+c)}\right)}{d^2}+\frac{b^2\log(\sin(c+dx^2))}{2d^2}-\frac{b^2x^2\cot(c+dx^2)}{2d}$$

```
[In] Int[x^3*(a + b*Csc[c + d*x^2])^2,x]
```

```
[Out] (a^2*x^4)/4 - (2*a*b*x^2*ArcTanh[E^(I*(c + d*x^2))])/d - (b^2*x^2*Cot[c + d
*x^2])/(2*d) + (b^2*Log[Sin[c + d*x^2]])/(2*d^2) + (I*a*b*PolyLog[2, -E^(I*
(c + d*x^2))])/d^2 - (I*a*b*PolyLog[2, E^(I*(c + d*x^2))])/d^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int x(a+b\csc(c+dx))^2\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(a^2x+2abx\csc(c+dx)+b^2x\csc^2(c+dx)\right)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}+(ab)\text{Subst}\left(\int x\csc(c+dx)\,dx,x,x^2\right)+\frac{1}{2}b^2\text{Subst}\left(\int x\csc^2(c+dx)\,dx,x,x^2\right)\\
&= \frac{a^2x^4}{4}-\frac{2abx^2\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{b^2x^2\cot\left(c+dx^2\right)}{2d}\\
&\quad -\frac{(ab)\text{Subst}\left(\int\log\left(1-e^{i(c+dx)}\right)\,dx,x,x^2\right)}{d}\\
&\quad +\frac{(ab)\text{Subst}\left(\int\log\left(1+e^{i(c+dx)}\right)\,dx,x,x^2\right)}{d}+\frac{b^2\text{Subst}\left(\int\cot(c+dx)\,dx,x,x^2\right)}{2d}\\
&= \frac{a^2x^4}{4}-\frac{2abx^2\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{b^2x^2\cot\left(c+dx^2\right)}{2d}+\frac{b^2\log\left(\sin\left(c+dx^2\right)\right)}{2d^2}\\
&\quad +\frac{(iab)\text{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{i(c+dx^2)}\right)}{d^2}-\frac{(iab)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{i(c+dx^2)}\right)}{d^2}\\
&= \frac{a^2x^4}{4}-\frac{2abx^2\text{arctanh}\left(e^{i(c+dx^2)}\right)}{d}-\frac{b^2x^2\cot\left(c+dx^2\right)}{2d}+\frac{b^2\log\left(\sin\left(c+dx^2\right)\right)}{2d^2}\\
&\quad +\frac{iab\,\text{PolyLog}\left(2,-e^{i(c+dx^2)}\right)}{d^2}-\frac{iab\,\text{PolyLog}\left(2,e^{i(c+dx^2)}\right)}{d^2}
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 268 vs. 2(125) = 250.

Time = 5.77 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.14

$$\int x^3\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$

$$=\frac{2b^2dx^2\cot(c)+dx^2(a^2dx^2-2b^2\cot(c))-2b^2(dx^2\cot(c)-\log\left(\sin\left(c+dx^2\right)\right))+4ab\Big(2\arctan(\tan(c))\text{arct}}{}$$

```
[In] Integrate[x^3*(a + b*Csc[c + d*x^2])^2,x]
```

```
[Out] (2*b^2*d*x^2*Cot[c] + d*x^2*(a^2*d*x^2 - 2*b^2*Cot[c]) - 2*b^2*(d*x^2*Cot[c
] - Log[Sin[c + d*x^2]]) + 4*a*b*(2*ArcTan[Tan[c]]*ArcTanh[Cos[c] - Sin[c]*
```

```
Tan[(d*x^2)/2]] + (((d*x^2 + ArcTan[Tan[c]])*(Log[1 - E^(I*(d*x^2 + ArcTan[
Tan[c]]))] - Log[1 + E^(I*(d*x^2 + ArcTan[Tan[c]]))]) + I*PolyLog[2, -E^(I*
(d*x^2 + ArcTan[Tan[c]]))] - I*PolyLog[2, E^(I*(d*x^2 + ArcTan[Tan[c]]))])*
Sec[c])/Sqrt[Sec[c]^2]) + b^2*d*x^2*Csc[c/2]*Csc[(c + d*x^2)/2]*Sin[(d*x^2)
/2] + b^2*d*x^2*Sec[c/2]*Sec[(c + d*x^2)/2]*Sin[(d*x^2)/2])/(4*d^2)
```

Maple [F]

$$\int x^{3}\left(a+b\csc\left(d\,x^{2}+c\right)\right)^{2}dx$$

```
[In] int(x^3*(a+b*csc(d*x^2+c))^2,x)
```

```
[Out] int(x^3*(a+b*csc(d*x^2+c))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(107) = 214$.

Time $= 0.29$ (sec) , antiderivative size $= 451$, normalized size of antiderivative $= 3.61$

$$\int x^{3}\left(a+b\csc\left(c+dx^{2}\right)\right)^{2}\,dx = \frac{a^{2}d^{2}x^{4}\sin\left(dx^{2}+c\right)-2\,b^{2}dx^{2}\cos\left(dx^{2}+c\right)-2i\,ab\mathrm{Li}_{2}(\cos\left(dx^{2}+c\right)+i\,\sin\left(dx^{2}+c\right))\sin\left(dx^{2}+c\right)+2i\,a}{}$$

```
[In] integrate(x^3*(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(a^2*d^2*x^4*sin(d*x^2 + c) - 2*b^2*d*x^2*cos(d*x^2 + c) - 2*I*a*b*dilo
g(cos(d*x^2 + c) + I*sin(d*x^2 + c))*sin(d*x^2 + c) + 2*I*a*b*dilog(cos(d*x
^2 + c) - I*sin(d*x^2 + c))*sin(d*x^2 + c) - 2*I*a*b*dilog(-cos(d*x^2 + c)
+ I*sin(d*x^2 + c))*sin(d*x^2 + c) + 2*I*a*b*dilog(-cos(d*x^2 + c) - I*sin(
d*x^2 + c))*sin(d*x^2 + c) - (2*a*b*d*x^2 - b^2)*log(cos(d*x^2 + c) + I*sin
(d*x^2 + c) + 1)*sin(d*x^2 + c) - (2*a*b*d*x^2 - b^2)*log(cos(d*x^2 + c) -
I*sin(d*x^2 + c) + 1)*sin(d*x^2 + c) - (2*a*b*c - b^2)*log(-1/2*cos(d*x^2 +
 c) + 1/2*I*sin(d*x^2 + c) + 1/2)*sin(d*x^2 + c) - (2*a*b*c - b^2)*log(-1/2
*cos(d*x^2 + c) - 1/2*I*sin(d*x^2 + c) + 1/2)*sin(d*x^2 + c) + 2*(a*b*d*x^2
 + a*b*c)*log(-cos(d*x^2 + c) + I*sin(d*x^2 + c) + 1)*sin(d*x^2 + c) + 2*(a
*b*d*x^2 + a*b*c)*log(-cos(d*x^2 + c) - I*sin(d*x^2 + c) + 1)*sin(d*x^2 + c
))/(d^2*sin(d*x^2 + c))
```

Sympy [F]

$$\int x^3 \left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int x^3 \left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x**3*(a+b*csc(d*x**2+c))**2,x)
[Out] Integral(x**3*(a + b*csc(c + d*x**2))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 604 vs. 2(107) = 214.

Time = 0.31 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.83

$$\int x^3 \left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\frac{1}{4}a^2x^4$$
$$-\frac{4\,b^2dx^2\cos\left(2\,dx^2+2\,c\right)+4i\,b^2dx^2\sin\left(2\,dx^2+2\,c\right)-2\left(2\,abdx^2-b^2-\left(2\,abdx^2-b^2\right)\cos\left(2\,dx^2+2\,c\right)+\dots\right.}{\dots}$$

```
[In] integrate(x^3*(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
[Out] 1/4*a^2*x^4 - (4*b^2*d*x^2*cos(2*d*x^2 + 2*c) + 4*I*b^2*d*x^2*sin(2*d*x^2 +
 2*c) - 2*(2*a*b*d*x^2 - b^2 - (2*a*b*d*x^2 - b^2)*cos(2*d*x^2 + 2*c) + (-2
*I*a*b*d*x^2 + I*b^2)*sin(2*d*x^2 + 2*c))*arctan2(sin(d*x^2 + c), cos(d*x^2
 + c) + 1) - 2*(b^2*cos(2*d*x^2 + 2*c) + I*b^2*sin(2*d*x^2 + 2*c) - b^2)*ar
ctan2(sin(d*x^2 + c), cos(d*x^2 + c) - 1) + 4*(a*b*d*x^2*cos(2*d*x^2 + 2*c)
 + I*a*b*d*x^2*sin(2*d*x^2 + 2*c) - a*b*d*x^2)*arctan2(sin(d*x^2 + c), -cos
(d*x^2 + c) + 1) - 4*(a*b*cos(2*d*x^2 + 2*c) + I*a*b*sin(2*d*x^2 + 2*c) - a
*b)*dilog(-e^(I*d*x^2 + I*c)) + 4*(a*b*cos(2*d*x^2 + 2*c) + I*a*b*sin(2*d*x
^2 + 2*c) - a*b)*dilog(e^(I*d*x^2 + I*c)) + (2*I*a*b*d*x^2 - I*b^2 + (-2*I*
a*b*d*x^2 + I*b^2)*cos(2*d*x^2 + 2*c) + (2*a*b*d*x^2 - b^2)*sin(2*d*x^2 + 2
*c))*log(cos(d*x^2 + c)^2 + sin(d*x^2 + c)^2 + 2*cos(d*x^2 + c) + 1) + (-2*
I*a*b*d*x^2 - I*b^2 + (2*I*a*b*d*x^2 + I*b^2)*cos(2*d*x^2 + 2*c) - (2*a*b*d
*x^2 + b^2)*sin(2*d*x^2 + 2*c))*log(cos(d*x^2 + c)^2 + sin(d*x^2 + c)^2 - 2
*cos(d*x^2 + c) + 1))/(-4*I*d^2*cos(2*d*x^2 + 2*c) + 4*d^2*sin(2*d*x^2 + 2*
c) + 4*I*d^2)
```

Giac [F]

$$\int x^3 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \int \left(b \csc \left(dx^2 + c\right) + a\right)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate((b*csc(d*x^2 + c) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \int x^3 \left(a + \frac{b}{\sin \left(d\, x^2 + c\right)}\right)^2 \, dx$$

```
[In] int(x^3*(a + b/sin(c + d*x^2))^2,x)
[Out] int(x^3*(a + b/sin(c + d*x^2))^2, x)
```

3.11 $\int x^2(a+b\csc(c+dx^2))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int x^2\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\text{Int}\left(x^2\left(a+b\csc\left(c+dx^2\right)\right)^2,x\right)$$

```
[Out] Unintegrable(x^2*(a+b*csc(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^2\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int x^2\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$

```
[In] Int[x^2*(a + b*Csc[c + d*x^2])^2,x]
```

```
[Out] Defer[Int][x^2*(a + b*Csc[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral}=\int x^2\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$

Mathematica [N/A]

Not integrable

Time = 35.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \int x^2 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx$$

```
[In] Integrate[x^2*(a + b*Csc[c + d*x^2])^2,x]
[Out] Integrate[x^2*(a + b*Csc[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int x^{2} \left(a + b \csc \left(d\, x^{2} + c\right)\right)^{2} dx$$

```
[In] int(x^2*(a+b*csc(d*x^2+c))^2,x)
[Out] int(x^2*(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int x^2 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \int \left(b \csc \left(dx^2 + c\right) + a\right)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*csc(d*x^2 + c)^2 + 2*a*b*x^2*csc(d*x^2 + c) + a^2*x^2, x)
```

Sympy [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int x^2 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx = \int x^2 \left(a + b \csc \left(c + dx^2\right)\right)^2 \, dx$$

```
[In] integrate(x**2*(a+b*csc(d*x**2+c))**2,x)
[Out] Integral(x**2*(a + b*csc(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 300, normalized size of antiderivative = 16.67

$$\int x^2 (a + b \csc(c + dx^2))^2 \, dx = \int (b \csc(dx^2 + c) + a)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 - (b^2*x*sin(2*d*x^2 + 2*c) - (d*cos(2*d*x^2 + 2*c)^2 + d*sin(2
*d*x^2 + 2*c)^2 - 2*d*cos(2*d*x^2 + 2*c) + d)*integrate(1/2*(4*a*b*d*x^2 -
b^2)*sin(d*x^2 + c)/(d*cos(d*x^2 + c)^2 + d*sin(d*x^2 + c)^2 + 2*d*cos(d*x^
2 + c) + d), x) - (d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 - 2*d*co
s(2*d*x^2 + 2*c) + d)*integrate(1/2*(4*a*b*d*x^2 + b^2)*sin(d*x^2 + c)/(d*c
os(d*x^2 + c)^2 + d*sin(d*x^2 + c)^2 - 2*d*cos(d*x^2 + c) + d), x))/(d*cos(
2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 - 2*d*cos(2*d*x^2 + 2*c) + d)
```

Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int x^2 (a + b \csc(c + dx^2))^2 \, dx = \int (b \csc(dx^2 + c) + a)^2 x^2 \, dx$$

```
[In] integrate(x^2*(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^2 + c) + a)^2*x^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int x^2 (a + b \csc(c + dx^2))^2 \, dx = \int x^2 \left(a + \frac{b}{\sin(d\,x^2 + c)}\right)^2 dx$$

```
[In] int(x^2*(a + b/sin(c + d*x^2))^2,x)
```

```
[Out] int(x^2*(a + b/sin(c + d*x^2))^2, x)
```

3.12 $\int x(a+b\csc(c+dx^2))^2 \, dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x(a+b\csc(c+dx^2))^2 \, dx = \frac{a^2x^2}{2} - \frac{ab\operatorname{arctanh}(\cos(c+dx^2))}{d} - \frac{b^2\cot(c+dx^2)}{2d}$$

```
[Out] 1/2*a^2*x^2-a*b*arctanh(cos(d*x^2+c))/d-1/2*b^2*cot(d*x^2+c)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4290, 3858, 3855, 3852, 8}

$$\int x(a+b\csc(c+dx^2))^2 \, dx = \frac{a^2x^2}{2} - \frac{ab\operatorname{arctanh}(\cos(c+dx^2))}{d} - \frac{b^2\cot(c+dx^2)}{2d}$$

```
[In] Int[x*(a + b*Csc[c + d*x^2])^2,x]
[Out] (a^2*x^2)/2 - (a*b*ArcTanh[Cos[c + d*x^2]])/d - (b^2*Cot[c + d*x^2])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int (a+b\csc(c+dx))^2\,dx, x, x^2\right)\\
&= \frac{a^2x^2}{2} + (ab)\text{Subst}\left(\int \csc(c+dx)\,dx, x, x^2\right) + \frac{1}{2}b^2\text{Subst}\left(\int \csc^2(c+dx)\,dx, x, x^2\right)\\
&= \frac{a^2x^2}{2} - \frac{ab\text{arctanh}(\cos(c+dx^2))}{d} - \frac{b^2\text{Subst}\left(\int 1\,dx, x, \cot(c+dx^2)\right)}{2d}\\
&= \frac{a^2x^2}{2} - \frac{ab\text{arctanh}(\cos(c+dx^2))}{d} - \frac{b^2\cot(c+dx^2)}{2d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.91

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$
$$= \frac{-b^2\cot\left(\frac{1}{2}(c+dx^2)\right) + 2a\left(ac+adx^2-2b\log\left(\cos\left(\frac{1}{2}(c+dx^2)\right)\right)+2b\log\left(\sin\left(\frac{1}{2}(c+dx^2)\right)\right)\right) + b^2\tan\left(\frac{1}{2}(c\right.}{4d}$$

```
[In] Integrate[x*(a + b*Csc[c + d*x^2])^2,x]
[Out] (-(b^2*Cot[(c + d*x^2)/2]) + 2*a*(a*c + a*d*x^2 - 2*b*Log[Cos[(c + d*x^2)/2
]] + 2*b*Log[Sin[(c + d*x^2)/2]]) + b^2*Tan[(c + d*x^2)/2])/(4*d)
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

method	result	size
parts	$\frac{a^{2} x^{2}}{2}-\frac{b^{2} \cot \left(d \,x^{2}+c \right)}{2 d}-\frac{a b \ln \left(\csc \left(d \,x^{2}+c \right)+\cot \left(d \,x^{2}+c \right)\right)}{d}$	51
derivativedivides	$\frac{a^{2} \left(d \,x^{2}+c \right)+2 a b \ln \left(\csc \left(d \,x^{2}+c \right)-\cot \left(d \,x^{2}+c \right)\right)-b^{2} \cot \left(d \,x^{2}+c \right)}{2 d}$	55
default	$\frac{a^{2} \left(d \,x^{2}+c \right)+2 a b \ln \left(\csc \left(d \,x^{2}+c \right)-\cot \left(d \,x^{2}+c \right)\right)-b^{2} \cot \left(d \,x^{2}+c \right)}{2 d}$	55
parallelrisch	$\frac{2 a^{2} x^{2} d -\cot \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right) b^{2}+\tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right) b^{2}+4 \ln \left(\tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)\right) a b}{4 d}$	63
risch	$\frac{a^{2} x^{2}}{2}-\frac{i b^{2}}{d \left({\mathrm e}^{2 i \left(d \,x^{2}+c \right)}-1\right)}+\frac{a b \ln \left({\mathrm e}^{i \left(d \,x^{2}+c \right)}-1\right)}{d}-\frac{a b \ln \left({\mathrm e}^{i \left(d \,x^{2}+c \right)}+1\right)}{d}$	75
norman	$\frac{-\frac{b^{2}}{4 d}+\frac{a^{2} x^{2} \tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)}{2}+\frac{b^{2} \tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)^{2}}{4 d}}{\tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)}+\frac{a b \ln \left(\tan \left(\frac{d \,x^{2}}{2}+\frac{c}{2}\right)\right)}{d}$	83

```
[In] int(x*(a+b*csc(d*x^2+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/2*a^2*x^2-1/2*b^2*cot(d*x^2+c)/d-a*b/d*ln(csc(d*x^2+c)+cot(d*x^2+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.09

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$
$$=\frac{a^2dx^2\sin\left(dx^2+c\right)-ab\log\left(\frac{1}{2}\cos\left(dx^2+c\right)+\frac{1}{2}\right)\sin\left(dx^2+c\right)+ab\log\left(-\frac{1}{2}\cos\left(dx^2+c\right)+\frac{1}{2}\right)\sin\left(dx^2+\right.}{2\,d\sin\left(dx^2+c\right)}$$

```
[In] integrate(x*(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
[Out] 1/2*(a^2*d*x^2*sin(d*x^2 + c) - a*b*log(1/2*cos(d*x^2 + c) + 1/2)*sin(d*x^2
 + c) + a*b*log(-1/2*cos(d*x^2 + c) + 1/2)*sin(d*x^2 + c) - b^2*cos(d*x^2 +
 c))/(d*sin(d*x^2 + c))
```

Sympy [F]

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx$$

```
[In] integrate(x*(a+b*csc(d*x**2+c))**2,x)
```

```
[Out] Integral(x*(a + b*csc(c + d*x**2))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(41) = 82$.

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.18

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx = \frac{1}{2}a^2x^2-\frac{ab\log\left(\cot\left(dx^2+c\right)+\csc\left(dx^2+c\right)\right)}{d} - \frac{b^2\sin\left(2\,dx^2+2\,c\right)}{d\cos\left(2\,dx^2+2\,c\right)^2+d\sin\left(2\,dx^2+2\,c\right)^2-2\,d\cos\left(2\,dx^2+2\,c\right)+d}$$

```
[In] integrate(x*(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 - a*b*log(cot(d*x^2 + c) + csc(d*x^2 + c))/d - b^2*sin(2*d*x^2
+ 2*c)/(d*cos(2*d*x^2 + 2*c)^2 + d*sin(2*d*x^2 + 2*c)^2 - 2*d*cos(2*d*x^2 +
 2*c) + d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(41) = 82$.

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.87

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx = \frac{2\,(dx^2+c)a^2+4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)\right|\right)+b^2\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)-\frac{4\,ab\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)+b^2}{\tan\left(\frac{1}{2}\,dx^2+\frac{1}{2}\,c\right)}}{4\,d}$$

```
[In] integrate(x*(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*(d*x^2 + c)*a^2 + 4*a*b*log(abs(tan(1/2*d*x^2 + 1/2*c))) + b^2*tan(1
/2*d*x^2 + 1/2*c) - (4*a*b*tan(1/2*d*x^2 + 1/2*c) + b^2)/tan(1/2*d*x^2 + 1/
2*c))/d
```

Mupad [B] (verification not implemented)

Time = 18.32 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.27

$$\int x\left(a+b\csc\left(c+dx^2\right)\right)^2\,dx=\frac{a^2\,x^2}{2}-\frac{b^2\,1\mathrm{i}}{d\,\left(\mathrm{e}^{2\mathrm{i}\,d\,x^2+c\,2\mathrm{i}}-1\right)}-\frac{a\,b\,\ln\left(-a\,b\,x\,4\mathrm{i}-a\,b\,x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,4\mathrm{i}\right)}{d}+\frac{a\,b\,\ln\left(a\,b\,x\,4\mathrm{i}-a\,b\,x\,\mathrm{e}^{d\,x^2\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,4\mathrm{i}\right)}{d}$$

```
[In] int(x*(a + b/sin(c + d*x^2))^2,x)
```

```
[Out] (a^2*x^2)/2 - (b^2*1i)/(d*(exp(c*2i + d*x^2*2i) - 1)) - (a*b*log(- a*b*x*4i
 - a*b*x*exp(d*x^2*1i)*exp(c*1i)*4i))/d + (a*b*log(a*b*x*4i - a*b*x*exp(d*x
^2*1i)*exp(c*1i)*4i))/d
```

3.13 $\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx = \text{Int}\left(\frac{(a+b\csc(c+dx^2))^2}{x},x\right)$$

```
[Out] Unintegrable((a+b*csc(d*x^2+c))^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx = \int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx$$

```
[In] Int[(a + b*Csc[c + d*x^2])^2/x,x]
[Out] Defer[Int][(a + b*Csc[c + d*x^2])^2/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 86.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx=\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*x^2])^2/x,x]
[Out] Integrate[(a + b*Csc[c + d*x^2])^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\csc(d\,x^2+c))^2}{x}dx$$

```
[In] int((a+b*csc(d*x^2+c))^2/x,x)
[Out] int((a+b*csc(d*x^2+c))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx=\int \frac{(b\csc(dx^2+c)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))^2/x,x, algorithm="fricas")
[Out] integral((b^2*csc(d*x^2 + c)^2 + 2*a*b*csc(d*x^2 + c) + a^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx=\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx$$

```
[In] integrate((a+b*csc(d*x**2+c))**2/x,x)
[Out] Integral((a + b*csc(c + d*x**2))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 360, normalized size of antiderivative = 20.00

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx=\int \frac{(b\csc(dx^2+c)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) - (b^2*sin(2*d*x^2 + 2*c) - (d*x^2*cos(2*d*x^2 + 2*c)^2 + d*x^2*
sin(2*d*x^2 + 2*c)^2 - 2*d*x^2*cos(2*d*x^2 + 2*c) + d*x^2)*integrate((2*a*b
*d*x^2 + b^2)*sin(d*x^2 + c)/(d*x^3*cos(d*x^2 + c)^2 + d*x^3*sin(d*x^2 + c)
^2 + 2*d*x^3*cos(d*x^2 + c) + d*x^3), x) - (d*x^2*cos(2*d*x^2 + 2*c)^2 + d*
x^2*sin(2*d*x^2 + 2*c)^2 - 2*d*x^2*cos(2*d*x^2 + 2*c) + d*x^2)*integrate((2
*a*b*d*x^2 - b^2)*sin(d*x^2 + c)/(d*x^3*cos(d*x^2 + c)^2 + d*x^3*sin(d*x^2
+ c)^2 - 2*d*x^3*cos(d*x^2 + c) + d*x^3), x))/(d*x^2*cos(2*d*x^2 + 2*c)^2 +
 d*x^2*sin(2*d*x^2 + 2*c)^2 - 2*d*x^2*cos(2*d*x^2 + 2*c) + d*x^2)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx=\int \frac{(b\csc(dx^2+c)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^2 + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 17.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\csc(c+dx^2))^2}{x}\,dx=\int \frac{\left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2}{x}\,dx$$

```
[In] int((a + b/sin(c + d*x^2))^2/x,x)
```

```
[Out] int((a + b/sin(c + d*x^2))^2/x, x)
```

3.14 $\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx = \mathrm{Int}\left(\frac{(a+b\csc(c+dx^2))^2}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*csc(d*x^2+c))^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx = \int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx$$

```
[In] Int[(a + b*Csc[c + d*x^2])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*Csc[c + d*x^2])^2/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 49.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx=\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*x^2])^2/x^2,x]
[Out] Integrate[(a + b*Csc[c + d*x^2])^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\csc(d\,x^2+c))^2}{x^2}dx$$

```
[In] int((a+b*csc(d*x^2+c))^2/x^2,x)
[Out] int((a+b*csc(d*x^2+c))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx=\int \frac{(b\csc(dx^2+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*csc(d*x^2 + c)^2 + 2*a*b*csc(d*x^2 + c) + a^2)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx=\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x**2+c))**2/x**2,x)
[Out] Integral((a + b*csc(c + d*x**2))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 366, normalized size of antiderivative = 20.33

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx=\int \frac{(b\csc(dx^2+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))^2/x^2,x, algorithm="maxima")
```

```
[Out] -a^2/x - (b^2*sin(2*d*x^2 + 2*c) - (d*x^3*cos(2*d*x^2 + 2*c)^2 + d*x^3*sin(
2*d*x^2 + 2*c)^2 - 2*d*x^3*cos(2*d*x^2 + 2*c) + d*x^3)*integrate(1/2*(4*a*b
*d*x^2 + 3*b^2)*sin(d*x^2 + c)/(d*x^4*cos(d*x^2 + c)^2 + d*x^4*sin(d*x^2 +
c)^2 + 2*d*x^4*cos(d*x^2 + c) + d*x^4), x) - (d*x^3*cos(2*d*x^2 + 2*c)^2 +
d*x^3*sin(2*d*x^2 + 2*c)^2 - 2*d*x^3*cos(2*d*x^2 + 2*c) + d*x^3)*integrate(
1/2*(4*a*b*d*x^2 - 3*b^2)*sin(d*x^2 + c)/(d*x^4*cos(d*x^2 + c)^2 + d*x^4*si
n(d*x^2 + c)^2 - 2*d*x^4*cos(d*x^2 + c) + d*x^4), x))/(d*x^3*cos(2*d*x^2 +
2*c)^2 + d*x^3*sin(2*d*x^2 + 2*c)^2 - 2*d*x^3*cos(2*d*x^2 + 2*c) + d*x^3)
```

Giac [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx=\int \frac{(b\csc(dx^2+c)+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^2 + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\csc(c+dx^2))^2}{x^2}\,dx=\int \frac{\left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2}{x^2}\,dx$$

```
[In] int((a + b/sin(c + d*x^2))^2/x^2,x)
```

```
[Out] int((a + b/sin(c + d*x^2))^2/x^2, x)
```

3.15 $\int x \csc^7\left(a+bx^2\right) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int x \csc^7\left(a+bx^2\right) \, dx = -\frac{5\text{arctanh}(\cos\left(a+bx^2\right))}{32b} - \frac{5\cot\left(a+bx^2\right)\csc\left(a+bx^2\right)}{32b} - \frac{5\cot\left(a+bx^2\right)\csc^3\left(a+bx^2\right)}{48b} - \frac{\cot\left(a+bx^2\right)\csc^5\left(a+bx^2\right)}{12b}$$

```
[Out] -5/32*arctanh(cos(b*x^2+a))/b-5/32*cot(b*x^2+a)*csc(b*x^2+a)/b-5/48*cot(b*x
^2+a)*csc(b*x^2+a)^3/b-1/12*cot(b*x^2+a)*csc(b*x^2+a)^5/b
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4290, 3853, 3855}

$$\int x \csc^7\left(a+bx^2\right) \, dx = -\frac{5\text{arctanh}(\cos\left(a+bx^2\right))}{32b} - \frac{\cot\left(a+bx^2\right)\csc^5\left(a+bx^2\right)}{12b} - \frac{5\cot\left(a+bx^2\right)\csc^3\left(a+bx^2\right)}{48b} - \frac{5\cot\left(a+bx^2\right)\csc\left(a+bx^2\right)}{32b}$$

```
[In] Int[x*Csc[a + b*x^2]^7,x]
```

```
[Out] (-5*ArcTanh[Cos[a + b*x^2]])/(32*b) - (5*Cot[a + b*x^2]*Csc[a + b*x^2])/(32
*b) - (5*Cot[a + b*x^2]*Csc[a + b*x^2]^3)/(48*b) - (Cot[a + b*x^2]*Csc[a +
b*x^2]^5)/(12*b)
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \csc^7(a+bx)\,dx, x, x^2\right)\\
&= -\frac{\cot\left(a+bx^2\right)\csc^5\left(a+bx^2\right)}{12b}+\frac{5}{12}\text{Subst}\left(\int \csc^5(a+bx)\,dx, x, x^2\right)\\
&= -\frac{5\cot\left(a+bx^2\right)\csc^3\left(a+bx^2\right)}{48b}-\frac{\cot\left(a+bx^2\right)\csc^5\left(a+bx^2\right)}{12b}\\
&\quad+\frac{5}{16}\text{Subst}\left(\int \csc^3(a+bx)\,dx, x, x^2\right)\\
&= -\frac{5\cot\left(a+bx^2\right)\csc\left(a+bx^2\right)}{32b}-\frac{5\cot\left(a+bx^2\right)\csc^3\left(a+bx^2\right)}{48b}\\
&\quad-\frac{\cot\left(a+bx^2\right)\csc^5\left(a+bx^2\right)}{12b}+\frac{5}{32}\text{Subst}\left(\int \csc(a+bx)\,dx, x, x^2\right)\\
&= -\frac{5\text{arctanh}\left(\cos\left(a+bx^2\right)\right)}{32b}-\frac{5\cot\left(a+bx^2\right)\csc\left(a+bx^2\right)}{32b}\\
&\quad-\frac{5\cot\left(a+bx^2\right)\csc^3\left(a+bx^2\right)}{48b}-\frac{\cot\left(a+bx^2\right)\csc^5\left(a+bx^2\right)}{12b}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.86

$$\int x\csc^7\left(a+bx^2\right)\,dx=-\frac{5\csc^2\left(\frac{1}{2}(a+bx^2)\right)}{128b}-\frac{\csc^4\left(\frac{1}{2}(a+bx^2)\right)}{128b}-\frac{\csc^6\left(\frac{1}{2}(a+bx^2)\right)}{768b}-\frac{5\log\left(\cos\left(\frac{1}{2}(a+bx^2)\right)\right)}{32b}+\frac{5\log\left(\sin\left(\frac{1}{2}(a+bx^2)\right)\right)}{32b}+\frac{5\sec^2\left(\frac{1}{2}(a+bx^2)\right)}{128b}+\frac{\sec^4\left(\frac{1}{2}(a+bx^2)\right)}{128b}+\frac{\sec^6\left(\frac{1}{2}(a+bx^2)\right)}{768b}$$

```
[In] Integrate[x*Csc[a + b*x^2]^7,x]
```

```
[Out] (-5*Csc[(a + b*x^2)/2]^2)/(128*b) - Csc[(a + b*x^2)/2]^4/(128*b) - Csc[(a +
 b*x^2)/2]^6/(768*b) - (5*Log[Cos[(a + b*x^2)/2]])/(32*b) + (5*Log[Sin[(a +
 b*x^2)/2]])/(32*b) + (5*Sec[(a + b*x^2)/2]^2)/(128*b) + Sec[(a + b*x^2)/2]
^4/(128*b) + Sec[(a + b*x^2)/2]^6/(768*b)
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\left(-\frac{\csc\left(b\,x^2+a\right)^5}{6}-\frac{5\csc\left(b\,x^2+a\right)^3}{24}-\frac{5\csc\left(b\,x^2+a\right)}{16}\right)\cot\left(b\,x^2+a\right)+\frac{5\ln\left(\csc\left(b\,x^2+a\right)-\cot\left(b\,x^2+a\right)\right)}{16}}{2b}$
default	$\frac{\left(-\frac{\csc\left(b\,x^2+a\right)^5}{6}-\frac{5\csc\left(b\,x^2+a\right)^3}{24}-\frac{5\csc\left(b\,x^2+a\right)}{16}\right)\cot\left(b\,x^2+a\right)+\frac{5\ln\left(\csc\left(b\,x^2+a\right)-\cot\left(b\,x^2+a\right)\right)}{16}}{2b}$
parallelrisch	$\frac{\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)^6-\cot\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)^6+9\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)^4-9\cot\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)^4+45\tan\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)^2-45\cot\left(\frac{a}{2}+\frac{b\,x^2}{2}\right)^2+120\ln\left(\tan\right.}{768b}$
risch	$\frac{15\,e^{11i\left(b\,x^2+a\right)}-85\,e^{9i\left(b\,x^2+a\right)}+198\,e^{7i\left(b\,x^2+a\right)}+198\,e^{5i\left(b\,x^2+a\right)}-85\,e^{3i\left(b\,x^2+a\right)}+15\,e^{i\left(b\,x^2+a\right)}}{48b\left(e^{2i\left(b\,x^2+a\right)}-1\right)^6}-\frac{5\ln\left(e^{i\left(b\,x^2+a\right)}+1\right.}{32b}$

```
[In] int(x*csc(b*x^2+a)^7,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/b*((-1/6*csc(b*x^2+a)^5-5/24*csc(b*x^2+a)^3-5/16*csc(b*x^2+a))*cot(b*x^
2+a)+5/16*ln(csc(b*x^2+a)-cot(b*x^2+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(82) = 164.

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.03

$$\int x\csc^7\left(a+bx^2\right)\,dx$$
$$=\frac{30\,\cos\left(bx^2+a\right)^5-80\,\cos\left(bx^2+a\right)^3-15\left(\cos\left(bx^2+a\right)^6-3\,\cos\left(bx^2+a\right)^4+3\,\cos\left(bx^2+a\right)^2-1\right)\log}{192\left(b\cos\left(bx^2+a\right)\right.}$$

```
[In] integrate(x*csc(b*x^2+a)^7,x, algorithm="fricas")
```

```
[Out] 1/192*(30*cos(b*x^2 + a)^5 - 80*cos(b*x^2 + a)^3 - 15*(cos(b*x^2 + a)^6 - 3
*cos(b*x^2 + a)^4 + 3*cos(b*x^2 + a)^2 - 1)*log(1/2*cos(b*x^2 + a) + 1/2) +
 15*(cos(b*x^2 + a)^6 - 3*cos(b*x^2 + a)^4 + 3*cos(b*x^2 + a)^2 - 1)*log(-1
/2*cos(b*x^2 + a) + 1/2) + 66*cos(b*x^2 + a))/(b*cos(b*x^2 + a)^6 - 3*b*cos
(b*x^2 + a)^4 + 3*b*cos(b*x^2 + a)^2 - b)
```

Sympy [F]

$$\int x\csc^7\left(a+bx^2\right)\,dx=\int x\csc^7\left(a+bx^2\right)dx$$

```
[In] integrate(x*csc(b*x**2+a)**7,x)
```

```
[Out] Integral(x*csc(a + b*x**2)**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3543 vs. 2(82) = 164.

Time = 0.28 (sec) , antiderivative size = 3543, normalized size of antiderivative = 39.37

$$\int x\csc^7\left(a+bx^2\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x*csc(b*x^2+a)^7,x, algorithm="maxima")
```

```
[Out] 1/192*(4*(15*cos(11*b*x^2 + 11*a) - 85*cos(9*b*x^2 + 9*a) + 198*cos(7*b*x^2
 + 7*a) + 198*cos(5*b*x^2 + 5*a) - 85*cos(3*b*x^2 + 3*a) + 15*cos(b*x^2 + a
))*cos(12*b*x^2 + 12*a) - 60*(6*cos(10*b*x^2 + 10*a) - 15*cos(8*b*x^2 + 8*a
) + 20*cos(6*b*x^2 + 6*a) - 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) -
1)*cos(11*b*x^2 + 11*a) + 24*(85*cos(9*b*x^2 + 9*a) - 198*cos(7*b*x^2 + 7*a
) - 198*cos(5*b*x^2 + 5*a) + 85*cos(3*b*x^2 + 3*a) - 15*cos(b*x^2 + a))*cos
```

```
(10*b*x^2 + 10*a) - 340*(15*cos(8*b*x^2 + 8*a) - 20*cos(6*b*x^2 + 6*a) + 15
*cos(4*b*x^2 + 4*a) - 6*cos(2*b*x^2 + 2*a) + 1)*cos(9*b*x^2 + 9*a) + 60*(19
8*cos(7*b*x^2 + 7*a) + 198*cos(5*b*x^2 + 5*a) - 85*cos(3*b*x^2 + 3*a) + 15*
cos(b*x^2 + a))*cos(8*b*x^2 + 8*a) - 792*(20*cos(6*b*x^2 + 6*a) - 15*cos(4*
b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) - 1)*cos(7*b*x^2 + 7*a) - 80*(198*cos(5
*b*x^2 + 5*a) - 85*cos(3*b*x^2 + 3*a) + 15*cos(b*x^2 + a))*cos(6*b*x^2 + 6*
a) + 792*(15*cos(4*b*x^2 + 4*a) - 6*cos(2*b*x^2 + 2*a) + 1)*cos(5*b*x^2 + 5
*a) - 300*(17*cos(3*b*x^2 + 3*a) - 3*cos(b*x^2 + a))*cos(4*b*x^2 + 4*a) + 3
40*(6*cos(2*b*x^2 + 2*a) - 1)*cos(3*b*x^2 + 3*a) - 360*cos(2*b*x^2 + 2*a)*c
os(b*x^2 + a) + 15*(2*(6*cos(10*b*x^2 + 10*a) - 15*cos(8*b*x^2 + 8*a) + 20*
cos(6*b*x^2 + 6*a) - 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) - 1)*cos(
12*b*x^2 + 12*a) - cos(12*b*x^2 + 12*a)^2 + 12*(15*cos(8*b*x^2 + 8*a) - 20*
cos(6*b*x^2 + 6*a) + 15*cos(4*b*x^2 + 4*a) - 6*cos(2*b*x^2 + 2*a) + 1)*cos(
10*b*x^2 + 10*a) - 36*cos(10*b*x^2 + 10*a)^2 + 30*(20*cos(6*b*x^2 + 6*a) -
15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) - 1)*cos(8*b*x^2 + 8*a) - 225*
cos(8*b*x^2 + 8*a)^2 + 40*(15*cos(4*b*x^2 + 4*a) - 6*cos(2*b*x^2 + 2*a) + 1
)*cos(6*b*x^2 + 6*a) - 400*cos(6*b*x^2 + 6*a)^2 + 30*(6*cos(2*b*x^2 + 2*a)
- 1)*cos(4*b*x^2 + 4*a) - 225*cos(4*b*x^2 + 4*a)^2 - 36*cos(2*b*x^2 + 2*a)^
2 + 2*(6*sin(10*b*x^2 + 10*a) - 15*sin(8*b*x^2 + 8*a) + 20*sin(6*b*x^2 + 6*
a) - 15*sin(4*b*x^2 + 4*a) + 6*sin(2*b*x^2 + 2*a))*sin(12*b*x^2 + 12*a) - s
in(12*b*x^2 + 12*a)^2 + 12*(15*sin(8*b*x^2 + 8*a) - 20*sin(6*b*x^2 + 6*a) +
 15*sin(4*b*x^2 + 4*a) - 6*sin(2*b*x^2 + 2*a))*sin(10*b*x^2 + 10*a) - 36*si
n(10*b*x^2 + 10*a)^2 + 30*(20*sin(6*b*x^2 + 6*a) - 15*sin(4*b*x^2 + 4*a) +
6*sin(2*b*x^2 + 2*a))*sin(8*b*x^2 + 8*a) - 225*sin(8*b*x^2 + 8*a)^2 + 120*(
5*sin(4*b*x^2 + 4*a) - 2*sin(2*b*x^2 + 2*a))*sin(6*b*x^2 + 6*a) - 400*sin(6
*b*x^2 + 6*a)^2 - 225*sin(4*b*x^2 + 4*a)^2 + 180*sin(4*b*x^2 + 4*a)*sin(2*b
*x^2 + 2*a) - 36*sin(2*b*x^2 + 2*a)^2 + 12*cos(2*b*x^2 + 2*a) - 1)*log(cos(
b*x^2)^2 + 2*cos(b*x^2)*cos(a) + cos(a)^2 + sin(b*x^2)^2 - 2*sin(b*x^2)*sin
(a) + sin(a)^2) - 15*(2*(6*cos(10*b*x^2 + 10*a) - 15*cos(8*b*x^2 + 8*a) + 2
0*cos(6*b*x^2 + 6*a) - 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) - 1)*co
s(12*b*x^2 + 12*a) - cos(12*b*x^2 + 12*a)^2 + 12*(15*cos(8*b*x^2 + 8*a) - 2
0*cos(6*b*x^2 + 6*a) + 15*cos(4*b*x^2 + 4*a) - 6*cos(2*b*x^2 + 2*a) + 1)*co
s(10*b*x^2 + 10*a) - 36*cos(10*b*x^2 + 10*a)^2 + 30*(20*cos(6*b*x^2 + 6*a)
- 15*cos(4*b*x^2 + 4*a) + 6*cos(2*b*x^2 + 2*a) - 1)*cos(8*b*x^2 + 8*a) - 22
5*cos(8*b*x^2 + 8*a)^2 + 40*(15*cos(4*b*x^2 + 4*a) - 6*cos(2*b*x^2 + 2*a) +
 1)*cos(6*b*x^2 + 6*a) - 400*cos(6*b*x^2 + 6*a)^2 + 30*(6*cos(2*b*x^2 + 2*a
) - 1)*cos(4*b*x^2 + 4*a) - 225*cos(4*b*x^2 + 4*a)^2 - 36*cos(2*b*x^2 + 2*a
)^2 + 2*(6*sin(10*b*x^2 + 10*a) - 15*sin(8*b*x^2 + 8*a) + 20*sin(6*b*x^2 +
6*a) - 15*sin(4*b*x^2 + 4*a) + 6*sin(2*b*x^2 + 2*a))*sin(12*b*x^2 + 12*a) -
 sin(12*b*x^2 + 12*a)^2 + 12*(15*sin(8*b*x^2 + 8*a) - 20*sin(6*b*x^2 + 6*a)
 + 15*sin(4*b*x^2 + 4*a) - 6*sin(2*b*x^2 + 2*a))*sin(10*b*x^2 + 10*a) - 36*
sin(10*b*x^2 + 10*a)^2 + 30*(20*sin(6*b*x^2 + 6*a) - 15*sin(4*b*x^2 + 4*a)
+ 6*sin(2*b*x^2 + 2*a))*sin(8*b*x^2 + 8*a) - 225*sin(8*b*x^2 + 8*a)^2 + 120
*(5*sin(4*b*x^2 + 4*a) - 2*sin(2*b*x^2 + 2*a))*sin(6*b*x^2 + 6*a) - 400*sin
(6*b*x^2 + 6*a)^2 - 225*sin(4*b*x^2 + 4*a)^2 + 180*sin(4*b*x^2 + 4*a)*sin(2
```

```
*b*x^2 + 2*a) - 36*sin(2*b*x^2 + 2*a)^2 + 12*cos(2*b*x^2 + 2*a) - 1)*log(co
s(b*x^2)^2 - 2*cos(b*x^2)*cos(a) + cos(a)^2 + sin(b*x^2)^2 + 2*sin(b*x^2)*s
in(a) + sin(a)^2) + 4*(15*sin(11*b*x^2 + 11*a) - 85*sin(9*b*x^2 + 9*a) + 19
8*sin(7*b*x^2 + 7*a) + 198*sin(5*b*x^2 + 5*a) - 85*sin(3*b*x^2 + 3*a) + 15*
sin(b*x^2 + a))*sin(12*b*x^2 + 12*a) - 60*(6*sin(10*b*x^2 + 10*a) - 15*sin(
8*b*x^2 + 8*a) + 20*sin(6*b*x^2 + 6*a) - 15*sin(4*b*x^2 + 4*a) + 6*sin(2*b*
x^2 + 2*a))*sin(11*b*x^2 + 11*a) + 24*(85*sin(9*b*x^2 + 9*a) - 198*sin(7*b*
x^2 + 7*a) - 198*sin(5*b*x^2 + 5*a) + 85*sin(3*b*x^2 + 3*a) - 15*sin(b*x^2
+ a))*sin(10*b*x^2 + 10*a) - 340*(15*sin(8*b*x^2 + 8*a) - 20*sin(6*b*x^2 +
6*a) + 15*sin(4*b*x^2 + 4*a) - 6*sin(2*b*x^2 + 2*a))*sin(9*b*x^2 + 9*a) + 6
0*(198*sin(7*b*x^2 + 7*a) + 198*sin(5*b*x^2 + 5*a) - 85*sin(3*b*x^2 + 3*a)
+ 15*sin(b*x^2 + a))*sin(8*b*x^2 + 8*a) - 792*(20*sin(6*b*x^2 + 6*a) - 15*s
in(4*b*x^2 + 4*a) + 6*sin(2*b*x^2 + 2*a))*sin(7*b*x^2 + 7*a) - 80*(198*sin(
5*b*x^2 + 5*a) - 85*sin(3*b*x^2 + 3*a) + 15*sin(b*x^2 + a))*sin(6*b*x^2 + 6
*a) + 2376*(5*sin(4*b*x^2 + 4*a) - 2*sin(2*b*x^2 + 2*a))*sin(5*b*x^2 + 5*a)
 - 300*(17*sin(3*b*x^2 + 3*a) - 3*sin(b*x^2 + a))*sin(4*b*x^2 + 4*a) + 2040
*sin(3*b*x^2 + 3*a)*sin(2*b*x^2 + 2*a) - 360*sin(2*b*x^2 + 2*a)*sin(b*x^2 +
 a) + 60*cos(b*x^2 + a))/(b*cos(12*b*x^2 + 12*a)^2 + 36*b*cos(10*b*x^2 + 10
*a)^2 + 225*b*cos(8*b*x^2 + 8*a)^2 + 400*b*cos(6*b*x^2 + 6*a)^2 + 225*b*cos
(4*b*x^2 + 4*a)^2 + 36*b*cos(2*b*x^2 + 2*a)^2 + b*sin(12*b*x^2 + 12*a)^2 +
36*b*sin(10*b*x^2 + 10*a)^2 + 225*b*sin(8*b*x^2 + 8*a)^2 + 400*b*sin(6*b*x^
2 + 6*a)^2 + 225*b*sin(4*b*x^2 + 4*a)^2 - 180*b*sin(4*b*x^2 + 4*a)*sin(2*b*
x^2 + 2*a) + 36*b*sin(2*b*x^2 + 2*a)^2 - 2*(6*b*cos(10*b*x^2 + 10*a) - 15*b
*cos(8*b*x^2 + 8*a) + 20*b*cos(6*b*x^2 + 6*a) - 15*b*cos(4*b*x^2 + 4*a) + 6
*b*cos(2*b*x^2 + 2*a) - b)*cos(12*b*x^2 + 12*a) - 12*(15*b*cos(8*b*x^2 + 8*
a) - 20*b*cos(6*b*x^2 + 6*a) + 15*b*cos(4*b*x^2 + 4*a) - 6*b*cos(2*b*x^2 +
2*a) + b)*cos(10*b*x^2 + 10*a) - 30*(20*b*cos(6*b*x^2 + 6*a) - 15*b*cos(4*b
*x^2 + 4*a) + 6*b*cos(2*b*x^2 + 2*a) - b)*cos(8*b*x^2 + 8*a) - 40*(15*b*cos
(4*b*x^2 + 4*a) - 6*b*cos(2*b*x^2 + 2*a) + b)*cos(6*b*x^2 + 6*a) - 30*(6*b*
cos(2*b*x^2 + 2*a) - b)*cos(4*b*x^2 + 4*a) - 12*b*cos(2*b*x^2 + 2*a) - 2*(6
*b*sin(10*b*x^2 + 10*a) - 15*b*sin(8*b*x^2 + 8*a) + 20*b*sin(6*b*x^2 + 6*a)
 - 15*b*sin(4*b*x^2 + 4*a) + 6*b*sin(2*b*x^2 + 2*a))*sin(12*b*x^2 + 12*a) -
 12*(15*b*sin(8*b*x^2 + 8*a) - 20*b*sin(6*b*x^2 + 6*a) + 15*b*sin(4*b*x^2 +
 4*a) - 6*b*sin(2*b*x^2 + 2*a))*sin(10*b*x^2 + 10*a) - 30*(20*b*sin(6*b*x^2
 + 6*a) - 15*b*sin(4*b*x^2 + 4*a) + 6*b*sin(2*b*x^2 + 2*a))*sin(8*b*x^2 + 8
*a) - 120*(5*b*sin(4*b*x^2 + 4*a) - 2*b*sin(2*b*x^2 + 2*a))*sin(6*b*x^2 + 6
*a) + b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(82) = 164$.

Time = 0.27 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.34

$$\int x\csc^7\left(a+bx^2\right)\,dx = -\frac{\frac{\left(\frac{9\left(\cos\left(bx^2+a\right)-1\right)}{\cos\left(bx^2+a\right)+1}-\frac{45\left(\cos\left(bx^2+a\right)-1\right)^2}{\left(\cos\left(bx^2+a\right)+1\right)^2}+\frac{110\left(\cos\left(bx^2+a\right)-1\right)^3}{\left(\cos\left(bx^2+a\right)+1\right)^3}-1\right)\left(\cos(bx^2+a)+1\right)^3}{\left(\cos(bx^2+a)-1\right)^3}+\frac{45\left(\cos(bx^2+a)-1\right)}{\cos(bx^2+a)+1}-\frac{9\left(\cos(bx^2+a)-1\right)^2}{\left(\cos(bx^2+a)+1\right)^2}+}{768\,b}$$

```
[In] integrate(x*csc(b*x^2+a)^7,x, algorithm="giac")
```

```
[Out] -1/768*((9*(cos(b*x^2 + a) - 1)/(cos(b*x^2 + a) + 1) - 45*(cos(b*x^2 + a) -
 1)^2/(cos(b*x^2 + a) + 1)^2 + 110*(cos(b*x^2 + a) - 1)^3/(cos(b*x^2 + a) +
 1)^3 - 1)*(cos(b*x^2 + a) + 1)^3/(cos(b*x^2 + a) - 1)^3 + 45*(cos(b*x^2 +
a) - 1)/(cos(b*x^2 + a) + 1) - 9*(cos(b*x^2 + a) - 1)^2/(cos(b*x^2 + a) + 1
)^2 + (cos(b*x^2 + a) - 1)^3/(cos(b*x^2 + a) + 1)^3 - 60*log(-(cos(b*x^2 +
a) - 1)/(cos(b*x^2 + a) + 1)))/b
```

Mupad [B] (verification not implemented)

Time = 28.55 (sec) , antiderivative size = 491, normalized size of antiderivative = 5.46

$$\int x\csc^7\left(a+bx^2\right)\,dx = -\frac{5\,\ln\left(-\frac{x\,5\mathrm{i}}{8}-\frac{x\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x^2\,1\mathrm{i}}\,5\mathrm{i}}{8}\right)}{32\,b}+\frac{5\,\ln\left(\frac{x\,5\mathrm{i}}{8}-\frac{x\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x^2\,1\mathrm{i}}\,5\mathrm{i}}{8}\right)}{32\,b}$$
$$+\frac{8\,\mathrm{e}^{3\mathrm{i}\,b\,x^2+a\,3\mathrm{i}}}{3\,b\,\left(5\,\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}-10\,\mathrm{e}^{4\mathrm{i}\,b\,x^2+a\,4\mathrm{i}}+10\,\mathrm{e}^{6\mathrm{i}\,b\,x^2+a\,6\mathrm{i}}-5\,\mathrm{e}^{8\mathrm{i}\,b\,x^2+a\,8\mathrm{i}}+\mathrm{e}^{10\mathrm{i}\,b\,x^2+a\,10\mathrm{i}}-1\right)}$$
$$+\frac{\mathrm{e}^{1\mathrm{i}\,b\,x^2+a\,1\mathrm{i}}}{6\,b\,\left(3\,\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}-3\,\mathrm{e}^{4\mathrm{i}\,b\,x^2+a\,4\mathrm{i}}+\mathrm{e}^{6\mathrm{i}\,b\,x^2+a\,6\mathrm{i}}-1\right)}+\frac{5\,\mathrm{e}^{1\mathrm{i}\,b\,x^2+a\,1\mathrm{i}}}{16\,b\,\left(\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}-1\right)}$$
$$+\frac{16\,\mathrm{e}^{5\mathrm{i}\,b\,x^2+a\,5\mathrm{i}}}{3\,b\,\left(1+15\,\mathrm{e}^{4\mathrm{i}\,b\,x^2+a\,4\mathrm{i}}-20\,\mathrm{e}^{6\mathrm{i}\,b\,x^2+a\,6\mathrm{i}}+15\,\mathrm{e}^{8\mathrm{i}\,b\,x^2+a\,8\mathrm{i}}-6\,\mathrm{e}^{10\mathrm{i}\,b\,x^2+a\,10\mathrm{i}}+\mathrm{e}^{12\mathrm{i}\,b\,x^2+a\,12\mathrm{i}}-6\,\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}\right)}$$
$$+\frac{\mathrm{e}^{1\mathrm{i}\,b\,x^2+a\,1\mathrm{i}}}{b\,\left(1+6\,\mathrm{e}^{4\mathrm{i}\,b\,x^2+a\,4\mathrm{i}}-4\,\mathrm{e}^{6\mathrm{i}\,b\,x^2+a\,6\mathrm{i}}+\mathrm{e}^{8\mathrm{i}\,b\,x^2+a\,8\mathrm{i}}-4\,\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}\right)}$$
$$-\frac{5\,\mathrm{e}^{1\mathrm{i}\,b\,x^2+a\,1\mathrm{i}}}{24\,b\,\left(1+\mathrm{e}^{4\mathrm{i}\,b\,x^2+a\,4\mathrm{i}}-2\,\mathrm{e}^{2\mathrm{i}\,b\,x^2+a\,2\mathrm{i}}\right)}$$

```
[In] int(x/sin(a + b*x^2)^7,x)
```

```
[Out] (5*log((x*5i)/8 - (x*exp(a*1i)*exp(b*x^2*1i)*5i)/8))/(32*b) - (5*log(- (x*5
i)/8 - (x*exp(a*1i)*exp(b*x^2*1i)*5i)/8))/(32*b) + (8*exp(a*3i + b*x^2*3i))
/(3*b*(5*exp(a*2i + b*x^2*2i) - 10*exp(a*4i + b*x^2*4i) + 10*exp(a*6i + b*x
```

```
^2*6i) - 5*exp(a*8i + b*x^2*8i) + exp(a*10i + b*x^2*10i) - 1)) + exp(a*1i +
 b*x^2*1i)/(6*b*(3*exp(a*2i + b*x^2*2i) - 3*exp(a*4i + b*x^2*4i) + exp(a*6i
 + b*x^2*6i) - 1)) + (5*exp(a*1i + b*x^2*1i))/(16*b*(exp(a*2i + b*x^2*2i) -
 1)) + (16*exp(a*5i + b*x^2*5i))/(3*b*(15*exp(a*4i + b*x^2*4i) - 6*exp(a*2i
 + b*x^2*2i) - 20*exp(a*6i + b*x^2*6i) + 15*exp(a*8i + b*x^2*8i) - 6*exp(a*
10i + b*x^2*10i) + exp(a*12i + b*x^2*12i) + 1)) + exp(a*1i + b*x^2*1i)/(b*(
6*exp(a*4i + b*x^2*4i) - 4*exp(a*2i + b*x^2*2i) - 4*exp(a*6i + b*x^2*6i) +
exp(a*8i + b*x^2*8i) + 1)) - (5*exp(a*1i + b*x^2*1i))/(24*b*(exp(a*4i + b*x
^2*4i) - 2*exp(a*2i + b*x^2*2i) + 1))
```

3.16 $\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 396

$$\begin{aligned}\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx &= \frac{x^6}{6a}+\frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}-\frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}\\ &+\frac{bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}\\ &+\frac{ib\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{ib\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}\end{aligned}$$

```
[Out] 1/6*x^6/a+1/2*I*b*x^4*ln(1-I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a/d/(
-a^2+b^2)^(1/2)-1/2*I*b*x^4*ln(1-I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))
/a/d/(-a^2+b^2)^(1/2)+b*x^2*polylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1
/2)))/a/d^2/(-a^2+b^2)^(1/2)-b*x^2*polylog(2,I*a*exp(I*(d*x^2+c))/(b+(-a^2+
b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+I*b*polylog(3,I*a*exp(I*(d*x^2+c))/(b-(
-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-I*b*polylog(3,I*a*exp(I*(d*x^2+c))
/(b+(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4290, 4276, 3404, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx = \frac{ib\,\text{PolyLog}\left(3,\frac{iae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{ib\,\text{PolyLog}\left(3,\frac{iae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} + \frac{bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{2ad\sqrt{b^2-a^2}} - \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{2ad\sqrt{b^2-a^2}} + \frac{x^6}{6a}$$

```
[In] Int[x^5/(a + b*Csc[c + d*x^2]),x]
```

```
[Out] x^6/(6*a) + ((I/2)*b*x^4*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b
^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((I/2)*b*x^4*Log[1 - (I*a*E^(I*(c + d*x^2))
)/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (b*x^2*PolyLog[2, (I*a*
E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) - (b*x
^2*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2
 + b^2]*d^2) + (I*b*PolyLog[3, (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2
])])/(a*Sqrt[-a^2 + b^2]*d^3) - (I*b*PolyLog[3, (I*a*E^(I*(c + d*x^2)))/(b
+ Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{a+b\csc(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x^2}{a}-\frac{bx^2}{a(b+a\sin(c+dx))}\right)\,dx, x, x^2\right)
\end{aligned}$$

$$= \frac{x^6}{6a} - \frac{b\text{Subst}\left(\int \frac{x^2}{b+a\sin(c+dx)}\,dx, x, x^2\right)}{2a}$$

$$= \frac{x^6}{6a} - \frac{b\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, x^2\right)}{a}$$

$$= \frac{x^6}{6a} + \frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{-a^2+b^2}} - \frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{-a^2+b^2}}$$

$$= \frac{x^6}{6a} + \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{(ib)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}d} + \frac{(ib)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}d}$$

$$= \frac{x^6}{6a} + \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{b\text{Subst}\left(\int \text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{b\text{Subst}\left(\int \text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{x^6}{6a} + \frac{ibx^4 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^4 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^2 \operatorname{PolyLog}\left(2, \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{bx^2 \operatorname{PolyLog}\left(2, \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(ib)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, \frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{(ib)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(2, \frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+dx^2)}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{x^6}{6a} + \frac{ibx^4 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^4 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{bx^2 \operatorname{PolyLog}\left(2, \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{bx^2 \operatorname{PolyLog}\left(2, \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{ib \operatorname{PolyLog}\left(3, \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{ib \operatorname{PolyLog}\left(3, \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.81

$$\int \frac{x^5}{a + b\csc(c + dx^2)}\,dx = \frac{\sqrt{a^2-b^2}d^3x^6 - 3bd^2x^4 \log\left(1 - \frac{ae^{i(c+dx^2)}}{-ib+\sqrt{a^2-b^2}}\right) + 3bd^2x^4 \log\left(1 + \frac{ae^{i(c+dx^2)}}{ib+\sqrt{a^2-b^2}}\right) + 6ibdx^2 \operatorname{PolyLog}\left(2, \frac{ae^{i(c+dx^2)}}{-ib+\sqrt{a^2-b^2}}\right) \cdots}{6a\sqrt{a^2-\cdots}}$$

```
[In] Integrate[x^5/(a + b*Csc[c + d*x^2]),x]
```

```
[Out] (Sqrt[a^2 - b^2]*d^3*x^6 - 3*b*d^2*x^4*Log[1 - (a*E^(I*(c + d*x^2)))/((-I)*
b + Sqrt[a^2 - b^2])] + 3*b*d^2*x^4*Log[1 + (a*E^(I*(c + d*x^2)))/(I*b + Sq
rt[a^2 - b^2])] + (6*I)*b*d*x^2*PolyLog[2, (a*E^(I*(c + d*x^2)))/((-I)*b +
Sqrt[a^2 - b^2])] - (6*I)*b*d*x^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(I*b +
 Sqrt[a^2 - b^2]))] - 6*b*PolyLog[3, (a*E^(I*(c + d*x^2)))/((-I)*b + Sqrt[a
^2 - b^2])] + 6*b*PolyLog[3, -((a*E^(I*(c + d*x^2)))/(I*b + Sqrt[a^2 - b^2]
))])/(6*a*Sqrt[a^2 - b^2]*d^3)
```

Maple [F]

$$\int \frac{x^5}{a + b\csc(d\,x^2 + c)}dx$$

```
[In] int(x^5/(a+b*csc(d*x^2+c)),x)
```

```
[Out] int(x^5/(a+b*csc(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1445 vs. 2(332) = 664.

Time = 0.38 (sec) , antiderivative size = 1445, normalized size of antiderivative = 3.65

$$\int \frac{x^5}{a + b\csc(c + dx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^5/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(2*(a^2 - b^2)*d^3*x^6 + 6*I*a*b*d*x^2*sqrt((a^2 - b^2)/a^2)*dilog((I*
b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c
))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) - 6*I*a*b*d*x^2*sqrt((a^2 - b^2)/a^2)*
dilog((I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(
d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) - 6*I*a*b*d*x^2*sqrt((a^2 - b
^2)/a^2)*dilog((-I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c)
- I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) + 6*I*a*b*d*x^2*sqr
t((a^2 - b^2)/a^2)*dilog((-I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d
*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) + 3*a*b*c
^2*sqrt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*
a*sqrt((a^2 - b^2)/a^2) + 2*I*b) + 3*a*b*c^2*sqrt((a^2 - b^2)/a^2)*log(2*a*
cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b)
- 3*a*b*c^2*sqrt((a^2 - b^2)/a^2)*log(-2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^2
 + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) - 3*a*b*c^2*sqrt((a^2 - b^2)/a^2
)*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2
) - 2*I*b) + 6*a*b*sqrt((a^2 - b^2)/a^2)*polylog(3, -(I*b*cos(d*x^2 + c) +
b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)
/a^2))/a) - 6*a*b*sqrt((a^2 - b^2)/a^2)*polylog(3, -(I*b*cos(d*x^2 + c) + b
*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/
a^2))/a) + 6*a*b*sqrt((a^2 - b^2)/a^2)*polylog(3, -(-I*b*cos(d*x^2 + c) + b
*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/
a^2))/a) - 6*a*b*sqrt((a^2 - b^2)/a^2)*polylog(3, -(-I*b*cos(d*x^2 + c) + b
*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/
a^2))/a) - 3*(a*b*d^2*x^4 - a*b*c^2)*sqrt((a^2 - b^2)/a^2)*log(-(I*b*cos(d*
x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt(
```

```
(a^2 - b^2)/a^2) - a)/a) + 3*(a*b*d^2*x^4 - a*b*c^2)*sqrt((a^2 - b^2)/a^2)*
log(-(I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d
*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) - 3*(a*b*d^2*x^4 - a*b*c^2)*sqrt((
a^2 - b^2)/a^2)*log(-(-I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2
 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) + 3*(a*b*d^2*x^4
- a*b*c^2)*sqrt((a^2 - b^2)/a^2)*log(-(-I*b*cos(d*x^2 + c) - b*sin(d*x^2 +
c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a))
/((a^3 - a*b^2)*d^3)
```

Sympy [F]

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^5}{a+b\csc(c+dx^2)}\,dx$$

[In] integrate(x**5/(a+b*csc(d*x**2+c)),x)

[Out] Integral(x**5/(a + b*csc(c + d*x**2)), x)

Maxima [F]

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^5}{b\csc(dx^2+c)+a}\,dx$$

[In] integrate(x^5/(a+b*csc(d*x^2+c)),x, algorithm="maxima")

```
[Out] 1/6*(x^6 - 12*a*b*integrate((2*b*x^5*cos(d*x^2 + c)^2 + a*x^5*cos(d*x^2 + c
)*sin(2*d*x^2 + 2*c) - a*x^5*cos(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^5*si
n(d*x^2 + c)^2 + a*x^5*sin(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*
cos(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + a^3*sin(2*d*
x^2 + 2*c)^2 + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*sin(d*x^2 + c) + a^3 - 2*
(2*a^2*b*sin(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [F]

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^5}{b\csc(dx^2+c)+a}\,dx$$

[In] integrate(x^5/(a+b*csc(d*x^2+c)),x, algorithm="giac")

[Out] integrate(x^5/(b*csc(d*x^2 + c) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^5}{a+\frac{b}{\sin(d\,x^2+c)}}\,dx$$

```
[In] int(x^5/(a + b/sin(c + d*x^2)),x)
[Out] int(x^5/(a + b/sin(c + d*x^2)), x)
```

3.17 $\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx = \text{Int}\left(\frac{x^4}{a+b\csc(c+dx^2)},x\right)$$

```
[Out] Unintegrable(x^4/(a+b*csc(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^4}{a+b\csc(c+dx^2)}\,dx$$

```
[In] Int[x^4/(a + b*Csc[c + d*x^2]),x]
```

```
[Out] Defer[Int][x^4/(a + b*Csc[c + d*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4}{a+b\csc(c+dx^2)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx$$

```
[In] Integrate[x^4/(a + b*Csc[c + d*x^2]),x]
[Out] Integrate[x^4/(a + b*Csc[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{a+b\csc(d\,x^2+c)}dx$$

```
[In] int(x^4/(a+b*csc(d*x^2+c)),x)
[Out] int(x^4/(a+b*csc(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^4}{b\csc(dx^2+c)+a}\,dx$$

```
[In] integrate(x^4/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
[Out] integral(x^4/(b*csc(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^4}{a+b\csc(c+dx^2)}\,dx$$

```
[In] integrate(x**4/(a+b*csc(d*x**2+c)),x)
[Out] Integral(x**4/(a + b*csc(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 253, normalized size of antiderivative = 14.06

$$\int \frac{x^4}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^4}{b\csc(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^4/(a+b*csc(d*x^2+c)),x, algorithm="maxima")
[Out] 1/5*(x^5 - 10*a*b*integrate((2*b*x^4*cos(d*x^2 + c)^2 + a*x^4*cos(d*x^2 + c
)*sin(2*d*x^2 + 2*c) - a*x^4*cos(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^4*si
n(d*x^2 + c)^2 + a*x^4*sin(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*
cos(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + a^3*sin(2*d*
x^2 + 2*c)^2 + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*sin(d*x^2 + c) + a^3 - 2*
(2*a^2*b*sin(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^4}{b\csc(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^4/(a+b*csc(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^4/(b*csc(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 18.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^4}{a + \frac{b}{\sin(d\,x^2+c)}}\,dx$$

```
[In] int(x^4/(a + b/sin(c + d*x^2)),x)
[Out] int(x^4/(a + b/sin(c + d*x^2)), x)
```

3.18 $\int \frac{x^3}{a+b\csc(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 271

$$\int \frac{x^3}{a+b\csc(c+dx^2)}\,dx = \frac{x^4}{4a}+\frac{ibx^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}\,d}-\frac{ibx^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}\,d}+\frac{b\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}\,d^2}-\frac{b\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}\,d^2}$$

```
[Out] 1/4*x^4/a+1/2*I*b*x^2*ln(1-I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a/d/(
-a^2+b^2)^(1/2)-1/2*I*b*x^2*ln(1-I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))
/a/d/(-a^2+b^2)^(1/2)+1/2*b*polylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1
/2)))/a/d^2/(-a^2+b^2)^(1/2)-1/2*b*polylog(2,I*a*exp(I*(d*x^2+c))/(b+(-a^2+
b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4290, 4276, 3404, 2296, 2221, 2317, 2438}

$$\int \frac{x^3}{a+b\csc(c+dx^2)}\,dx = \frac{b\,\text{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}}-\frac{b\,\text{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{2ad^2\sqrt{b^2-a^2}}+\frac{ibx^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{b^2-a^2}}\right)}{2ad\sqrt{b^2-a^2}}-\frac{ibx^2\log\left(1-\frac{iae^{i(c+dx^2)}}{\sqrt{b^2-a^2}+b}\right)}{2ad\sqrt{b^2-a^2}}+\frac{x^4}{4a}$$

```
[In] Int[x^3/(a + b*Csc[c + d*x^2]),x]
```

```
[Out] x^4/(4*a) + ((I/2)*b*x^2*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b
^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((I/2)*b*x^2*Log[1 - (I*a*E^(I*(c + d*x^2))
)/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (b*PolyLog[2, (I*a*E^(I
*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])/(2*a*Sqrt[-a^2 + b^2]*d^2) - (b*Pol
yLog[2, (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(2*a*Sqrt[-a^2 + b
^2]*d^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
```

```
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{a+b\csc(c+dx)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x}{a}-\frac{bx}{a(b+a\sin(c+dx))}\right)\,dx, x, x^2\right)\\
&= \frac{x^4}{4a}-\frac{b\text{Subst}\left(\int \frac{x}{b+a\sin(c+dx)}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^4}{4a}-\frac{b\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, x^2\right)}{a}\\
&= \frac{x^4}{4a}+\frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{-a^2+b^2}}\\
&\quad -\frac{(ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{\sqrt{-a^2+b^2}}\\
&= \frac{x^4}{4a}+\frac{ibx^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}-\frac{ibx^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d}\\
&\quad -\frac{(ib)\text{Subst}\left(\int \log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a\sqrt{-a^2+b^2}d}\\
&\quad +\frac{(ib)\text{Subst}\left(\int \log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a\sqrt{-a^2+b^2}d}
\end{aligned}$$

$$= \frac{x^4}{4a} + \frac{ibx^2 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^2 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{b\text{Subst}\left(\int \frac{\log\left(1-\frac{2iax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+dx^2)}\right)}{2a\sqrt{-a^2+b^2}d^2} + \frac{b\text{Subst}\left(\int \frac{\log\left(1-\frac{2iax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+dx^2)}\right)}{2a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{x^4}{4a} + \frac{ibx^2 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} - \frac{ibx^2 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d} + \frac{b\,\text{PolyLog}\left(2, \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2} - \frac{b\,\text{PolyLog}\left(2, \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a\sqrt{-a^2+b^2}d^2}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 987 vs. $2(271) = 542$.

Time = 5.40 (sec) , antiderivative size = 987, normalized size of antiderivative = 3.64

$$\int \frac{x^3}{a + b\csc(c + dx^2)}\,dx$$

$$= \frac{\csc(c+dx^2)\left(x^4 - \frac{2b\left(\frac{\pi \arctan\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{2\left(c-\arccos\left(-\frac{b}{a}\right)\right)\text{arctanh}\left(\frac{(a-b)\cot\left(\frac{1}{4}(2c+\pi+2dx^2)\right)}{\sqrt{a^2-b^2}}\right) + (-2c+\pi-2dx^2)\text{arcta}\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

```
[In] Integrate[x^3/(a + b*Csc[c + d*x^2]),x]
```

```
[Out] (Csc[c + d*x^2]*(x^4 - (2*b*((Pi*ArcTan[(a + b*Tan[(c + d*x^2)/2])/Sqrt[-a^
2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(c - ArcCos[-(b/a)])*ArcTanh[((a - b)*Cot[
(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]] + (-2*c + Pi - 2*d*x^2)*ArcTanh[(
(a + b)*Tan[(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]] - (ArcCos[-(b/a)] - (
2*I)*ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]])*Log[((
```

```
a + b)*(a - b - I*Sqrt[a^2 - b^2])*(1 + I*Cot[(2*c + Pi + 2*d*x^2)/4]))/(a*
(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi + 2*d*x^2)/4]))] + (ArcCos[-(b/a)] +
 (2*I)*(-ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]] + A
rcTanh[((a + b)*Tan[(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]]))*Log[((-1)^(
1/4)*Sqrt[a^2 - b^2])/(Sqrt[2]*Sqrt[a]*E^((I/2)*(c + d*x^2))*Sqrt[b + a*Sin
[c + d*x^2]])] + (ArcCos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Cot[(2*c + Pi + 2
*d*x^2)/4])/Sqrt[a^2 - b^2]] - (2*I)*ArcTanh[((a + b)*Tan[(2*c + Pi + 2*d*x
^2)/4])/Sqrt[a^2 - b^2]])*Log[-(((-1)^(3/4)*Sqrt[a^2 - b^2]*E^((I/2)*(c + d
*x^2)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Sin[c + d*x^2]]))] - (ArcCos[-(b/a)] +
(2*I)*ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^2)/4])/Sqrt[a^2 - b^2]])*Log[1
 + (I*(I*b + Sqrt[a^2 - b^2])*(a + b + Sqrt[a^2 - b^2]*Tan[(2*c - Pi + 2*d*
x^2)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi + 2*d*x^2)/4]))] + I*(P
olyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b + Sqrt[a^2 - b^2]*Tan[(2*c - Pi +
 2*d*x^2)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi + 2*d*x^2)/4]))] -
 PolyLog[2, ((b + I*Sqrt[a^2 - b^2])*(a + b + Sqrt[a^2 - b^2]*Tan[(2*c - Pi
 + 2*d*x^2)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi + 2*d*x^2)/4]))]
))/Sqrt[a^2 - b^2]))/d^2)*(b + a*Sin[c + d*x^2]))/(4*a*(a + b*Csc[c + d*x^2
]))
```

Maple [F]

$$\int \frac{x^3}{a + b\csc\left(d\,x^2 + c\right)}dx$$

```
[In] int(x^3/(a+b*csc(d*x^2+c)),x)
[Out] int(x^3/(a+b*csc(d*x^2+c)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1050 vs. 2(223) = 446.

Time = 0.36 (sec) , antiderivative size = 1050, normalized size of antiderivative = 3.87

$$\int \frac{x^3}{a + b\csc\left(c + dx^2\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
[Out] 1/4*((a^2 - b^2)*d^2*x^4 - a*b*c*sqrt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^2 +
c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) - a*b*c*sqrt
((a^2 - b^2)/a^2)*log(2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt(
(a^2 - b^2)/a^2) - 2*I*b) + a*b*c*sqrt((a^2 - b^2)/a^2)*log(-2*a*cos(d*x^2
+ c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) + a*b*c*sq
rt((a^2 - b^2)/a^2)*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sq
```

```
rt((a^2 - b^2)/a^2) - 2*I*b) + I*a*b*sqrt((a^2 - b^2)/a^2)*dilog((I*b*cos(d
*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt
((a^2 - b^2)/a^2) - a)/a + 1) - I*a*b*sqrt((a^2 - b^2)/a^2)*dilog((I*b*cos(
d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqr
t((a^2 - b^2)/a^2) - a)/a + 1) - I*a*b*sqrt((a^2 - b^2)/a^2)*dilog((-I*b*co
s(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*s
qrt((a^2 - b^2)/a^2) - a)/a + 1) + I*a*b*sqrt((a^2 - b^2)/a^2)*dilog((-I*b*
cos(d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))
*sqrt((a^2 - b^2)/a^2) - a)/a + 1) - (a*b*d*x^2 + a*b*c)*sqrt((a^2 - b^2)/a
^2)*log(-(I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*s
in(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) + (a*b*d*x^2 + a*b*c)*sqrt((a^
2 - b^2)/a^2)*log(-(I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 +
c) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) - (a*b*d*x^2 + a*b*c
)*sqrt((a^2 - b^2)/a^2)*log(-(-I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*c
os(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) + (a*b*d*
x^2 + a*b*c)*sqrt((a^2 - b^2)/a^2)*log(-(-I*b*cos(d*x^2 + c) - b*sin(d*x^2
+ c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a
))/((a^3 - a*b^2)*d^2)
```

Sympy [F]

$$\int \frac{x^3}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^3}{a + b\csc(c + dx^2)}\,dx$$

```
[In] integrate(x**3/(a+b*csc(d*x**2+c)),x)
[Out] Integral(x**3/(a + b*csc(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{x^3}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^3}{b\csc(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^3/(a+b*csc(d*x^2+c)),x, algorithm="maxima")
[Out] 1/4*(x^4 - 8*a*b*integrate((2*b*x^3*cos(d*x^2 + c)^2 + a*x^3*cos(d*x^2 + c)
*sin(2*d*x^2 + 2*c) - a*x^3*cos(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^3*sin
(d*x^2 + c)^2 + a*x^3*sin(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*c
os(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + a^3*sin(2*d*x
^2 + 2*c)^2 + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*sin(d*x^2 + c) + a^3 - 2*(
2*a^2*b*sin(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [F]

$$\int \frac{x^3}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^3}{b\csc(dx^2 + c) + a}\,dx$$

```
[In] integrate(x^3/(a+b*csc(d*x^2+c)),x, algorithm="giac")
[Out] integrate(x^3/(b*csc(d*x^2 + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b\csc(c + dx^2)}\,dx = \int \frac{x^3}{a + \frac{b}{\sin(d\,x^2+c)}}\,dx$$

```
[In] int(x^3/(a + b/sin(c + d*x^2)),x)
[Out] int(x^3/(a + b/sin(c + d*x^2)), x)
```

3.19 $\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx = \text{Int}\left(\frac{x^2}{a+b\csc(c+dx^2)},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*csc(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^2}{a+b\csc(c+dx^2)}\,dx$$

```
[In] Int[x^2/(a + b*Csc[c + d*x^2]),x]
[Out] Defer[Int][x^2/(a + b*Csc[c + d*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{a+b\csc(c+dx^2)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx$$

```
[In] Integrate[x^2/(a + b*Csc[c + d*x^2]),x]
[Out] Integrate[x^2/(a + b*Csc[c + d*x^2]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{a+b\csc(d\,x^2+c)}dx$$

```
[In] int(x^2/(a+b*csc(d*x^2+c)),x)
[Out] int(x^2/(a+b*csc(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^2}{b\csc(dx^2+c)+a}\,dx$$

```
[In] integrate(x^2/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
[Out] integral(x^2/(b*csc(d*x^2 + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx=\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx$$

```
[In] integrate(x**2/(a+b*csc(d*x**2+c)),x)
[Out] Integral(x**2/(a + b*csc(c + d*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 253, normalized size of antiderivative = 14.06

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^2}{b\csc(dx^2+c)+a}\,dx$$

```
[In] integrate(x^2/(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] 1/3*(x^3 - 6*a*b*integrate((2*b*x^2*cos(d*x^2 + c)^2 + a*x^2*cos(d*x^2 + c)
*sin(2*d*x^2 + 2*c) - a*x^2*cos(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*x^2*sin
(d*x^2 + c)^2 + a*x^2*sin(d*x^2 + c))/(a^3*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*c
os(d*x^2 + c)^2 + 4*a^2*b*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + a^3*sin(2*d*x
^2 + 2*c)^2 + 4*a*b^2*sin(d*x^2 + c)^2 + 4*a^2*b*sin(d*x^2 + c) + a^3 - 2*(
2*a^2*b*sin(d*x^2 + c) + a^3)*cos(2*d*x^2 + 2*c)), x))/a
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^2}{b\csc(dx^2+c)+a}\,dx$$

```
[In] integrate(x^2/(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(x^2/(b*csc(d*x^2 + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 18.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{a+b\csc(c+dx^2)}\,dx = \int \frac{x^2}{a+\frac{b}{\sin(d\,x^2+c)}}\,dx$$

```
[In] int(x^2/(a + b/sin(c + d*x^2)),x)
```

```
[Out] int(x^2/(a + b/sin(c + d*x^2)), x)
```

3.20 $\int \frac{x}{a+b\csc(c+dx^2)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 63

$$\int \frac{x}{a+b\csc(c+dx^2)}\,dx=\frac{x^2}{2a}+\frac{b\,\text{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d}$$

```
[Out] 1/2*x^2/a+b*arctanh((a+b*tan(1/2*d*x^2+1/2*c))/(a^2-b^2)^(1/2))/a/d/(a^2-b^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4290, 3868, 2739, 632, 212}

$$\int \frac{x}{a+b\csc(c+dx^2)}\,dx=\frac{b\,\text{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}+\frac{x^2}{2a}$$

```
[In] Int[x/(a + b*Csc[c + d*x^2]),x]
```

```
[Out] x^2/(2*a) + (b*ArcTanh[(a + b*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]])/(a*Sqrt
[a^2 - b^2]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{a+b\csc(c+dx)}\,dx, x, x^2\right)\\
&= \frac{x^2}{2a} - \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a\sin(c+dx)}{b}}\,dx, x, x^2\right)}{2a}\\
&= \frac{x^2}{2a} - \frac{\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{ad}\\
&= \frac{x^2}{2a} + \frac{2\text{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}+2\tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{ad}\\
&= \frac{x^2}{2a} + \frac{b\,\text{arctanh}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}\,d}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{x}{a+b\csc\left(c+dx^2\right)}\,dx = \frac{\frac{c}{d}+x^2-\frac{2b\arctan\left(\frac{a+b\tan\left(\frac{1}{2}\left(c+dx^2\right)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}d}}{2a}$$

```
[In] Integrate[x/(a + b*Csc[c + d*x^2]),x]
```

```
[Out] (c/d + x^2 - (2*b*ArcTan[(a + b*Tan[(c + d*x^2)/2])/Sqrt[-a^2 + b^2]])/(Sqr
t[-a^2 + b^2]*d))/(2*a)
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{-\frac{2b\arctan\left(\frac{2b\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}+\frac{2\arctan\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)}{a}}{2d}$	73
default	$\frac{-\frac{2b\arctan\left(\frac{2b\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}+\frac{2\arctan\left(\tan\left(\frac{dx^2}{2}+\frac{c}{2}\right)\right)}{a}}{2d}$	73
risch	$\frac{x^2}{2a}+\frac{b\ln\left(e^{i\left(dx^2+c\right)}+\frac{ib\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,a}\right)}{2\sqrt{a^2-b^2}\,da}-\frac{b\ln\left(e^{i\left(dx^2+c\right)}+\frac{ib\sqrt{a^2-b^2}-a^2+b^2}{\sqrt{a^2-b^2}\,a}\right)}{2\sqrt{a^2-b^2}\,da}$	154

```
[In] int(x/(a+b*csc(d*x^2+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(-2*b/a/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*d*x^2+1/2*c)+2*a)/(-
a^2+b^2)^(1/2))+2/a*arctan(tan(1/2*d*x^2+1/2*c)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 261, normalized size of antiderivative = 4.14

$$\int \frac{x}{a+b\csc\left(c+dx^2\right)}\,dx$$
$$= \left[\frac{2\left(a^2-b^2\right)dx^2+\sqrt{a^2-b^2}\,b\log\left(\frac{\left(a^2-2b^2\right)\cos\left(dx^2+c\right)^2+2ab\sin\left(dx^2+c\right)+a^2+b^2+2\left(b\cos\left(dx^2+c\right)\sin\left(dx^2+c\right)+a\cos\left(dx^2+c\right)\right)\sqrt{}}{a^2\cos\left(dx^2+c\right)^2-2ab\sin\left(dx^2+c\right)-a^2-b^2}\right.}{4\left(a^3-ab^2\right)d}\right.$$

```
[In] integrate(x/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2 - b^2)*d*x^2 + sqrt(a^2 - b^2)*b*log(((a^2 - 2*b^2)*cos(d*x^2
+ c)^2 + 2*a*b*sin(d*x^2 + c) + a^2 + b^2 + 2*(b*cos(d*x^2 + c)*sin(d*x^2 +
 c) + a*cos(d*x^2 + c))*sqrt(a^2 - b^2))/(a^2*cos(d*x^2 + c)^2 - 2*a*b*sin(
d*x^2 + c) - a^2 - b^2)))/((a^3 - a*b^2)*d), 1/2*((a^2 - b^2)*d*x^2 + sqrt(
-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*sin(d*x^2 + c) + a)/((a^2 - b^2)*
cos(d*x^2 + c))))/((a^3 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{x}{a+b\csc(c+dx^2)}\,dx=\int \frac{x}{a+b\csc(c+dx^2)}\,dx$$

```
[In] integrate(x/(a+b*csc(d*x**2+c)),x)
```

```
[Out] Integral(x/(a + b*csc(c + d*x**2)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x}{a+b\csc(c+dx^2)}\,dx=\text{Timed out}$$

```
[In] integrate(x/(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.33

$$\int \frac{x}{a+b\csc(c+dx^2)}\,dx=-\frac{\left(\pi\left\lfloor\frac{dx^2+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}ad}+\frac{dx^2+c}{2ad}$$

```
[In] integrate(x/(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

```
[Out] -(pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*d*x^2 + 1/2
*c) + a)/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2)*a*d) + 1/2*(d*x^2 + c)/(a*d
)
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.59

$$\int \frac{x}{a+b\csc\left(c+dx^2\right)}\,dx=\frac{x^2}{2\,a}-\frac{b\,\ln\left(b\,x\,{\mathrm{e}}^{d\,x^2\,1{}\mathrm{i}}\,{\mathrm{e}}^{c\,1{}\mathrm{i}}\,2{}\mathrm{i}-\frac{2\,b\,x\,\left(a\,1{}\mathrm{i}+b\,{\mathrm{e}}^{d\,x^2\,1{}\mathrm{i}}\,{\mathrm{e}}^{c\,1{}\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{2\,a\,d\,\sqrt{a+b}\,\sqrt{a-b}}+\frac{b\,\ln\left(b\,x\,{\mathrm{e}}^{d\,x^2\,1{}\mathrm{i}}\,{\mathrm{e}}^{c\,1{}\mathrm{i}}\,2{}\mathrm{i}+\frac{2\,b\,x\,\left(a\,1{}\mathrm{i}+b\,{\mathrm{e}}^{d\,x^2\,1{}\mathrm{i}}\,{\mathrm{e}}^{c\,1{}\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{2\,a\,d\,\sqrt{a+b}\,\sqrt{a-b}}$$

```
[In] int(x/(a + b/sin(c + d*x^2)),x)
```

```
[Out] x^2/(2*a) - (b*log(b*x*exp(d*x^2*1i)*exp(c*1i)*2i - (2*b*x*(a*1i + b*exp(d*
x^2*1i)*exp(c*1i)))/((a + b)^(1/2)*(a - b)^(1/2))))/(2*a*d*(a + b)^(1/2)*(a
 - b)^(1/2)) + (b*log(b*x*exp(d*x^2*1i)*exp(c*1i)*2i + (2*b*x*(a*1i + b*exp
(d*x^2*1i)*exp(c*1i)))/((a + b)^(1/2)*(a - b)^(1/2))))/(2*a*d*(a + b)^(1/2)
*(a - b)^(1/2))
```

3.21 $\int \frac{1}{x(a+b\csc(c+dx^2))} dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)} dx = \text{Int}\left(\frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}, x\right)$$

```
[Out] Unintegrable(1/x/(a+b*csc(d*x^2+c)),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)} dx = \int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)} dx$$

```
[In] Int[1/(x*(a + b*Csc[c + d*x^2])),x]
[Out] Defer[Int][1/(x*(a + b*Csc[c + d*x^2])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)} dx$$

Mathematica [N/A]

Not integrable

Time = 3.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*Csc[c + d*x^2])),x]
[Out] Integrate[1/(x*(a + b*Csc[c + d*x^2])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\csc\left(d\,x^2+c\right)\right)}dx$$

```
[In] int(1/x/(a+b*csc(d*x^2+c)),x)
[Out] int(1/x/(a+b*csc(d*x^2+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{(b\csc\left(dx^2+c\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(d*x^2+c)),x, algorithm="fricas")
[Out] integral(1/(b*x*csc(d*x^2 + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*csc(d*x**2+c)),x)
[Out] Integral(1/(x*(a + b*csc(c + d*x**2))), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 250, normalized size of antiderivative = 13.89

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(d*x^2+c)),x, algorithm="maxima")
```

```
[Out] -(2*a*b*integrate((2*b*cos(d*x^2 + c)^2 + a*cos(d*x^2 + c)*sin(2*d*x^2 + 2*
c) - a*cos(2*d*x^2 + 2*c)*sin(d*x^2 + c) + 2*b*sin(d*x^2 + c)^2 + a*sin(d*x
^2 + c))/(a^3*x*cos(2*d*x^2 + 2*c)^2 + 4*a*b^2*x*cos(d*x^2 + c)^2 + 4*a^2*b
*x*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + a^3*x*sin(2*d*x^2 + 2*c)^2 + 4*a*b^2
*x*sin(d*x^2 + c)^2 + 4*a^2*b*x*sin(d*x^2 + c) + a^3*x - 2*(2*a^2*b*x*sin(d
*x^2 + c) + a^3*x)*cos(2*d*x^2 + 2*c)), x) - log(x))/a
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(d*x^2+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*csc(d*x^2 + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 20.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)}\,dx=\int \frac{1}{x\left(a+\frac{b}{\sin\left(d\,x^2+c\right)}\right)}\,dx$$

```
[In] int(1/(x*(a + b/sin(c + d*x^2))),x)
```

```
[Out] int(1/(x*(a + b/sin(c + d*x^2))), x)
```

3.22 $\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=-\frac{a}{x}+b\text{Int}\left(\frac{\csc\left(c+dx^2\right)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(csc(d*x^2+c)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx$$

```
[In] Int[(a + b*Csc[c + d*x^2])/x^2,x]
```

```
[Out] -(a/x) + b*Defer[Int][Csc[c + d*x^2]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int\left(\frac{a}{x^2}+\frac{b\csc\left(c+dx^2\right)}{x^2}\right)\,dx\\&=-\frac{a}{x}+b\int\frac{\csc\left(c+dx^2\right)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*x^2])/x^2,x]
[Out] Integrate[(a + b*Csc[c + d*x^2])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(d\,x^{2}+c\right)}{x^{2}}dx$$

```
[In] int((a+b*csc(d*x^2+c))/x^2,x)
[Out] int((a+b*csc(d*x^2+c))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=\int \frac{b\csc\left(dx^2+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="fricas")
[Out] integral((b*csc(d*x^2 + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx=\int \frac{a+b\csc\left(c+dx^2\right)}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x**2+c))/x**2,x)
[Out] Integral((a + b*csc(c + d*x**2))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 7.88

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = \int \frac{b\csc(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="maxima")
```

```
[Out] b*(integrate(sin(d*x^2 + c)/(x^2*cos(d*x^2 + c)^2 + x^2*sin(d*x^2 + c)^2 +
2*x^2*cos(d*x^2 + c) + x^2), x) + integrate(sin(d*x^2 + c)/(x^2*cos(d*x^2 +
 c)^2 + x^2*sin(d*x^2 + c)^2 - 2*x^2*cos(d*x^2 + c) + x^2), x)) - a/x
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = \int \frac{b\csc(dx^2+c)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csc(d*x^2+c))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^2 + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{a+b\csc(c+dx^2)}{x^2}\,dx = \int \frac{a+\frac{b}{\sin(d\,x^2+c)}}{x^2}\,dx$$

```
[In] int((a + b/sin(c + d*x^2))/x^2,x)
```

```
[Out] int((a + b/sin(c + d*x^2))/x^2, x)
```

3.23 $\int \frac{x^5}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1124

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = -\frac{ib^2x^4}{2a^2(a^2-b^2)d}+\frac{x^6}{6a^2}+\frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}+\frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}-\frac{ib^3x^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}+\frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}+\frac{ib^3x^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}-\frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}-\frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}-\frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}-\frac{b^3x^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}+\frac{2bx^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}+\frac{b^3x^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}-\frac{2bx^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}-\frac{ib^3\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}+\frac{2ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}+\frac{ib^3\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}-\frac{2ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}-\frac{b^2x^4\cos(c+dx^2)}{2a(a^2-b^2)d(b+a\sin(c+dx^2))}$$

```
[Out] -I*b^2*polylog(2,-a*exp(I*(d*x^2+c))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d
^3+1/6*x^6/a^2+b^2*x^2*ln(1+a*exp(I*(d*x^2+c))/(I*b-(a^2-b^2)^(1/2)))/a^2/(
```

```
a^2-b^2)/d^2+b^2*x^2*ln(1+a*exp(I*(d*x^2+c))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^
2-b^2)/d^2-2*I*b*polylog(3,I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/d
^3/(-a^2+b^2)^(1/2)+1/2*I*b^3*x^4*ln(1-I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(
1/2)))/a^2/(-a^2+b^2)^(3/2)/d-1/2*I*b^2*x^4/a^2/(a^2-b^2)/d+I*b*x^4*ln(1-I*
a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-b^3*x^2*pol
ylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+
b^3*x^2*polylog(2,I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)
^(3/2)/d^2-I*b^2*polylog(2,-a*exp(I*(d*x^2+c))/(I*b-(a^2-b^2)^(1/2)))/a^2/(
a^2-b^2)/d^3+I*b^3*polylog(3,I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2
/(-a^2+b^2)^(3/2)/d^3-1/2*b^2*x^4*cos(d*x^2+c)/a/(a^2-b^2)/d/(b+a*sin(d*x^2
+c))-I*b^3*polylog(3,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b
^2)^(3/2)/d^3-I*b*x^4*ln(1-I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/d
/(-a^2+b^2)^(1/2)+2*b*x^2*polylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2
)))/a^2/d^2/(-a^2+b^2)^(1/2)-2*b*x^2*polylog(2,I*a*exp(I*(d*x^2+c))/(b+(-a^
2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+2*I*b*polylog(3,I*a*exp(I*(d*x^2+c)
)/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-1/2*I*b^3*x^4*ln(1-I*a*exp
(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d
```

Rubi [A] (verified)

Time = 2.81 (sec) , antiderivative size = 1124, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules

used = {4290, 4276, 3405, 3404, 2296, 2221, 2611, 2320, 6724, 4617, 2317, 2438}

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = \frac{x^6}{6a^2} + \frac{ib\log\left(1-\frac{iae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)x^4}{a^2\sqrt{b^2-a^2}d} - \frac{ib^3\log\left(1-\frac{iae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)x^4}{2a^2(b^2-a^2)^{3/2}d} - \frac{ib\log\left(1-\frac{iae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)x^4}{a^2\sqrt{b^2-a^2}d} + \frac{ib^3\log\left(1-\frac{iae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)x^4}{2a^2(b^2-a^2)^{3/2}d} - \frac{ib^2x^4}{2a^2(a^2-b^2)d} - \frac{b^2\cos(dx^2+c)x^4}{2a(a^2-b^2)d(b+a\sin(dx^2+c))} + \frac{b^2\log\left(\frac{e^{i(dx^2+c)}a}{ib-\sqrt{a^2-b^2}}+1\right)x^2}{a^2(a^2-b^2)d^2} + \frac{b^2\log\left(\frac{e^{i(dx^2+c)}a}{ib+\sqrt{a^2-b^2}}+1\right)x^2}{a^2(a^2-b^2)d^2} + \frac{2b\,\mathrm{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)x^2}{a^2\sqrt{b^2-a^2}d^2} - \frac{b^3\,\mathrm{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)x^2}{a^2(b^2-a^2)^{3/2}d^2} - \frac{2b\,\mathrm{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)x^2}{a^2\sqrt{b^2-a^2}d^2} + \frac{b^3\,\mathrm{PolyLog}\left(2,\frac{iae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)x^2}{a^2(b^2-a^2)^{3/2}d^2} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(dx^2+c)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} + \frac{2ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2\sqrt{b^2-a^2}d^3} - \frac{ib^3\,\mathrm{PolyLog}\left(3,\frac{iae^{i(dx^2+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2(b^2-a^2)^{3/2}d^3} - \frac{2ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2\sqrt{b^2-a^2}d^3} + \frac{ib^3\,\mathrm{PolyLog}\left(3,\frac{iae^{i(dx^2+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2(b^2-a^2)^{3/2}d^3}$$

```
[In] Int[x^5/(a + b*Csc[c + d*x^2])^2,x]
[Out] ((-1/2*I)*b^2*x^4)/(a^2*(a^2 - b^2)*d) + x^6/(6*a^2) + (b^2*x^2*Log[1 + (a*
E^(I*(c + d*x^2)))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) + (b^2*x
```

```
^2*Log[1 + (a*E^(I*(c + d*x^2)))/(I*b + Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)
*d^2) - ((I/2)*b^3*x^4*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2
])])/(a^2*(-a^2 + b^2)^(3/2)*d) + (I*b*x^4*Log[1 - (I*a*E^(I*(c + d*x^2)))/
(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + ((I/2)*b^3*x^4*Log[1 -
(I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d)
 - (I*b*x^4*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*S
qrt[-a^2 + b^2]*d) - (I*b^2*PolyLog[2, -((a*E^(I*(c + d*x^2)))/(I*b - Sqrt[
a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (I*b^2*PolyLog[2, -((a*E^(I*(c + d*x
^2)))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (b^3*x^2*PolyLog[2
, (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*
d^2) + (2*b*x^2*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])
/(a^2*Sqrt[-a^2 + b^2]*d^2) + (b^3*x^2*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/(
b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (2*b*x^2*PolyLog[2,
(I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2)
 - (I*b^3*PolyLog[3, (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])/(a^2*
(-a^2 + b^2)^(3/2)*d^3) + ((2*I)*b*PolyLog[3, (I*a*E^(I*(c + d*x^2)))/(b -
Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (I*b^3*PolyLog[3, (I*a*E^(
I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((2
*I)*b*PolyLog[3, (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt
[-a^2 + b^2]*d^3) - (b^2*x^4*Cos[c + d*x^2])/(2*a*(a^2 - b^2)*d*(b + a*Sin[
c + d*x^2]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
```

```
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
```

```
1)/n], 0] && IntegerQ[p]
```

Rule 4617

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2
] + b*E^(I*(c + d*x)))), x], x] + Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/
(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(a+b\csc(c+dx))^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\sin(c+dx))^2}-\frac{2bx^2}{a^2(b+a\sin(c+dx))}\right)\,dx, x, x^2\right)\\
&= \frac{x^6}{6a^2}-\frac{b\text{Subst}\left(\int \frac{x^2}{b+a\sin(c+dx)}\,dx, x, x^2\right)}{a^2}+\frac{b^2\text{Subst}\left(\int \frac{x^2}{(b+a\sin(c+dx))^2}\,dx, x, x^2\right)}{2a^2}\\
&= \frac{x^6}{6a^2}-\frac{b^2x^4\cos\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\sin\left(c+dx^2\right)\right)}\\
&\quad-\frac{(2b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, x^2\right)}{a^2}\\
&\quad-\frac{b^3\text{Subst}\left(\int \frac{x^2}{b+a\sin(c+dx)}\,dx, x, x^2\right)}{2a^2\left(a^2-b^2\right)}+\frac{b^2\text{Subst}\left(\int \frac{x\cos(c+dx)}{b+a\sin(c+dx)}\,dx, x, x^2\right)}{a\left(a^2-b^2\right)d}
\end{aligned}$$

$$= -\frac{ib^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} - \frac{b^2x^4\cos(c+dx^2)}{2a(a^2-b^2)d(b+a\sin(c+dx^2))}$$
$$- \frac{b^3\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, x^2\right)}{a^2(a^2-b^2)}$$
$$+ \frac{(2ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}}$$
$$- \frac{(2ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}}$$
$$+ \frac{(ib^2)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib-\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, x^2\right)}{a(a^2-b^2)d}$$
$$+ \frac{(ib^2)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, x^2\right)}{a(a^2-b^2)d}$$

$$= -\frac{ib^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{b^2x^4\cos(c+dx^2)}{2a(a^2-b^2)d(b+a\sin(c+dx^2))}$$
$$- \frac{(ib^3)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{a(-a^2+b^2)^{3/2}}$$
$$+ \frac{(ib^3)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{a(-a^2+b^2)^{3/2}}$$
$$- \frac{b^2\text{Subst}\left(\int \log\left(1+\frac{ae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx, x, x^2\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{b^2\text{Subst}\left(\int \log\left(1+\frac{ae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx, x, x^2\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{(2ib)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+ \frac{(2ib)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$= -\frac{ib^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$

$$+ \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{ib^3x^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$

$$+ \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d}$$

$$- \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2bx^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$- \frac{2bx^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{b^2x^4\cos(c+dx^2)}{2a(a^2-b^2)d(b+a\sin(c+dx^2))}$$

$$+ \frac{(ib^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{ax}{ib-\sqrt{a^2-b^2}}\right)}{x}dx, x, e^{i(c+dx^2)}\right)}{a^2(a^2-b^2)d^3}$$

$$+ \frac{(ib^2)\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{ax}{ib+\sqrt{a^2-b^2}}\right)}{x}dx, x, e^{i(c+dx^2)}\right)}{a^2(a^2-b^2)d^3}$$

$$- \frac{(2b)\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$+ \frac{(2b)\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$+ \frac{(ib^3)\,\mathrm{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$- \frac{(ib^3)\,\mathrm{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^2\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$
\begin{aligned}
&= -\frac{ib^2x^4}{2a^2\left(a^2-b^2\right)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2} \\
&+ \frac{b^2x^2\log\left(1+\frac{ae^{i\left(c+dx^2\right)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2} - \frac{ib^3x^4\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} \\
&+ \frac{ibx^4\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^4\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} \\
&- \frac{ibx^4\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i\left(c+dx^2\right)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3} \\
&- \frac{ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i\left(c+dx^2\right)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3} - \frac{b^3x^2\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2} \\
&+ \frac{2bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^3x^2\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2} \\
&- \frac{2bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{b^2x^4\cos\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\sin\left(c+dx^2\right)\right)} \\
&+ \frac{(2ib)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i\left(c+dx^2\right)}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&- \frac{(2ib)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i\left(c+dx^2\right)}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&+ \frac{b^3\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2} \\
&- \frac{b^3\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^2\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ib^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} \\
&+ \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{ib^3x^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} \\
&+ \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} \\
&- \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} \\
&- \frac{ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{b^3x^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
&+ \frac{2bx^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^3x^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
&- \frac{2bx^2\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{2ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&- \frac{2ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} - \frac{b^2x^4\cos(c+dx^2)}{2a(a^2-b^2)d(b+a\sin(c+dx^2))} \\
&- \frac{(ib^3)\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^2)}\right)}{a^2(-a^2+b^2)^{3/2}d^3} \\
&+ \frac{(ib^3)\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^2)}\right)}{a^2(-a^2+b^2)^{3/2}d^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{ib^2x^4}{2a^2(a^2-b^2)d} + \frac{x^6}{6a^2} + \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} \\
&+ \frac{b^2x^2\log\left(1+\frac{ae^{i(c+dx^2)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{ib^3x^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} \\
&+ \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} \\
&- \frac{ibx^4\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{ib^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} \\
&- \frac{ib^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+dx^2)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{b^3x^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
&+ \frac{2bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^3x^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
&- \frac{2bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{ib^3\,\text{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3} \\
&+ \frac{2ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{ib^3\,\text{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3} \\
&- \frac{2ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} - \frac{b^2x^4\cos(c+dx^2)}{2a(a^2-b^2)d(b+a\sin(c+dx^2))}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 10.60 (sec) , antiderivative size = 2033, normalized size of antiderivative = 1.81

$$
\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = \text{Result too large to show}
$$

```
[In] Integrate[x^5/(a + b*Csc[c + d*x^2])^2,x]
```

```
[Out] (Csc[c/2]*Csc[c + d*x^2]^2*Sec[c/2]*(-(b^3*x^4*Cos[c]) - a*b^2*x^4*Sin[d*x^
2])*(b + a*Sin[c + d*x^2]))/(4*a^2*(-a + b)*(a + b)*d*(a + b*Csc[c + d*x^2]
)^2) + (x^6*Csc[c + d*x^2]^2*(b + a*Sin[c + d*x^2])^2)/(6*a^2*(a + b*Csc[c
+ d*x^2])^2) + (b*E^((2*I)*c)*Csc[c + d*x^2]^2*((-2*I)*b*d^2*E^((2*I)*c)*Sq
```

```
rt[(a^2 - b^2)*E^((2*I)*c)]*x^4 - 2*b*d*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*L
og[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)]
)] + 2*b*d*E^((2*I)*c)*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*Log[1 + (a*E^(I*(2
*c + d*x^2)))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 2*a^2*d^2*E^
(I*c)*x^4*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E
^((2*I)*c)])] - b^2*d^2*E^(I*c)*x^4*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E^
(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 2*a^2*d^2*E^((3*I)*c)*x^4*Log[1 +
 (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + b
^2*d^2*E^((3*I)*c)*x^4*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) - Sqrt[
(a^2 - b^2)*E^((2*I)*c)])] - 2*b*d*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*Log[1
+ (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] +
2*b*d*E^((2*I)*c)*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*Log[1 + (a*E^(I*(2*c +
d*x^2)))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 2*a^2*d^2*E^(I*c)
*x^4*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*
I)*c)])] + b^2*d^2*E^(I*c)*x^4*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c)
 + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 2*a^2*d^2*E^((3*I)*c)*x^4*Log[1 + (a*E
^(I*(2*c + d*x^2)))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - b^2*d^
2*E^((3*I)*c)*x^4*Log[1 + (a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) + Sqrt[(a^2
- b^2)*E^((2*I)*c)])] - (2*I)*(-1 + E^((2*I)*c))*(b*Sqrt[(a^2 - b^2)*E^((2*
I)*c)] - 2*a^2*d*E^(I*c)*x^2 + b^2*d*E^(I*c)*x^2)*PolyLog[2, (I*a*E^(I*(2*c
 + d*x^2)))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (2*I)*(-1 + E^
((2*I)*c))*(-(b*Sqrt[(a^2 - b^2)*E^((2*I)*c)]) - 2*a^2*d*E^(I*c)*x^2 + b^2*
d*E^(I*c)*x^2)*PolyLog[2, -((a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) + Sqrt[(a^
2 - b^2)*E^((2*I)*c)]))] + 4*a^2*E^(I*c)*PolyLog[3, (I*a*E^(I*(2*c + d*x^2)
))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 2*b^2*E^(I*c)*PolyLog[3
, (I*a*E^(I*(2*c + d*x^2)))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])]
- 4*a^2*E^((3*I)*c)*PolyLog[3, (I*a*E^(I*(2*c + d*x^2)))/(b*E^(I*c) + I*Sqr
t[(a^2 - b^2)*E^((2*I)*c)])] + 2*b^2*E^((3*I)*c)*PolyLog[3, (I*a*E^(I*(2*c
+ d*x^2)))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 4*a^2*E^(I*c)*P
olyLog[3, -((a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I
)*c)]))] + 2*b^2*E^(I*c)*PolyLog[3, -((a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c)
+ Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + 4*a^2*E^((3*I)*c)*PolyLog[3, -((a*E^(I
*(2*c + d*x^2)))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - 2*b^2*E^
((3*I)*c)*PolyLog[3, -((a*E^(I*(2*c + d*x^2)))/(I*b*E^(I*c) + Sqrt[(a^2 - b
^2)*E^((2*I)*c)]))])*(b + a*Sin[c + d*x^2])^2)/(2*a^2*d^3*((a^2 - b^2)*E^((
2*I)*c))^(3/2)*(-1 + E^((2*I)*c))*(a + b*Csc[c + d*x^2])^2)
```

Maple [F]

$$\int \frac{x^5}{(a + b\csc(d\,x^2 + c))^2}dx$$

[In] int(x^5/(a+b*csc(d*x^2+c))^2,x)

[Out] int(x^5/(a+b*csc(d*x^2+c))^2,x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3032 vs. 2(966) = 1932.

Time = 0.47 (sec) , antiderivative size = 3032, normalized size of antiderivative = 2.70

$$\int \frac{x^5}{(a + b\csc(c + dx^2))^2}\,dx = \text{Too large to display}$$

[In] integrate(x^5/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")

```
[Out] 1/12*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d^3*x^6*sin(d*x^2 + c) + 2*(a^4*b - 2*a^2
*b^3 + b^5)*d^3*x^6 - 6*(a^3*b^2 - a*b^4)*d^2*x^4*cos(d*x^2 + c) + 6*(2*a^3
*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*po
lylog(3, -(I*b*cos(d*x^2 + c) + b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*
sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2))/a) - 6*(2*a^3*b^2 - a*b^4 + (2*a^4*b
 - a^2*b^3)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*polylog(3, -(I*b*cos(d*x^
2 + c) + b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a
^2 - b^2)/a^2))/a) + 6*(2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*sin(d*x^2 +
 c))*sqrt((a^2 - b^2)/a^2)*polylog(3, -(-I*b*cos(d*x^2 + c) + b*sin(d*x^2 +
 c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2))/a) - 6
*(2*a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/
a^2)*polylog(3, -(-I*b*cos(d*x^2 + c) + b*sin(d*x^2 + c) - (a*cos(d*x^2 + c
) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2))/a) - 6*(I*a^2*b^3 - I*b^5 +
(I*a^3*b^2 - I*a*b^4)*sin(d*x^2 + c) + (-I*(2*a^4*b - a^2*b^3)*d*x^2*sin(d*
x^2 + c) - I*(2*a^3*b^2 - a*b^4)*d*x^2)*sqrt((a^2 - b^2)/a^2))*dilog((I*b*c
os(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*
sqrt((a^2 - b^2)/a^2) - a)/a + 1) - 6*(I*a^2*b^3 - I*b^5 + (I*a^3*b^2 - I*a
*b^4)*sin(d*x^2 + c) + (I*(2*a^4*b - a^2*b^3)*d*x^2*sin(d*x^2 + c) + I*(2*a
^3*b^2 - a*b^4)*d*x^2)*sqrt((a^2 - b^2)/a^2))*dilog((I*b*cos(d*x^2 + c) - b
*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/
a^2) - a)/a + 1) - 6*(-I*a^2*b^3 + I*b^5 + (-I*a^3*b^2 + I*a*b^4)*sin(d*x^2
 + c) + (I*(2*a^4*b - a^2*b^3)*d*x^2*sin(d*x^2 + c) + I*(2*a^3*b^2 - a*b^4)
*d*x^2)*sqrt((a^2 - b^2)/a^2))*dilog((-I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c
) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a +
1) - 6*(-I*a^2*b^3 + I*b^5 + (-I*a^3*b^2 + I*a*b^4)*sin(d*x^2 + c) + (-I*(2
```

```
*a^4*b - a^2*b^3)*d*x^2*sin(d*x^2 + c) - I*(2*a^3*b^2 - a*b^4)*d*x^2)*sqrt(
(a^2 - b^2)/a^2))*dilog((-I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*
x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) - 3*(2*(a^
3*b^2 - a*b^4)*c*sin(d*x^2 + c) + 2*(a^2*b^3 - b^5)*c - ((2*a^4*b - a^2*b^3
)*c^2*sin(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*c^2)*sqrt((a^2 - b^2)/a^2))*log(
2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I
*b) - 3*(2*(a^3*b^2 - a*b^4)*c*sin(d*x^2 + c) + 2*(a^2*b^3 - b^5)*c - ((2*a
^4*b - a^2*b^3)*c^2*sin(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*c^2)*sqrt((a^2 - b
^2)/a^2))*log(2*a*cos(d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b
^2)/a^2) - 2*I*b) - 3*(2*(a^3*b^2 - a*b^4)*c*sin(d*x^2 + c) + 2*(a^2*b^3 -
b^5)*c + ((2*a^4*b - a^2*b^3)*c^2*sin(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*c^2)
*sqrt((a^2 - b^2)/a^2))*log(-2*a*cos(d*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*
a*sqrt((a^2 - b^2)/a^2) + 2*I*b) - 3*(2*(a^3*b^2 - a*b^4)*c*sin(d*x^2 + c)
+ 2*(a^2*b^3 - b^5)*c + ((2*a^4*b - a^2*b^3)*c^2*sin(d*x^2 + c) + (2*a^3*b^
2 - a*b^4)*c^2)*sqrt((a^2 - b^2)/a^2))*log(-2*a*cos(d*x^2 + c) - 2*I*a*sin(
d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b) + 3*(2*(a^2*b^3 - b^5)*d*x^
2 + 2*(a^2*b^3 - b^5)*c + 2*((a^3*b^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)
*sin(d*x^2 + c) - ((2*a^3*b^2 - a*b^4)*d^2*x^4 - (2*a^3*b^2 - a*b^4)*c^2 +
((2*a^4*b - a^2*b^3)*d^2*x^4 - (2*a^4*b - a^2*b^3)*c^2)*sin(d*x^2 + c))*sqr
t((a^2 - b^2)/a^2))*log(-(I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*
x^2 + c) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) + 3*(2*(a^2*b^
3 - b^5)*d*x^2 + 2*(a^2*b^3 - b^5)*c + 2*((a^3*b^2 - a*b^4)*d*x^2 + (a^3*b^
2 - a*b^4)*c)*sin(d*x^2 + c) + ((2*a^3*b^2 - a*b^4)*d^2*x^4 - (2*a^3*b^2 -
a*b^4)*c^2 + ((2*a^4*b - a^2*b^3)*d^2*x^4 - (2*a^4*b - a^2*b^3)*c^2)*sin(d*
x^2 + c))*sqrt((a^2 - b^2)/a^2))*log(-(I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c
) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) +
 3*(2*(a^2*b^3 - b^5)*d*x^2 + 2*(a^2*b^3 - b^5)*c + 2*((a^3*b^2 - a*b^4)*d*
x^2 + (a^3*b^2 - a*b^4)*c)*sin(d*x^2 + c) - ((2*a^3*b^2 - a*b^4)*d^2*x^4 -
(2*a^3*b^2 - a*b^4)*c^2 + ((2*a^4*b - a^2*b^3)*d^2*x^4 - (2*a^4*b - a^2*b^3
)*c^2)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2))*log(-(-I*b*cos(d*x^2 + c) - b
*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/
a^2) - a)/a) + 3*(2*(a^2*b^3 - b^5)*d*x^2 + 2*(a^2*b^3 - b^5)*c + 2*((a^3*b
^2 - a*b^4)*d*x^2 + (a^3*b^2 - a*b^4)*c)*sin(d*x^2 + c) + ((2*a^3*b^2 - a*b
^4)*d^2*x^4 - (2*a^3*b^2 - a*b^4)*c^2 + ((2*a^4*b - a^2*b^3)*d^2*x^4 - (2*a
^4*b - a^2*b^3)*c^2)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2))*log(-(-I*b*cos(
d*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqr
t((a^2 - b^2)/a^2) - a)/a))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d^3*sin(d*x^2 + c)
 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^3)
```

Sympy [F]

$$\int \frac{x^5}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{x^5}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] integrate(x**5/(a+b*csc(d*x**2+c))**2,x)
```

```
[Out] Integral(x**5/(a + b*csc(c + d*x**2))**2, x)
```

Maxima [F]

$$\int \frac{x^5}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{x^5}{\left(b\csc\left(dx^2+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^5/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*((a^4 - a^2*b^2)*d*x^6*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^6*c
os(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^6*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 -
 b^4)*d*x^6*sin(d*x^2 + c)^2 - 6*a*b^3*x^4*cos(d*x^2 + c) + 4*(a^3*b - a*b^
3)*d*x^6*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^6 - 2*(3*a*b^3*x^4*cos(d*x^2
+ c) + 2*(a^3*b - a*b^3)*d*x^6*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^6)*cos(
2*d*x^2 + 2*c) - 6*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a
^2*b^4)*d*cos(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*sin(2*d*x
^2 + 2*c) + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*
d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)
*d - 2*(2*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x
^2 + 2*c))*integrate(2*(2*(2*a^2*b^2 - b^4)*d*x^5*cos(d*x^2 + c)^2 + 2*(2*a
^2*b^2 - b^4)*d*x^5*sin(d*x^2 + c)^2 - 2*a*b^3*x^3*cos(d*x^2 + c) + (2*a^3*
b - a*b^3)*d*x^5*sin(d*x^2 + c) - (2*a*b^3*x^3*cos(d*x^2 + c) + (2*a^3*b -
a*b^3)*d*x^5*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + ((2*a^3*b - a*b^3)*d*x^5*
cos(d*x^2 + c) - 2*a*b^3*x^3*sin(d*x^2 + c) - 2*a^2*b^2*x^3)*sin(2*d*x^2 +
2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*cos
(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (
a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x^2 +
 c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d - 2*(2*(a^
5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c)), x
) + 2*(2*(a^3*b - a*b^3)*d*x^6*cos(d*x^2 + c) - 3*a*b^3*x^4*sin(d*x^2 + c)
- 3*a^2*b^2*x^4)*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^
2 + 4*(a^4*b^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*
x^2 + c)*sin(2*d*x^2 + 2*c) + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a
^4*b^2 - a^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c)
 + (a^6 - a^4*b^2)*d - 2*(2*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4
*b^2)*d)*cos(2*d*x^2 + 2*c))
```

Giac [F]

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^5}{(b\csc(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^5/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^5/(b*csc(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^5}{\left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^5/(a + b/sin(c + d*x^2))^2,x)
[Out] int(x^5/(a + b/sin(c + d*x^2))^2, x)
```

3.24 $\int \frac{x^4}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx = \operatorname{Int}\left(\frac{x^4}{(a+b\csc(c+dx^2))^2},x\right)$$

```
[Out] Unintegrable(x^4/(a+b*csc(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx$$

```
[In] Int[x^4/(a + b*Csc[c + d*x^2])^2,x]
[Out] Defer[Int][x^4/(a + b*Csc[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 12.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx=\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx$$

```
[In] Integrate[x^4/(a + b*Csc[c + d*x^2])^2,x]
[Out] Integrate[x^4/(a + b*Csc[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(a+b\csc(d\,x^2+c))^2}dx$$

```
[In] int(x^4/(a+b*csc(d*x^2+c))^2,x)
[Out] int(x^4/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx=\int \frac{x^4}{(b\csc(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(x^4/(b^2*csc(d*x^2 + c)^2 + 2*a*b*csc(d*x^2 + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx=\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx$$

```
[In] integrate(x**4/(a+b*csc(d*x**2+c))**2,x)
[Out] Integral(x**4/(a + b*csc(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 1286, normalized size of antiderivative = 71.44

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^4}{(b\csc(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/5*((a^4 - a^2*b^2)*d*x^5*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^5*c
os(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^5*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 -
 b^4)*d*x^5*sin(d*x^2 + c)^2 - 5*a*b^3*x^3*cos(d*x^2 + c) + 4*(a^3*b - a*b^
3)*d*x^5*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^5 - (5*a*b^3*x^3*cos(d*x^2 +
c) + 4*(a^3*b - a*b^3)*d*x^5*sin(d*x^2 + c) + 2*(a^4 - a^2*b^2)*d*x^5)*cos(
2*d*x^2 + 2*c) - 5*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a
^2*b^4)*d*cos(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*sin(2*d*x
^2 + 2*c) + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*
d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)
*d - 2*(2*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x
^2 + 2*c))*integrate((4*(2*a^2*b^2 - b^4)*d*x^4*cos(d*x^2 + c)^2 + 4*(2*a^2
*b^2 - b^4)*d*x^4*sin(d*x^2 + c)^2 - 3*a*b^3*x^2*cos(d*x^2 + c) + 2*(2*a^3*
b - a*b^3)*d*x^4*sin(d*x^2 + c) - (3*a*b^3*x^2*cos(d*x^2 + c) + 2*(2*a^3*b
- a*b^3)*d*x^4*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (2*(2*a^3*b - a*b^3)*d*
x^4*cos(d*x^2 + c) - 3*a*b^3*x^2*sin(d*x^2 + c) - 3*a^2*b^2*x^2)*sin(2*d*x^
2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d
*cos(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c)
 + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x
^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d - 2*(2
*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c)
), x) + (4*(a^3*b - a*b^3)*d*x^5*cos(d*x^2 + c) - 5*a*b^3*x^3*sin(d*x^2 + c
) - 5*a^2*b^2*x^3)*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c
)^2 + 4*(a^4*b^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(
d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*
(a^4*b^2 - a^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 +
c) + (a^6 - a^4*b^2)*d - 2*(2*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a
^4*b^2)*d)*cos(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^4}{(b\csc(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^4/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^4/(b*csc(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 21.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^4}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^4}{\left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^4/(a + b/sin(c + d*x^2))^2,x)
[Out] int(x^4/(a + b/sin(c + d*x^2))^2, x)
```

3.25 $\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 616

$$\begin{aligned}\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx &= \frac{x^4}{4a^2} - \frac{ib^3x^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} \\
&\quad + \frac{ibx^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} \\
&\quad - \frac{ibx^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{b^2\log(b+a\sin(c+dx^2))}{2a^2(a^2-b^2)d^2} \\
&\quad - \frac{b^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} + \frac{b\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&\quad + \frac{b^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} - \frac{b\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&\quad - \frac{b^2x^2\cos(c+dx^2)}{2a(a^2-b^2)d(b+a\sin(c+dx^2))}\end{aligned}$$

```
[Out] 1/4*x^4/a^2+1/2*b^2*ln(b+a*sin(d*x^2+c))/a^2/(a^2-b^2)/d^2-1/2*I*b^3*x^2*ln
(1-I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+1/2*I*
b^3*x^2*ln(1-I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2
)/d-1/2*b^3*polylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+
b^2)^(3/2)/d^2+1/2*b^3*polylog(2,I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))
/a^2/(-a^2+b^2)^(3/2)/d^2-1/2*b^2*x^2*cos(d*x^2+c)/a/(a^2-b^2)/d/(b+a*sin(d
```

```
*x^2+c))+I*b*x^2*ln(1-I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^
2+b^2)^(1/2)-I*b*x^2*ln(1-I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2)))/a^2/d/
(-a^2+b^2)^(1/2)+b*polylog(2,I*a*exp(I*(d*x^2+c))/(b-(-a^2+b^2)^(1/2)))/a^2
/d^2/(-a^2+b^2)^(1/2)-b*polylog(2,I*a*exp(I*(d*x^2+c))/(b+(-a^2+b^2)^(1/2))
)/a^2/d^2/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 616, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4290, 4276, 3405, 3404, 2296, 2221, 2317, 2438, 2747, 31}

$$\begin{aligned}\int \frac{x^3}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx = {}& \frac{b\,\text{PolyLog}\left(2,\frac{iae^{i\left(dx^2+c\right)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}}-\frac{b\,\text{PolyLog}\left(2,\frac{iae^{i\left(dx^2+c\right)}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2\sqrt{b^2-a^2}}\\&+\frac{b^2\log\left(a\sin\left(c+dx^2\right)+b\right)}{2a^2d^2\left(a^2-b^2\right)}\\&+\frac{ibx^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d\sqrt{b^2-a^2}}-\frac{ibx^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{\sqrt{b^2-a^2}+b}\right)}{a^2d\sqrt{b^2-a^2}}\\&-\frac{b^2x^2\cos\left(c+dx^2\right)}{2ad\left(a^2-b^2\right)\left(a\sin\left(c+dx^2\right)+b\right)}\\&-\frac{b^3\,\text{PolyLog}\left(2,\frac{iae^{i\left(dx^2+c\right)}}{b-\sqrt{b^2-a^2}}\right)}{2a^2d^2\left(b^2-a^2\right)^{3/2}}\\&+\frac{b^3\,\text{PolyLog}\left(2,\frac{iae^{i\left(dx^2+c\right)}}{b+\sqrt{b^2-a^2}}\right)}{2a^2d^2\left(b^2-a^2\right)^{3/2}}-\frac{ib^3x^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{b^2-a^2}}\right)}{2a^2d\left(b^2-a^2\right)^{3/2}}\\&+\frac{ib^3x^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{\sqrt{b^2-a^2}+b}\right)}{2a^2d\left(b^2-a^2\right)^{3/2}}+\frac{x^4}{4a^2}\end{aligned}$$

```
[In] Int[x^3/(a + b*Csc[c + d*x^2])^2,x]
```

```
[Out] x^4/(4*a^2) - ((I/2)*b^3*x^2*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2
 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + (I*b*x^2*Log[1 - (I*a*E^(I*(c + d*x
^2)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) + ((I/2)*b^3*x^2*Lo
g[1 - (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3
/2)*d) - (I*b*x^2*Log[1 - (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/
(a^2*Sqrt[-a^2 + b^2]*d) + (b^2*Log[b + a*Sin[c + d*x^2]])/(2*a^2*(a^2 - b^
2)*d^2) - (b^3*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/(b - Sqrt[-a^2 + b^2])])/
(2*a^2*(-a^2 + b^2)^(3/2)*d^2) + (b*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/(b -
 Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (b^3*PolyLog[2, (I*a*E^(I
```

```
*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(2*a^2*(-a^2 + b^2)^(3/2)*d^2) - (b
*PolyLog[2, (I*a*E^(I*(c + d*x^2)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2
 + b^2]*d^2) - (b^2*x^2*Cos[c + d*x^2])/(2*a*(a^2 - b^2)*d*(b + a*Sin[c + d
*x^2]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
```

```
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(a+b\csc(c+dx))^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{x}{a^2}+\frac{b^2x}{a^2(b+a\sin(c+dx))^2}-\frac{2bx}{a^2(b+a\sin(c+dx))}\right)\,dx, x, x^2\right)\\
&= \frac{x^4}{4a^2}-\frac{b\text{Subst}\left(\int \frac{x}{b+a\sin(c+dx)}\,dx, x, x^2\right)}{a^2}+\frac{b^2\text{Subst}\left(\int \frac{x}{(b+a\sin(c+dx))^2}\,dx, x, x^2\right)}{2a^2}\\
&= \frac{x^4}{4a^2}-\frac{b^2x^2\cos\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\sin\left(c+dx^2\right)\right)}\\
&\quad -\frac{(2b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, x^2\right)}{a^2}\\
&\quad -\frac{b^3\text{Subst}\left(\int \frac{x}{b+a\sin(c+dx)}\,dx, x, x^2\right)}{2a^2\left(a^2-b^2\right)}+\frac{b^2\text{Subst}\left(\int \frac{\cos(c+dx)}{b+a\sin(c+dx)}\,dx, x, x^2\right)}{2a\left(a^2-b^2\right)d}
\end{aligned}$$

$$= \frac{x^4}{4a^2} - \frac{b^2x^2\cos(c+dx^2)}{2a(a^2-b^2)d(b+a\sin(c+dx^2))}$$
$$- \frac{b^3\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, x^2\right)}{a^2(a^2-b^2)}$$
$$+ \frac{(2ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}}$$
$$- \frac{(2ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{a\sqrt{-a^2+b^2}}$$
$$+ \frac{b^2\text{Subst}\left(\int \frac{1}{b+x}\,dx, x, a\sin(c+dx^2)\right)}{2a^2(a^2-b^2)d^2}$$

$$= \frac{x^4}{4a^2} + \frac{ibx^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{ibx^2\log\left(1-\frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+ \frac{b^2\log(b+a\sin(c+dx^2))}{2a^2(a^2-b^2)d^2} - \frac{b^2x^2\cos(c+dx^2)}{2a(a^2-b^2)d(b+a\sin(c+dx^2))}$$
$$- \frac{(ib^3)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{a(-a^2+b^2)^{3/2}}$$
$$+ \frac{(ib^3)\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^2\right)}{a(-a^2+b^2)^{3/2}}$$
$$- \frac{(ib)\text{Subst}\left(\int \log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+ \frac{(ib)\text{Subst}\left(\int \log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$= \frac{x^4}{4a^2} - \frac{ib^3x^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} + \frac{ibx^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} - \frac{ibx^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{b^2\log\left(b+a\sin\left(c+dx^2\right)\right)}{2a^2\left(a^2-b^2\right)d^2} - \frac{b^2x^2\cos\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\sin\left(c+dx^2\right)\right)} - \frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{2iax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i\left(c+dx^2\right)}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b\text{Subst}\left(\int\frac{\log\left(1-\frac{2iax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i\left(c+dx^2\right)}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{\left(ib^3\right)\text{Subst}\left(\int\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} - \frac{\left(ib^3\right)\text{Subst}\left(\int\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^2\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d}$$

$$= \frac{x^4}{4a^2} - \frac{ib^3x^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} + \frac{ibx^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{ib^3x^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d} - \frac{ibx^2\log\left(1-\frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{b^2\log\left(b+a\sin\left(c+dx^2\right)\right)}{2a^2\left(a^2-b^2\right)d^2} + \frac{b\,\text{PolyLog}\left(2, \frac{iae^{i\left(c+dx^2\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{b\,\text{PolyLog}\left(2, \frac{iae^{i\left(c+dx^2\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{b^2x^2\cos\left(c+dx^2\right)}{2a\left(a^2-b^2\right)d\left(b+a\sin\left(c+dx^2\right)\right)} + \frac{b^3\text{Subst}\left(\int\frac{\log\left(1-\frac{2iax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i\left(c+dx^2\right)}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d^2} - \frac{b^3\text{Subst}\left(\int\frac{\log\left(1-\frac{2iax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i\left(c+dx^2\right)}\right)}{2a^2\left(-a^2+b^2\right)^{3/2}d^2}$$

$$
\begin{aligned}
&= \frac{x^4}{4a^2} - \frac{ib^3x^2 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} + \frac{ibx^2 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&+ \frac{ib^3x^2 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d} - \frac{ibx^2 \log\left(1 - \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} \\
&+ \frac{b^2 \log(b + a\sin(c+dx^2))}{2a^2(a^2-b^2)d^2} - \frac{b^3 \operatorname{PolyLog}\left(2, \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} \\
&+ \frac{b \operatorname{PolyLog}\left(2, \frac{iae^{i(c+dx^2)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{b^3 \operatorname{PolyLog}\left(2, \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{2a^2(-a^2+b^2)^{3/2}d^2} \\
&- \frac{b \operatorname{PolyLog}\left(2, \frac{iae^{i(c+dx^2)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{b^2x^2\cos(c+dx^2)}{2a(a^2-b^2)d(b+a\sin(c+dx^2))}
\end{aligned}
$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2566 vs. $2(616) = 1232$.

Time = 16.93 (sec) , antiderivative size = 2566, normalized size of antiderivative = 4.17

$$
\int \frac{x^3}{(a + b\csc(c+dx^2))^2} \, dx = \text{Result too large to show}
$$

```
[In] Integrate[x^3/(a + b*Csc[c + d*x^2])^2,x]
```

```
[Out] ((-(b^2*c*Cos[c + d*x^2]) + b^2*(c + d*x^2)*Cos[c + d*x^2])*Csc[c + d*x^2]^
2*(b + a*Sin[c + d*x^2]))/(2*a*(-a + b)*(a + b)*d^2*(a + b*Csc[c + d*x^2])^
2) + ((-c + d*x^2)*(c + d*x^2)*Csc[c + d*x^2]^2*(b + a*Sin[c + d*x^2])^2)/(
4*a^2*d^2*(a + b*Csc[c + d*x^2])^2) + (Csc[c + d*x^2]^2*(-2*a*b*ArcTanh[(a
+ b*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]] + 2*(a*b + 2*a^2*c - b^2*c)*ArcTan
h[(a + b*Tan[(c + d*x^2)/2])/Sqrt[a^2 - b^2]] + b*Sqrt[a^2 - b^2]*Log[Sec[(
c + d*x^2)/2]^2] - b*Sqrt[a^2 - b^2]*Log[Sec[(c + d*x^2)/2]^2*(b + a*Sin[c
+ d*x^2])] + I*(2*a^2 - b^2)*Log[1 - I*Tan[(c + d*x^2)/2]]*Log[(a - Sqrt[a^
2 - b^2] + b*Tan[(c + d*x^2)/2])/(a - I*b - Sqrt[a^2 - b^2])] - I*(2*a^2 -
b^2)*Log[1 + I*Tan[(c + d*x^2)/2]]*Log[(a - Sqrt[a^2 - b^2] + b*Tan[(c + d*
x^2)/2])/(a + I*b - Sqrt[a^2 - b^2])] - I*(2*a^2 - b^2)*Log[1 - I*Tan[(c +
d*x^2)/2]]*Log[(a + Sqrt[a^2 - b^2] + b*Tan[(c + d*x^2)/2])/(a - I*b + Sqrt
[a^2 - b^2])] + I*(2*a^2 - b^2)*Log[1 + I*Tan[(c + d*x^2)/2]]*Log[(a + Sqrt
[a^2 - b^2] + b*Tan[(c + d*x^2)/2])/(a + I*b + Sqrt[a^2 - b^2])] - I*(2*a^2
 - b^2)*PolyLog[2, (b*(1 + I*Tan[(c + d*x^2)/2]))/((-I)*a + b + I*Sqrt[a^2
- b^2])] + I*(2*a^2 - b^2)*PolyLog[2, (b*(1 + I*Tan[(c + d*x^2)/2]))/(b - I
```

```
*(a + Sqrt[a^2 - b^2]))] - I*(2*a^2 - b^2)*PolyLog[2, -((b*(I + Tan[(c + d*
x^2)/2]))/(a - I*b + Sqrt[a^2 - b^2]))] + I*(2*a^2 - b^2)*PolyLog[2, (b*(I
+ Tan[(c + d*x^2)/2]))/(-a + I*b + Sqrt[a^2 - b^2])])*(b + a*Sin[c + d*x^2]
)^2*((2*b*c)/((a^2 - b^2)*d*(b + a*Sin[c + d*x^2])) - (b^3*c)/(a^2*(a^2 - b
^2)*d*(b + a*Sin[c + d*x^2])) - (2*b*(c + d*x^2))/((a^2 - b^2)*d*(b + a*Sin
[c + d*x^2])) + (b^3*(c + d*x^2))/(a^2*(a^2 - b^2)*d*(b + a*Sin[c + d*x^2])
) + (b^2*Cos[c + d*x^2])/(a*(a^2 - b^2)*d*(b + a*Sin[c + d*x^2]))))/(2*d*(a
 + b*Csc[c + d*x^2])^2*(((2*a^2 - b^2)*Log[(a - Sqrt[a^2 - b^2] + b*Tan[(c
+ d*x^2)/2])/(a - I*b - Sqrt[a^2 - b^2])]*Sec[(c + d*x^2)/2]^2)/(2*(1 - I*T
an[(c + d*x^2)/2])) - ((2*a^2 - b^2)*Log[(a + Sqrt[a^2 - b^2] + b*Tan[(c +
d*x^2)/2])/(a - I*b + Sqrt[a^2 - b^2])]*Sec[(c + d*x^2)/2]^2)/(2*(1 - I*Tan
[(c + d*x^2)/2])) - ((2*a^2 - b^2)*Log[1 - (b*(1 + I*Tan[(c + d*x^2)/2]))/(
(-I)*a + b + I*Sqrt[a^2 - b^2])]*Sec[(c + d*x^2)/2]^2)/(2*(1 + I*Tan[(c + d
*x^2)/2])) + ((2*a^2 - b^2)*Log[1 - (b*(1 + I*Tan[(c + d*x^2)/2]))/(b - I*(
a + Sqrt[a^2 - b^2]))]*Sec[(c + d*x^2)/2]^2)/(2*(1 + I*Tan[(c + d*x^2)/2]))
 + ((2*a^2 - b^2)*Log[(a - Sqrt[a^2 - b^2] + b*Tan[(c + d*x^2)/2])/(a + I*b
 - Sqrt[a^2 - b^2])]*Sec[(c + d*x^2)/2]^2)/(2*(1 + I*Tan[(c + d*x^2)/2])) -
 ((2*a^2 - b^2)*Log[(a + Sqrt[a^2 - b^2] + b*Tan[(c + d*x^2)/2])/(a + I*b +
 Sqrt[a^2 - b^2])]*Sec[(c + d*x^2)/2]^2)/(2*(1 + I*Tan[(c + d*x^2)/2])) + b
*Sqrt[a^2 - b^2]*Tan[(c + d*x^2)/2] + ((I/2)*(2*a^2 - b^2)*Log[1 + (b*(I +
Tan[(c + d*x^2)/2]))/(a - I*b + Sqrt[a^2 - b^2])]*Sec[(c + d*x^2)/2]^2)/(I
+ Tan[(c + d*x^2)/2]) - ((I/2)*(2*a^2 - b^2)*Log[1 - (b*(I + Tan[(c + d*x^2
)/2]))/(-a + I*b + Sqrt[a^2 - b^2])]*Sec[(c + d*x^2)/2]^2)/(I + Tan[(c + d*
x^2)/2]) + ((I/2)*b*(2*a^2 - b^2)*Log[1 - I*Tan[(c + d*x^2)/2]]*Sec[(c + d*
x^2)/2]^2)/(a - Sqrt[a^2 - b^2] + b*Tan[(c + d*x^2)/2]) - ((I/2)*b*(2*a^2 -
 b^2)*Log[1 + I*Tan[(c + d*x^2)/2]]*Sec[(c + d*x^2)/2]^2)/(a - Sqrt[a^2 - b
^2] + b*Tan[(c + d*x^2)/2]) - ((I/2)*b*(2*a^2 - b^2)*Log[1 - I*Tan[(c + d*x
^2)/2]]*Sec[(c + d*x^2)/2]^2)/(a + Sqrt[a^2 - b^2] + b*Tan[(c + d*x^2)/2])
+ ((I/2)*b*(2*a^2 - b^2)*Log[1 + I*Tan[(c + d*x^2)/2]]*Sec[(c + d*x^2)/2]^2
)/(a + Sqrt[a^2 - b^2] + b*Tan[(c + d*x^2)/2]) - (b*Sqrt[a^2 - b^2]*Cos[(c
+ d*x^2)/2]^2*(a*Cos[c + d*x^2]*Sec[(c + d*x^2)/2]^2 + Sec[(c + d*x^2)/2]^2
*(b + a*Sin[c + d*x^2])*Tan[(c + d*x^2)/2]))/(b + a*Sin[c + d*x^2]) - (a*b^
2*Sec[(c + d*x^2)/2]^2)/(Sqrt[a^2 - b^2]*(1 - (a + b*Tan[(c + d*x^2)/2])^2/
(a^2 - b^2))) + (b*(a*b + 2*a^2*c - b^2*c)*Sec[(c + d*x^2)/2]^2)/(Sqrt[a^2
- b^2]*(1 - (a + b*Tan[(c + d*x^2)/2])^2/(a^2 - b^2)))))
```

Maple [F]

$$\int \frac{x^3}{(a + b\csc(d\,x^2 + c))^2}dx$$

[In] int(x^3/(a+b*csc(d*x^2+c))^2,x)

[Out] int(x^3/(a+b*csc(d*x^2+c))^2,x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1906 vs. $2(526) = 1052$.

Time = 0.42 (sec) , antiderivative size = 1906, normalized size of antiderivative = 3.09

$$\int \frac{x^3}{(a + b\csc(c + dx^2))^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((a^5 - 2*a^3*b^2 + a*b^4)*d^2*x^4*sin(d*x^2 + c) + (a^4*b - 2*a^2*b^3
+ b^5)*d^2*x^4 - 2*(a^3*b^2 - a*b^4)*d*x^2*cos(d*x^2 + c) + (2*I*a^3*b^2 -
I*a*b^4 + (2*I*a^4*b - I*a^2*b^3)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*dil
og((I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*x
^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a + 1) + (-2*I*a^3*b^2 + I*a*b^4 + (-2*
I*a^4*b + I*a^2*b^3)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*dilog((I*b*cos(d
*x^2 + c) - b*sin(d*x^2 + c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt
((a^2 - b^2)/a^2) - a)/a + 1) + (-2*I*a^3*b^2 + I*a*b^4 + (-2*I*a^4*b + I*a
^2*b^3)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*dilog((-I*b*cos(d*x^2 + c) -
b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)
/a^2) - a)/a + 1) + (2*I*a^3*b^2 - I*a*b^4 + (2*I*a^4*b - I*a^2*b^3)*sin(d*
x^2 + c))*sqrt((a^2 - b^2)/a^2)*dilog((-I*b*cos(d*x^2 + c) - b*sin(d*x^2 +
c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a +
 1) - ((2*a^3*b^2 - a*b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c + ((2*a^4*b - a^2*
b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*l
og(-(I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) + I*a*sin(d*
x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) + ((2*a^3*b^2 - a*b^4)*d*x^2 + (2*a
^3*b^2 - a*b^4)*c + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*sin
(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*log(-(I*b*cos(d*x^2 + c) - b*sin(d*x^2 +
 c) - (a*cos(d*x^2 + c) + I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a)
 - ((2*a^3*b^2 - a*b^4)*d*x^2 + (2*a^3*b^2 - a*b^4)*c + ((2*a^4*b - a^2*b^3
)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2)*log(
-(-I*b*cos(d*x^2 + c) - b*sin(d*x^2 + c) + (a*cos(d*x^2 + c) - I*a*sin(d*x^
2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a) + ((2*a^3*b^2 - a*b^4)*d*x^2 + (2*a^3
*b^2 - a*b^4)*c + ((2*a^4*b - a^2*b^3)*d*x^2 + (2*a^4*b - a^2*b^3)*c)*sin(d
*x^2 + c))*sqrt((a^2 - b^2)/a^2)*log(-(-I*b*cos(d*x^2 + c) - b*sin(d*x^2 +
c) - (a*cos(d*x^2 + c) - I*a*sin(d*x^2 + c))*sqrt((a^2 - b^2)/a^2) - a)/a)
+ (a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*sin(d*x^2 + c) - ((2*a^4*b - a^2*b^3)*
c*sin(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*c)*sqrt((a^2 - b^2)/a^2))*log(2*a*co
s(d*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) +
(a^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*sin(d*x^2 + c) - ((2*a^4*b - a^2*b^3)*c*
sin(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*c)*sqrt((a^2 - b^2)/a^2))*log(2*a*cos(
d*x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b) + (a
^2*b^3 - b^5 + (a^3*b^2 - a*b^4)*sin(d*x^2 + c) + ((2*a^4*b - a^2*b^3)*c*si
```

```
n(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*c)*sqrt((a^2 - b^2)/a^2))*log(-2*a*cos(d
*x^2 + c) + 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) + (a^
2*b^3 - b^5 + (a^3*b^2 - a*b^4)*sin(d*x^2 + c) + ((2*a^4*b - a^2*b^3)*c*sin
(d*x^2 + c) + (2*a^3*b^2 - a*b^4)*c)*sqrt((a^2 - b^2)/a^2))*log(-2*a*cos(d*
x^2 + c) - 2*I*a*sin(d*x^2 + c) + 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b))/((a^7
 - 2*a^5*b^2 + a^3*b^4)*d^2*sin(d*x^2 + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*
d^2)
```

Sympy [F]

$$\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx=\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx$$

```
[In] integrate(x**3/(a+b*csc(d*x**2+c))**2,x)
[Out] Integral(x**3/(a + b*csc(c + d*x**2))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^3}{(a+b\csc(c+dx^2))^2}\,dx=\int \frac{x^3}{(b\csc(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^3/(b*csc(d*x^2 + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{x^3}{\left(a+\frac{b}{\sin\left(d\,x^2+c\right)}\right)^2}\,dx$$

```
[In] int(x^3/(a + b/sin(c + d*x^2))^2,x)
[Out] int(x^3/(a + b/sin(c + d*x^2))^2, x)
```

3.26 $\int \frac{x^2}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x^2}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx = \text{Int}\left(\frac{x^2}{\left(a+b\csc\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(x^2/(a+b*csc(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx = \int \frac{x^2}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[x^2/(a + b*Csc[c + d*x^2])^2,x]
[Out] Defer[Int][x^2/(a + b*Csc[c + d*x^2])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{x^2}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx=\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx$$

```
[In] Integrate[x^2/(a + b*Csc[c + d*x^2])^2,x]
[Out] Integrate[x^2/(a + b*Csc[c + d*x^2])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a+b\csc(d\,x^2+c))^2}dx$$

```
[In] int(x^2/(a+b*csc(d*x^2+c))^2,x)
[Out] int(x^2/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx=\int \frac{x^2}{(b\csc(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(x^2/(b^2*csc(d*x^2 + c)^2 + 2*a*b*csc(d*x^2 + c) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx=\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx$$

```
[In] integrate(x**2/(a+b*csc(d*x**2+c))**2,x)
[Out] Integral(x**2/(a + b*csc(c + d*x**2))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 1265, normalized size of antiderivative = 70.28

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\csc(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((a^4 - a^2*b^2)*d*x^3*cos(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*x^3*c
os(d*x^2 + c)^2 + (a^4 - a^2*b^2)*d*x^3*sin(2*d*x^2 + 2*c)^2 + 4*(a^2*b^2 -
 b^4)*d*x^3*sin(d*x^2 + c)^2 - 3*a*b^3*x*cos(d*x^2 + c) + 4*(a^3*b - a*b^3)
*d*x^3*sin(d*x^2 + c) + (a^4 - a^2*b^2)*d*x^3 - (3*a*b^3*x*cos(d*x^2 + c) +
 4*(a^3*b - a*b^3)*d*x^3*sin(d*x^2 + c) + 2*(a^4 - a^2*b^2)*d*x^3)*cos(2*d*
x^2 + 2*c) - 3*((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b
^4)*d*cos(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 +
 2*c) + (a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*si
n(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d -
 2*(2*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 +
 2*c))*integrate((4*(2*a^2*b^2 - b^4)*d*x^2*cos(d*x^2 + c)^2 + 4*(2*a^2*b^2
 - b^4)*d*x^2*sin(d*x^2 + c)^2 - a*b^3*cos(d*x^2 + c) + 2*(2*a^3*b - a*b^3)
*d*x^2*sin(d*x^2 + c) - (a*b^3*cos(d*x^2 + c) + 2*(2*a^3*b - a*b^3)*d*x^2*s
in(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (2*(2*a^3*b - a*b^3)*d*x^2*cos(d*x^2 +
c) - a*b^3*sin(d*x^2 + c) - a^2*b^2)*sin(2*d*x^2 + 2*c))/((a^6 - a^4*b^2)*d
*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*cos(d*x^2 + c)^2 + 4*(a^5*b
 - a^3*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) + (a^6 - a^4*b^2)*d*sin(2*d
*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x^2 + c)^2 + 4*(a^5*b - a^3*b
^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d - 2*(2*(a^5*b - a^3*b^3)*d*sin(d*x
^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c)), x) + (4*(a^3*b - a*b^3)*d
*x^3*cos(d*x^2 + c) - 3*a*b^3*x*sin(d*x^2 + c) - 3*a^2*b^2*x)*sin(2*d*x^2 +
 2*c))/((a^6 - a^4*b^2)*d*cos(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*co
s(d*x^2 + c)^2 + 4*(a^5*b - a^3*b^3)*d*cos(d*x^2 + c)*sin(2*d*x^2 + 2*c) +
(a^6 - a^4*b^2)*d*sin(2*d*x^2 + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*sin(d*x^2
+ c)^2 + 4*(a^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d - 2*(2*(a
^5*b - a^3*b^3)*d*sin(d*x^2 + c) + (a^6 - a^4*b^2)*d)*cos(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^2}{(b\csc(dx^2+c)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(x^2/(b*csc(d*x^2 + c) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 26.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x^2}{(a+b\csc(c+dx^2))^2}\,dx = \int \frac{x^2}{\left(a+\frac{b}{\sin(d\,x^2+c)}\right)^2}\,dx$$

```
[In] int(x^2/(a + b/sin(c + d*x^2))^2,x)
[Out] int(x^2/(a + b/sin(c + d*x^2))^2, x)
```

3.27 $\int \frac{x}{\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 120

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx=\frac{x^2}{2a^2}+\frac{b(2a^2-b^2)\,\text{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{3/2}\,d}-\frac{b^2\cot(c+dx^2)}{2a(a^2-b^2)\,d\,(a+b\csc(c+dx^2))}$$

```
[Out] 1/2*x^2/a^2+b*(2*a^2-b^2)*arctanh((a+b*tan(1/2*d*x^2+1/2*c))/(a^2-b^2)^(1/2
))/a^2/(a^2-b^2)^(3/2)/d-1/2*b^2*cot(d*x^2+c)/a/(a^2-b^2)/d/(a+b*csc(d*x^2+
c))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4290, 3870, 4004, 3916, 2739, 632, 212}

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx=\frac{b(2a^2-b^2)\,\text{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d(a^2-b^2)^{3/2}}-\frac{b^2\cot(c+dx^2)}{2ad(a^2-b^2)(a+b\csc(c+dx^2))}+\frac{x^2}{2a^2}$$

```
[In] Int[x/(a + b*Csc[c + d*x^2])^2,x]
```

```
[Out] x^2/(2*a^2) + (b*(2*a^2 - b^2)*ArcTanh[(a + b*Tan[(c + d*x^2)/2])/Sqrt[a^2
- b^2]])/(a^2*(a^2 - b^2)^(3/2)*d) - (b^2*Cot[c + d*x^2])/(2*a*(a^2 - b^2)*
d*(a + b*Csc[c + d*x^2]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
```

```
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(a+b\csc(c+dx))^2}\,dx,x,x^2\right)$$

$$= -\frac{b^2\cot(c+dx^2)}{2a(a^2-b^2)\,d\,(a+b\csc(c+dx^2))} - \frac{\text{Subst}\left(\int \frac{-a^2+b^2+ab\csc(c+dx)}{a+b\csc(c+dx)}\,dx,x,x^2\right)}{2a(a^2-b^2)}$$

$$= \frac{x^2}{2a^2} - \frac{b^2\cot(c+dx^2)}{2a(a^2-b^2)\,d\,(a+b\csc(c+dx^2))} - \frac{(b(2a^2-b^2))\,\text{Subst}\left(\int \frac{\csc(c+dx)}{a+b\csc(c+dx)}\,dx,x,x^2\right)}{2a^2(a^2-b^2)}$$

$$= \frac{x^2}{2a^2} - \frac{b^2\cot(c+dx^2)}{2a(a^2-b^2)\,d\,(a+b\csc(c+dx^2))} - \frac{(2a^2-b^2)\,\text{Subst}\left(\int \frac{1}{1+\frac{a\sin(c+dx)}{b}}\,dx,x,x^2\right)}{2a^2(a^2-b^2)}$$

$$= \frac{x^2}{2a^2} - \frac{b^2\cot(c+dx^2)}{2a(a^2-b^2)\,d\,(a+b\csc(c+dx^2))} - \frac{(2a^2-b^2)\,\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{a^2(a^2-b^2)\,d}$$

$$= \frac{x^2}{2a^2} - \frac{b^2\cot(c+dx^2)}{2a(a^2-b^2)\,d\,(a+b\csc(c+dx^2))} + \frac{(2(2a^2-b^2))\,\text{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx,x,\frac{2a}{b}+2\tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{a^2(a^2-b^2)\,d}$$

$$= \frac{x^2}{2a^2} + \frac{b(2a^2-b^2)\,\text{arctanh}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{1}{2}(c+dx^2)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{3/2}\,d} - \frac{b^2\cot(c+dx^2)}{2a(a^2-b^2)\,d\,(a+b\csc(c+dx^2))}$$

Mathematica [A] (verified)

Time = 1.29 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.32

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx$$

$$= \frac{\csc(c+dx^2)\left(\frac{ab^2\cot(c+dx^2)}{(-a+b)(a+b)} + (c+dx^2)(a+b\csc(c+dx^2)) - \frac{2b(-2a^2+b^2)\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^2)\right)}{\sqrt{-a^2+b^2}}\right)(a+b\csc(c+dx^2}{(-a^2+b^2)^{3/2}}\right.}{2a^2d(a+b\csc(c+dx^2))^2}$$

```
[In] Integrate[x/(a + b*Csc[c + d*x^2])^2,x]
```

```
[Out] (Csc[c + d*x^2]*((a*b^2*Cot[c + d*x^2])/((-a + b)*(a + b)) + (c + d*x^2)*(a
 + b*Csc[c + d*x^2]) - (2*b*(-2*a^2 + b^2)*ArcTan[(a + b*Tan[(c + d*x^2)/2]
)/Sqrt[-a^2 + b^2]]*(a + b*Csc[c + d*x^2]))/(-a^2 + b^2)^(3/2))*(b + a*Sin[
c + d*x^2]))/(2*a^2*d*(a + b*Csc[c + d*x^2])^2)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{-\frac{2b\left(\frac{\frac{a^{2}\tan\left(\frac{d x^{2}}{2}+\frac{c}{2}\right)}{2a^{2}-2b^{2}}+\frac{ab}{2a^{2}-2b^{2}}}{\frac{\tan\left(\frac{d x^{2}}{2}+\frac{c}{2}\right)^{2} b}{2}+a\tan\left(\frac{d x^{2}}{2}+\frac{c}{2}\right)+\frac{b}{2}}+\frac{2\left(2a^{2}-b^{2}\right)\arctan\left(\frac{2b\tan\left(\frac{d x^{2}}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{\left(2a^{2}-2b^{2}\right)\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}}+\frac{2\arctan\left(\tan\left(\frac{d x^{2}}{2}+\frac{c}{2}\right)\right)}{a^{2}}}{2d}$
default	$\frac{-\frac{2b\left(\frac{\frac{a^{2}\tan\left(\frac{d x^{2}}{2}+\frac{c}{2}\right)}{2a^{2}-2b^{2}}+\frac{ab}{2a^{2}-2b^{2}}}{\frac{\tan\left(\frac{d x^{2}}{2}+\frac{c}{2}\right)^{2} b}{2}+a\tan\left(\frac{d x^{2}}{2}+\frac{c}{2}\right)+\frac{b}{2}}+\frac{2\left(2a^{2}-b^{2}\right)\arctan\left(\frac{2b\tan\left(\frac{d x^{2}}{2}+\frac{c}{2}\right)+2a}{2\sqrt{-a^{2}+b^{2}}}\right)}{\left(2a^{2}-2b^{2}\right)\sqrt{-a^{2}+b^{2}}}\right)}{a^{2}}+\frac{2\arctan\left(\tan\left(\frac{d x^{2}}{2}+\frac{c}{2}\right)\right)}{a^{2}}}{2d}$
risch	$\frac{x^{2}}{2a^{2}}-\frac{i b^{2}\left(i a+b\,e^{i\left(d x^{2}+c\right)}\right)}{a^{2}\left(-a^{2}+b^{2}\right) d\left(2b\,e^{i\left(d x^{2}+c\right)}-i a\,e^{2 i\left(d x^{2}+c\right)}+i a\right)}+\frac{b\ln\left(e^{i\left(d x^{2}+c\right)}+\frac{i b\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{\sqrt{a^{2}-b^{2}}\,a}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b) d}-\frac{b^{3}\ln\left(e^{i\left(d x^{2}+\cdots\right.}\right.}{2\sqrt{a^{2}-b^{\cdots}}}$

```
[In] int(x/(a+b*csc(d*x^2+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(-2/a^2*b*((1/2*a^2/(a^2-b^2)*tan(1/2*d*x^2+1/2*c)+1/2*a*b/(a^2-b^2))
/(1/2*tan(1/2*d*x^2+1/2*c)^2*b+a*tan(1/2*d*x^2+1/2*c)+1/2*b)+2*(2*a^2-b^2)/
(2*a^2-2*b^2)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*d*x^2+1/2*c)+2*a)/(-
a^2+b^2)^(1/2)))+2/a^2*arctan(tan(1/2*d*x^2+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(111) = 222.

Time = 0.29 (sec) , antiderivative size = 536, normalized size of antiderivative = 4.47

$$\int \frac{x}{\left(a+b\csc\left(c+dx^{2}\right)\right)^{2}}\,dx$$
$$=\left[\frac{2\left(a^{5}-2a^{3}b^{2}+ab^{4}\right)dx^{2}\sin\left(dx^{2}+c\right)+2\left(a^{4}b-2a^{2}b^{3}+b^{5}\right)dx^{2}+\left(2a^{2}b^{2}-b^{4}+\left(2a^{3}b-ab^{3}\right)\sin\left(dx^{2}+\cdots\right.\right.}{4\left(\left(a^{7}-2a^{5}b^{2}+a^{3}b^{\cdots}\right.\right.}\right.$$

```
[In] integrate(x/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*sin(d*x^2 + c) + 2*(a^4*b - 2*a^2*b
^3 + b^5)*d*x^2 + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*sin(d*x^2 + c))*sqrt
(a^2 - b^2)*log(((a^2 - 2*b^2)*cos(d*x^2 + c)^2 + 2*a*b*sin(d*x^2 + c) + a^
2 + b^2 + 2*(b*cos(d*x^2 + c)*sin(d*x^2 + c) + a*cos(d*x^2 + c))*sqrt(a^2 -
 b^2))/(a^2*cos(d*x^2 + c)^2 - 2*a*b*sin(d*x^2 + c) - a^2 - b^2)) - 2*(a^3*
b^2 - a*b^4)*cos(d*x^2 + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*sin(d*x^2 + c)
+ (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), 1/2*((a^5 - 2*a^3*b^2 + a*b^4)*d*x^2*si
n(d*x^2 + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*x^2 + (2*a^2*b^2 - b^4 + (2*a^3*
b - a*b^3)*sin(d*x^2 + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin
(d*x^2 + c) + a)/((a^2 - b^2)*cos(d*x^2 + c))) - (a^3*b^2 - a*b^4)*cos(d*x^
2 + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*sin(d*x^2 + c) + (a^6*b - 2*a^4*b^3
+ a^2*b^5)*d)]
```

Sympy [F]

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx=\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx$$

```
[In] integrate(x/(a+b*csc(d*x**2+c))**2,x)
[Out] Integral(x/(a + b*csc(c + d*x**2))**2, x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx=\text{Timed out}$$

```
[In] integrate(x/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.45

$$\begin{aligned}\int \frac{x}{(a+b\csc(c+dx^2))^2}\,dx &= -\frac{(2a^2b-b^3)\left(\pi\left\lfloor\frac{dx^2+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4d-a^2b^2d)\sqrt{-a^2+b^2}}\\&\quad-\frac{ab\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+b^2}{(a^3d-ab^2d)\left(b\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)^2+2a\tan\left(\frac{1}{2}dx^2+\frac{1}{2}c\right)+b\right)}+\frac{dx^2+c}{2a^2d}\end{aligned}$$

```
[In] integrate(x/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*b - b^3)*(pi*floor(1/2*(d*x^2 + c)/pi + 1/2)*sgn(b) + arctan((b*tan
(1/2*d*x^2 + 1/2*c) + a)/sqrt(-a^2 + b^2)))/((a^4*d - a^2*b^2*d)*sqrt(-a^2
+ b^2)) - (a*b*tan(1/2*d*x^2 + 1/2*c) + b^2)/((a^3*d - a*b^2*d)*(b*tan(1/2*
d*x^2 + 1/2*c)^2 + 2*a*tan(1/2*d*x^2 + 1/2*c) + b)) + 1/2*(d*x^2 + c)/(a^2*
d)
```

Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 2755, normalized size of antiderivative = 22.96

$$\int \frac{x}{(a + b\csc(c + dx^2))^2}\,dx = \text{Too large to display}$$

```
[In] int(x/(a + b/sin(c + d*x^2))^2,x)
```

```
[Out] - atan((8*a^3*b^3*tan(c/2 + (d*x^2)/2))/((8*a^3*b^9)/(a^6 + a^2*b^4 - 2*a^4
*b^2) - (24*a^5*b^7)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (16*a^7*b^5)/(a^6 + a^2*
b^4 - 2*a^4*b^2) + (8*a^9*b^3)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (8*a^11*b)/(a^
6 + a^2*b^4 - 2*a^4*b^2)) - (8*a*b^5*tan(c/2 + (d*x^2)/2))/((8*a^3*b^9)/(a^
6 + a^2*b^4 - 2*a^4*b^2) - (24*a^5*b^7)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (16*a
^7*b^5)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (8*a^9*b^3)/(a^6 + a^2*b^4 - 2*a^4*b^
2) - (8*a^11*b)/(a^6 + a^2*b^4 - 2*a^4*b^2)) + (8*a^5*b*tan(c/2 + (d*x^2)/2
))/((8*a^3*b^9)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (24*a^5*b^7)/(a^6 + a^2*b^4 -
 2*a^4*b^2) + (16*a^7*b^5)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (8*a^9*b^3)/(a^6 +
 a^2*b^4 - 2*a^4*b^2) - (8*a^11*b)/(a^6 + a^2*b^4 - 2*a^4*b^2)))/(a^2*d) -
(b^2/(a*(a^2 - b^2)) + (b*tan(c/2 + (d*x^2)/2))/(a^2 - b^2))/(d*(b + b*tan(
c/2 + (d*x^2)/2)^2 + 2*a*tan(c/2 + (d*x^2)/2))) - (b*atan(((b*(2*a^2 - b^2)
*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x^2)/2)*(2*a*b^7 - 2*a^7*b -
8*a^3*b^5 + 9*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (4*(2*a*b^6 - 4*a^3*b
^4 + 2*a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (b*(2*a^2 - b^2)*((a + b)^3*
(a - b)^3)^(1/2)*((4*(4*a^8*b - 4*a^6*b^3))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (
8*tan(c/2 + (d*x^2)/2)*(4*a^4*b^6 - 12*a^6*b^4 + 8*a^8*b^2))/(a^7 + a^3*b^4
 - 2*a^5*b^2) - (b*((4*(8*a^5*b^6 - 16*a^7*b^4 + 8*a^9*b^2))/(a^6 + a^2*b^4
 - 2*a^4*b^2) + (8*tan(c/2 + (d*x^2)/2)*(12*a^11*b - 8*a^5*b^7 + 28*a^7*b^5
 - 32*a^9*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(2*a^2 - b^2)*((a + b)^3*(a -
b)^3)^(1/2))/(2*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))))/(2*(a^8 - a^2*b^
6 + 3*a^4*b^4 - 3*a^6*b^2)))*1i)/(2*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)
) - (b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((4*(2*a*b^6 - 4*a^3*b^4 +
 2*a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) - (8*tan(c/2 + (d*x^2)/2)*(2*a*b^7
 - 2*a^7*b - 8*a^3*b^5 + 9*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b*(2*a^
2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((4*(4*a^8*b - 4*a^6*b^3))/(a^6 + a^2*
b^4 - 2*a^4*b^2) + (8*tan(c/2 + (d*x^2)/2)*(4*a^4*b^6 - 12*a^6*b^4 + 8*a^8*
b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b*((4*(8*a^5*b^6 - 16*a^7*b^4 + 8*a^9*
```

```
b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (8*tan(c/2 + (d*x^2)/2)*(12*a^11*b - 8*
a^5*b^7 + 28*a^7*b^5 - 32*a^9*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(2*a^2 - b
^2)*((a + b)^3*(a - b)^3)^(1/2))/(2*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)
)))/(2*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*1i)/(2*(a^8 - a^2*b^6 + 3*
a^4*b^4 - 3*a^6*b^2)))/((8*(b^5 - 2*a^2*b^3))/(a^6 + a^2*b^4 - 2*a^4*b^2) +
 (16*tan(c/2 + (d*x^2)/2)*(b^6 - 3*a^2*b^4 + 2*a^4*b^2))/(a^7 + a^3*b^4 - 2
*a^5*b^2) + (b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x
^2)/2)*(2*a*b^7 - 2*a^7*b - 8*a^3*b^5 + 9*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*
b^2) - (4*(2*a*b^6 - 4*a^3*b^4 + 2*a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) +
(b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((4*(4*a^8*b - 4*a^6*b^3))/(a^
6 + a^2*b^4 - 2*a^4*b^2) + (8*tan(c/2 + (d*x^2)/2)*(4*a^4*b^6 - 12*a^6*b^4
+ 8*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (b*((4*(8*a^5*b^6 - 16*a^7*b^4
+ 8*a^9*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (8*tan(c/2 + (d*x^2)/2)*(12*a^1
1*b - 8*a^5*b^7 + 28*a^7*b^5 - 32*a^9*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(2
*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2))/(2*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*
a^6*b^2))))/(2*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))))/(2*(a^8 - a^2*b^6
 + 3*a^4*b^4 - 3*a^6*b^2)) + (b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*(
(4*(2*a*b^6 - 4*a^3*b^4 + 2*a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) - (8*tan(
c/2 + (d*x^2)/2)*(2*a*b^7 - 2*a^7*b - 8*a^3*b^5 + 9*a^5*b^3))/(a^7 + a^3*b^
4 - 2*a^5*b^2) + (b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((4*(4*a^8*b
- 4*a^6*b^3))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (8*tan(c/2 + (d*x^2)/2)*(4*a^4*
b^6 - 12*a^6*b^4 + 8*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b*((4*(8*a^5*
b^6 - 16*a^7*b^4 + 8*a^9*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (8*tan(c/2 + (
d*x^2)/2)*(12*a^11*b - 8*a^5*b^7 + 28*a^7*b^5 - 32*a^9*b^3))/(a^7 + a^3*b^4
 - 2*a^5*b^2))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2))/(2*(a^8 - a^2*b^6
 + 3*a^4*b^4 - 3*a^6*b^2))))/(2*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))))/
(2*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))))*(2*a^2 - b^2)*((a + b)^3*(a -
 b)^3)^(1/2)*1i)/(d*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))
```

3.28 $\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*csc(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x*(a + b*Csc[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Csc[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 18.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*Csc[c + d*x^2])^2),x]
[Out] Integrate[1/(x*(a + b*Csc[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\csc\left(d\,x^2+c\right))^2}dx$$

```
[In] int(1/x/(a+b*csc(d*x^2+c))^2,x)
[Out] int(1/x/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*csc(d*x^2 + c)^2 + 2*a*b*x*csc(d*x^2 + c) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*csc(d*x**2+c))**2,x)
[Out] Integral(1/(x*(a + b*csc(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 5.46 (sec) , antiderivative size = 4629, normalized size of antiderivative = 257.17

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] (a^6*d*x^2*cos(2*d*x^2 + 2*c)^2*log(x) + a^6*d*x^2*log(x)*sin(2*d*x^2 + 2*c
)^2 + (a^2*b^4*cos(2*c)^2 + a^2*b^4*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2*log(x)
 + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(c)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*si
n(c)^2)*d*x^2*cos(d*x^2)^2*log(x) + (a^2*b^4*cos(2*c)^2 + a^2*b^4*sin(2*c)^
2)*d*x^2*log(x)*sin(2*d*x^2)^2 + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(c)^2 +
(a^4*b^2 - 2*a^2*b^4 + b^6)*sin(c)^2)*d*x^2*log(x)*sin(d*x^2)^2 + (a^6 - 2*
a^4*b^2 + a^2*b^4)*d*x^2*log(x) + (a^2*b^4*sin(2*c) - 4*((a^3*b^3 - a*b^5)*
cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c))*d*x^2*cos(d*x^2)*log(x
) + 2*(a^4*b^2 - a^2*b^4)*d*x^2*cos(2*c)*log(x) + 4*((a^3*b^3 - a*b^5)*cos(
2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2*log(x)*sin(d*x^2))*c
os(2*d*x^2) - (2*a^4*b^2*d*x^2*cos(2*d*x^2)*cos(2*c)*log(x) - 2*a^4*b^2*d*x
^2*log(x)*sin(2*d*x^2)*sin(2*c) + a^3*b^3*cos(d*x^2 + c) + 4*(a^5*b - a^3*b
^3)*d*x^2*cos(c)*log(x)*sin(d*x^2) + 4*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2)*l
og(x)*sin(c) + 2*(a^6 - a^4*b^2)*d*x^2*log(x))*cos(2*d*x^2 + 2*c) + (a*b^5*
cos(2*d*x^2)*cos(2*c) - a*b^5*sin(2*d*x^2)*sin(2*c) + a^3*b^3 - a*b^5 + 2*(
a^2*b^4 - b^6)*cos(c)*sin(d*x^2) + 2*(a^2*b^4 - b^6)*cos(d*x^2)*sin(c))*cos
(d*x^2 + c) + 2*(2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*log(x)*sin(c) - (a^3*b
^3 - a*b^5)*cos(c))*cos(d*x^2) - (a^8*d*x^2*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^
2*sin(2*d*x^2 + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^2*co
s(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a
^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^2*cos(d*x^2)^2 + (a^4*b^4*cos(2*c)^2 + a^4*
b^4*sin(2*c)^2)*d*x^2*sin(2*d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^
2*cos(c)*sin(d*x^2) + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^
2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^2*sin(d*x^2)^2 + 4*(a^7*b - 2*a^5*b^
3 + a^3*b^5)*d*x^2*cos(d*x^2)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*x^2 -
2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*si
n(c))*d*x^2*cos(d*x^2) - (a^6*b^2 - a^4*b^4)*d*x^2*cos(2*c) - 2*((a^5*b^3 -
 a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^2*sin(
d*x^2))*cos(2*d*x^2) - 2*(a^6*b^2*d*x^2*cos(2*d*x^2)*cos(2*c) - a^6*b^2*d*x
^2*sin(2*d*x^2)*sin(2*c) + 2*(a^7*b - a^5*b^3)*d*x^2*cos(c)*sin(d*x^2) + 2*
(a^7*b - a^5*b^3)*d*x^2*cos(d*x^2)*sin(c) + (a^8 - a^6*b^2)*d*x^2)*cos(2*d*
x^2 + 2*c) - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5
)*sin(2*c)*sin(c))*d*x^2*cos(d*x^2) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c
) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^2*sin(d*x^2) + (a^6*b^2 - a^4*
b^4)*d*x^2*sin(2*c))*sin(2*d*x^2) - 2*(a^6*b^2*d*x^2*cos(2*c)*sin(2*d*x^2)
```

```
+ a^6*b^2*d*x^2*cos(2*d*x^2)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*x^2*cos(d*x^2
)*cos(c) + 2*(a^7*b - a^5*b^3)*d*x^2*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))
*integrate(-2*(a^2*b^4*cos(2*c)*sin(2*d*x^2) + a^2*b^4*cos(2*d*x^2)*sin(2*c
) - 2*(a^3*b^3 - a*b^5)*cos(d*x^2)*cos(c) + 2*(a^3*b^3 - a*b^5)*sin(d*x^2)*
sin(c) - (a^3*b^3*cos(d*x^2 + c) - (2*a^5*b - a^3*b^3)*d*x^2*sin(d*x^2 + c)
)*cos(2*d*x^2 + 2*c) + (a^3*b^3 - a*b^5 + (a*b^5*cos(2*c) + (2*a^3*b^3 - a*
b^5)*d*x^2*sin(2*c))*cos(2*d*x^2) - 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*
cos(c) - (a^2*b^4 - b^6)*sin(c))*cos(d*x^2) - (a*b^5*sin(2*c) - (2*a^3*b^3
- a*b^5)*d*x^2*cos(2*c))*sin(2*d*x^2) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*
x^2*sin(c) + (a^2*b^4 - b^6)*cos(c))*sin(d*x^2))*cos(d*x^2 + c) - (a^3*b^3*
sin(d*x^2 + c) + a^4*b^2 + (2*a^5*b - a^3*b^3)*d*x^2*cos(d*x^2 + c))*sin(2*
d*x^2 + 2*c) - ((2*a^5*b - 3*a^3*b^3 + a*b^5)*d*x^2 - (a*b^5*sin(2*c) - (2*
a^3*b^3 - a*b^5)*d*x^2*cos(2*c))*cos(2*d*x^2) + 2*((2*a^4*b^2 - 3*a^2*b^4 +
 b^6)*d*x^2*sin(c) + (a^2*b^4 - b^6)*cos(c))*cos(d*x^2) - (a*b^5*cos(2*c) +
 (2*a^3*b^3 - a*b^5)*d*x^2*sin(2*c))*sin(2*d*x^2) + 2*((2*a^4*b^2 - 3*a^2*b
^4 + b^6)*d*x^2*cos(c) - (a^2*b^4 - b^6)*sin(c))*sin(d*x^2))*sin(d*x^2 + c)
)/(a^8*d*x^3*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^3*sin(2*d*x^2 + 2*c)^2 + (a^4*b
^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6*b^2 - 2*
a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x
^3*cos(d*x^2)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^3*sin(2*d*x
^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*cos(c)*sin(d*x^2) + 4*((a^6*b
^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)
^2)*d*x^3*sin(d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*cos(d*x^2)*s
in(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*x^3 - 2*(2*((a^5*b^3 - a^3*b^5)*cos(c
)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^3*cos(d*x^2) - (a^6*b
^2 - a^4*b^4)*d*x^3*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^
5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^3*sin(d*x^2))*cos(2*d*x^2) - 2*(a^6*b
^2*d*x^3*cos(2*d*x^2)*cos(2*c) - a^6*b^2*d*x^3*sin(2*d*x^2)*sin(2*c) + 2*(a
^7*b - a^5*b^3)*d*x^3*cos(c)*sin(d*x^2) + 2*(a^7*b - a^5*b^3)*d*x^3*cos(d*x
^2)*sin(c) + (a^8 - a^6*b^2)*d*x^3)*cos(2*d*x^2 + 2*c) - 2*(2*((a^5*b^3 - a
^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^3*cos(d*
x^2) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c
)*sin(c))*d*x^3*sin(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^3*sin(2*c))*sin(2*d*x^
2) - 2*(a^6*b^2*d*x^3*cos(2*c)*sin(2*d*x^2) + a^6*b^2*d*x^3*cos(2*d*x^2)*si
n(2*c) - 2*(a^7*b - a^5*b^3)*d*x^3*cos(d*x^2)*cos(c) + 2*(a^7*b - a^5*b^3)*
d*x^3*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c)), x) + (a^2*b^4*cos(2*c) - 4*((
a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*x^2
*cos(d*x^2)*log(x) - 4*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^
5)*cos(2*c)*sin(c))*d*x^2*log(x)*sin(d*x^2) - 2*(a^4*b^2 - a^2*b^4)*d*x^2*l
og(x)*sin(2*c))*sin(2*d*x^2) - (2*a^4*b^2*d*x^2*cos(2*c)*log(x)*sin(2*d*x^2
) + 2*a^4*b^2*d*x^2*cos(2*d*x^2)*log(x)*sin(2*c) - 4*(a^5*b - a^3*b^3)*d*x^
2*cos(d*x^2)*cos(c)*log(x) + a^3*b^3*sin(d*x^2 + c) + 4*(a^5*b - a^3*b^3)*d
*x^2*log(x)*sin(d*x^2)*sin(c) + a^4*b^2)*sin(2*d*x^2 + 2*c) + (a*b^5*cos(2*
c)*sin(2*d*x^2) + a*b^5*cos(2*d*x^2)*sin(2*c) - 2*(a^2*b^4 - b^6)*cos(d*x^2
)*cos(c) + 2*(a^2*b^4 - b^6)*sin(d*x^2)*sin(c))*sin(d*x^2 + c) + 2*(2*(a^5*
```

```
b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(c)*log(x) + (a^3*b^3 - a*b^5)*sin(c))*sin(
d*x^2))/(a^8*d*x^2*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^2*sin(2*d*x^2 + 2*c)^2 +
(a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^2*cos(2*d*x^2)^2 + 4*((a^6*b^
2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^
2)*d*x^2*cos(d*x^2)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^2*sin
(2*d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^2*cos(c)*sin(d*x^2) + 4*(
(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*
sin(c)^2)*d*x^2*sin(d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^2*cos(d*
x^2)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*x^2 - 2*(2*((a^5*b^3 - a^3*b^5)
*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^2*cos(d*x^2) -
(a^6*b^2 - a^4*b^4)*d*x^2*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c)
 + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^2*sin(d*x^2))*cos(2*d*x^2) - 2*
(a^6*b^2*d*x^2*cos(2*d*x^2)*cos(2*c) - a^6*b^2*d*x^2*sin(2*d*x^2)*sin(2*c)
+ 2*(a^7*b - a^5*b^3)*d*x^2*cos(c)*sin(d*x^2) + 2*(a^7*b - a^5*b^3)*d*x^2*c
os(d*x^2)*sin(c) + (a^8 - a^6*b^2)*d*x^2)*cos(2*d*x^2 + 2*c) - 2*(2*((a^5*b
^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^2*
cos(d*x^2) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*c
os(2*c)*sin(c))*d*x^2*sin(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^2*sin(2*c))*sin(
2*d*x^2) - 2*(a^6*b^2*d*x^2*cos(2*c)*sin(2*d*x^2) + a^6*b^2*d*x^2*cos(2*d*x
^2)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*x^2*cos(d*x^2)*cos(c) + 2*(a^7*b - a^5
*b^3)*d*x^2*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*csc(d*x^2 + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 18.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x\left(a+\frac{b}{\sin\left(d\,x^2+c\right)}\right)^2}\,dx$$

```
[In] int(1/(x*(a + b/sin(c + d*x^2))^2),x)
[Out] int(1/(x*(a + b/sin(c + d*x^2))^2), x)
```

3.29 $\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\operatorname{Int}\left(\frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*csc(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*Csc[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Csc[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 14.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 \left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^2 \left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^2*(a + b*Csc[c + d*x^2])^2),x]
[Out] Integrate[1/(x^2*(a + b*Csc[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a+b\csc\left(d\,x^2+c\right)\right)^2}dx$$

```
[In] int(1/x^2/(a+b*csc(d*x^2+c))^2,x)
[Out] int(1/x^2/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^2 \left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)^2 x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*csc(d*x^2 + c)^2 + 2*a*b*x^2*csc(d*x^2 + c) + a^2*x^2),
 x)
```

Sympy [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2 \left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{x^2 \left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*csc(d*x**2+c))**2,x)
[Out] Integral(1/(x**2*(a + b*csc(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 5.41 (sec) , antiderivative size = 4560, normalized size of antiderivative = 253.33

$$\int \frac{1}{x^2 \left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx = \int \frac{1}{\left(b\csc\left(dx^2+c\right)+a\right)^2 x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
[Out] -((a^6 - a^4*b^2)*d*x^2*cos(2*d*x^2 + 2*c)^2 + (a^6 - a^4*b^2)*d*x^2*sin(2*
d*x^2 + 2*c)^2 + (a^6 - 2*a^4*b^2 + a^2*b^4)*d*x^2 - (a^2*b^4*sin(2*c) - (a
^4*b^2 - a^2*b^4)*d*x^2*cos(2*c))*cos(2*d*x^2) + (a^3*b^3*cos(d*x^2 + c) -
(a^4*b^2 - a^2*b^4)*d*x^2*cos(2*d*x^2)*cos(2*c) - 2*(a^5*b - 2*a^3*b^3 + a*
b^5)*d*x^2*cos(c)*sin(d*x^2) + (a^4*b^2 - a^2*b^4)*d*x^2*sin(2*d*x^2)*sin(2
*c) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(d*x^2)*sin(c) - 2*(a^5*b - a^
3*b^3)*d*x^2*sin(d*x^2 + c) - (2*a^6 - 3*a^4*b^2 + a^2*b^4)*d*x^2)*cos(2*d*
x^2 + 2*c) - (a^3*b^3 - a*b^5 + (a*b^5*cos(2*c) + 2*(a^3*b^3 - a*b^5)*d*x^2
*sin(2*c))*cos(2*d*x^2) - 2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2*cos(c) - (
a^2*b^4 - b^6)*sin(c))*cos(d*x^2) - (a*b^5*sin(2*c) - 2*(a^3*b^3 - a*b^5)*d
*x^2*cos(2*c))*sin(2*d*x^2) + 2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2*sin(c)
 + (a^2*b^4 - b^6)*cos(c))*sin(d*x^2))*cos(d*x^2 + c) + 2*((a^5*b - 2*a^3*b
^3 + a*b^5)*d*x^2*sin(c) + (a^3*b^3 - a*b^5)*cos(c))*cos(d*x^2) + (a^8*d*x^
3*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^3*sin(2*d*x^2 + 2*c)^2 + (a^4*b^4*cos(2*c)
^2 + a^4*b^4*sin(2*c)^2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a
^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^3*cos(d*x^
2)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^3*sin(2*d*x^2)^2 + 4*(
a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*cos(c)*sin(d*x^2) + 4*((a^6*b^2 - 2*a^4*
b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^3*s
in(d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*cos(d*x^2)*sin(c) + (a^
8 - 2*a^6*b^2 + a^4*b^4)*d*x^3 - 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c)
- (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^3*cos(d*x^2) - (a^6*b^2 - a^4*b^
```

```
4)*d*x^3*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3
*b^5)*sin(2*c)*sin(c))*d*x^3*sin(d*x^2))*cos(2*d*x^2) - 2*(a^6*b^2*d*x^3*co
s(2*d*x^2)*cos(2*c) - a^6*b^2*d*x^3*sin(2*d*x^2)*sin(2*c) + 2*(a^7*b - a^5*
b^3)*d*x^3*cos(c)*sin(d*x^2) + 2*(a^7*b - a^5*b^3)*d*x^3*cos(d*x^2)*sin(c)
+ (a^8 - a^6*b^2)*d*x^3)*cos(2*d*x^2 + 2*c) - 2*(2*((a^5*b^3 - a^3*b^5)*cos
(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^3*cos(d*x^2) + 2*((
a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d
*x^3*sin(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^3*sin(2*c))*sin(2*d*x^2) - 2*(a^6
*b^2*d*x^3*cos(2*c)*sin(2*d*x^2) + a^6*b^2*d*x^3*cos(2*d*x^2)*sin(2*c) - 2*
(a^7*b - a^5*b^3)*d*x^3*cos(d*x^2)*cos(c) + 2*(a^7*b - a^5*b^3)*d*x^3*sin(d
*x^2)*sin(c))*sin(2*d*x^2 + 2*c))*integrate(-(3*a^2*b^4*cos(2*c)*sin(2*d*x^
2) + 3*a^2*b^4*cos(2*d*x^2)*sin(2*c) - 6*(a^3*b^3 - a*b^5)*cos(d*x^2)*cos(c
) + 6*(a^3*b^3 - a*b^5)*sin(d*x^2)*sin(c) - (3*a^3*b^3*cos(d*x^2 + c) - 2*(
2*a^5*b - a^3*b^3)*d*x^2*sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (3*a^3*b^3 -
3*a*b^5 + (3*a*b^5*cos(2*c) + 2*(2*a^3*b^3 - a*b^5)*d*x^2*sin(2*c))*cos(2*d
*x^2) - 2*(2*(2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*cos(c) - 3*(a^2*b^4 - b^6)
*sin(c))*cos(d*x^2) - (3*a*b^5*sin(2*c) - 2*(2*a^3*b^3 - a*b^5)*d*x^2*cos(2
*c))*sin(2*d*x^2) + 2*(2*(2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*sin(c) + 3*(a^
2*b^4 - b^6)*cos(c))*sin(d*x^2))*cos(d*x^2 + c) - (3*a^3*b^3*sin(d*x^2 + c)
 + 3*a^4*b^2 + 2*(2*a^5*b - a^3*b^3)*d*x^2*cos(d*x^2 + c))*sin(2*d*x^2 + 2*
c) - (2*(2*a^5*b - 3*a^3*b^3 + a*b^5)*d*x^2 - (3*a*b^5*sin(2*c) - 2*(2*a^3*
b^3 - a*b^5)*d*x^2*cos(2*c))*cos(2*d*x^2) + 2*(2*(2*a^4*b^2 - 3*a^2*b^4 + b
^6)*d*x^2*sin(c) + 3*(a^2*b^4 - b^6)*cos(c))*cos(d*x^2) - (3*a*b^5*cos(2*c)
 + 2*(2*a^3*b^3 - a*b^5)*d*x^2*sin(2*c))*sin(2*d*x^2) + 2*(2*(2*a^4*b^2 - 3
*a^2*b^4 + b^6)*d*x^2*cos(c) - 3*(a^2*b^4 - b^6)*sin(c))*sin(d*x^2))*sin(d*
x^2 + c))/(a^8*d*x^4*cos(2*d*x^2 + 2*c)^2 + a^8*d*x^4*sin(2*d*x^2 + 2*c)^2
+ (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^4*cos(2*d*x^2)^2 + 4*((a^6*
b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c
)^2)*d*x^4*cos(d*x^2)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^4*s
in(2*d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^4*cos(c)*sin(d*x^2) + 4
*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6
)*sin(c)^2)*d*x^4*sin(d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^4*cos(
d*x^2)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*x^4 - 2*(2*((a^5*b^3 - a^3*b^
5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^4*cos(d*x^2)
- (a^6*b^2 - a^4*b^4)*d*x^4*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(
c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^4*sin(d*x^2))*cos(2*d*x^2) -
2*(a^6*b^2*d*x^4*cos(2*d*x^2)*cos(2*c) - a^6*b^2*d*x^4*sin(2*d*x^2)*sin(2*c
) + 2*(a^7*b - a^5*b^3)*d*x^4*cos(c)*sin(d*x^2) + 2*(a^7*b - a^5*b^3)*d*x^4
*cos(d*x^2)*sin(c) + (a^8 - a^6*b^2)*d*x^4)*cos(2*d*x^2 + 2*c) - 2*(2*((a^5
*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^
4*cos(d*x^2) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)
*cos(2*c)*sin(c))*d*x^4*sin(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^4*sin(2*c))*si
n(2*d*x^2) - 2*(a^6*b^2*d*x^4*cos(2*c)*sin(2*d*x^2) + a^6*b^2*d*x^4*cos(2*d
*x^2)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*x^4*cos(d*x^2)*cos(c) + 2*(a^7*b - a
^5*b^3)*d*x^4*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c)), x) - (a^2*b^4*cos(2*c
```

```
) + (a^4*b^2 - a^2*b^4)*d*x^2*sin(2*c))*sin(2*d*x^2) + (a^3*b^3*sin(d*x^2 +
 c) + a^4*b^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*cos(d*x^2)*cos(c) - (a^
4*b^2 - a^2*b^4)*d*x^2*cos(2*c)*sin(2*d*x^2) - (a^4*b^2 - a^2*b^4)*d*x^2*co
s(2*d*x^2)*sin(2*c) - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*x^2*sin(d*x^2)*sin(c)
 + 2*(a^5*b - a^3*b^3)*d*x^2*cos(d*x^2 + c))*sin(2*d*x^2 + 2*c) + (2*(a^5*b
 - 2*a^3*b^3 + a*b^5)*d*x^2 - (a*b^5*sin(2*c) - 2*(a^3*b^3 - a*b^5)*d*x^2*c
os(2*c))*cos(2*d*x^2) + 2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2*sin(c) + (a^
2*b^4 - b^6)*cos(c))*cos(d*x^2) - (a*b^5*cos(2*c) + 2*(a^3*b^3 - a*b^5)*d*x
^2*sin(2*c))*sin(2*d*x^2) + 2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*x^2*cos(c) -
 (a^2*b^4 - b^6)*sin(c))*sin(d*x^2))*sin(d*x^2 + c) + 2*((a^5*b - 2*a^3*b^3
 + a*b^5)*d*x^2*cos(c) - (a^3*b^3 - a*b^5)*sin(c))*sin(d*x^2))/(a^8*d*x^3*c
os(2*d*x^2 + 2*c)^2 + a^8*d*x^3*sin(2*d*x^2 + 2*c)^2 + (a^4*b^4*cos(2*c)^2
+ a^4*b^4*sin(2*c)^2)*d*x^3*cos(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*
b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^3*cos(d*x^2)^
2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*x^3*sin(2*d*x^2)^2 + 4*(a^7
*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*cos(c)*sin(d*x^2) + 4*((a^6*b^2 - 2*a^4*b^4
 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^3*sin(
d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*x^3*cos(d*x^2)*sin(c) + (a^8 -
 2*a^6*b^2 + a^4*b^4)*d*x^3 - 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (
a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^3*cos(d*x^2) - (a^6*b^2 - a^4*b^4)*
d*x^3*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^
5)*sin(2*c)*sin(c))*d*x^3*sin(d*x^2))*cos(2*d*x^2) - 2*(a^6*b^2*d*x^3*cos(2
*d*x^2)*cos(2*c) - a^6*b^2*d*x^3*sin(2*d*x^2)*sin(2*c) + 2*(a^7*b - a^5*b^3
)*d*x^3*cos(c)*sin(d*x^2) + 2*(a^7*b - a^5*b^3)*d*x^3*cos(d*x^2)*sin(c) + (
a^8 - a^6*b^2)*d*x^3)*cos(2*d*x^2 + 2*c) - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*
c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x^3*cos(d*x^2) + 2*((a^5
*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^
3*sin(d*x^2) + (a^6*b^2 - a^4*b^4)*d*x^3*sin(2*c))*sin(2*d*x^2) - 2*(a^6*b^
2*d*x^3*cos(2*c)*sin(2*d*x^2) + a^6*b^2*d*x^3*cos(2*d*x^2)*sin(2*c) - 2*(a^
7*b - a^5*b^3)*d*x^3*cos(d*x^2)*cos(c) + 2*(a^7*b - a^5*b^3)*d*x^3*sin(d*x^
2)*sin(c))*sin(2*d*x^2 + 2*c))
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+b\csc(c+dx^2))^2}\,dx = \int \frac{1}{(b\csc(dx^2+c)+a)^2 x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*csc(d*x^2 + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 17.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^2 \left(a + b \csc \left(c + dx^2\right)\right)^2} \, dx = \int \frac{1}{x^2 \left(a + \frac{b}{\sin(d\,x^2+c)}\right)^2} \, dx$$

```
[In] int(1/(x^2*(a + b/sin(c + d*x^2))^2),x)
[Out] int(1/(x^2*(a + b/sin(c + d*x^2))^2), x)
```

3.30 $\int \frac{1}{x^3\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^3\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^3\left(a+b\csc\left(c+dx^2\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^3/(a+b*csc(d*x^2+c))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^3\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Int[1/(x^3*(a + b*Csc[c + d*x^2])^2),x]
```

```
[Out] Defer[Int][1/(x^3*(a + b*Csc[c + d*x^2])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^3\left(a+b\csc\left(c+dx^2\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 15.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 \left(a+b\csc \left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{x^3 \left(a+b\csc \left(c+dx^2\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^3*(a + b*Csc[c + d*x^2])^2),x]
[Out] Integrate[1/(x^3*(a + b*Csc[c + d*x^2])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a+b\csc \left(d\,x^2+c\right))^2}dx$$

```
[In] int(1/x^3/(a+b*csc(d*x^2+c))^2,x)
[Out] int(1/x^3/(a+b*csc(d*x^2+c))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{1}{x^3 \left(a+b\csc \left(c+dx^2\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc \left(dx^2+c\right)+a\right)^2 x^3}\,dx$$

```
[In] integrate(1/x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^3*csc(d*x^2 + c)^2 + 2*a*b*x^3*csc(d*x^2 + c) + a^2*x^3),
 x)
```

Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^3 (a + b \csc(c + dx^2))^2} \, dx = \int \frac{1}{x^3 (a + b \csc(c + dx^2))^2} \, dx$$

```
[In] integrate(1/x**3/(a+b*csc(d*x**2+c))**2,x)
```

```
[Out] Integral(1/(x**3*(a + b*csc(c + d*x**2))**2), x)
```

Maxima [N/A]

Not integrable

Time = 5.37 (sec) , antiderivative size = 3530, normalized size of antiderivative = 196.11

$$\int \frac{1}{x^3 (a + b \csc(c + dx^2))^2} \, dx = \int \frac{1}{(b \csc(dx^2 + c) + a)^2 x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*((a^4 - a^2*b^2)*d*x^2 - ((a^4 - a^2*b^2)*d*x^2*cos(2*c) - 2*a^2*b^2*s
in(2*c))*cos(2*d*x^2) + ((a^4 - a^2*b^2)*d*x^2*cos(2*d*x^2)*cos(2*c) - 2*(a
^3*b - a*b^3)*d*x^2*cos(c)*sin(d*x^2) - (a^4 - a^2*b^2)*d*x^2*sin(2*d*x^2)*
sin(2*c) - 2*(a^3*b - a*b^3)*d*x^2*cos(d*x^2)*sin(c) - (a^4 - a^2*b^2)*d*x^
2)*cos(2*d*x^2 + 2*c) - 2*(a*b^3 - (a*b^3*cos(2*c) + (a^3*b - a*b^3)*d*x^2*
sin(2*c))*cos(2*d*x^2) - 2*((a^2*b^2 - b^4)*d*x^2*cos(c) - b^4*sin(c))*cos(
d*x^2) - ((a^3*b - a*b^3)*d*x^2*cos(2*c) - a*b^3*sin(2*c))*sin(2*d*x^2) + 2
*(b^4*cos(c) + (a^2*b^2 - b^4)*d*x^2*sin(c))*sin(d*x^2))*cos(d*x^2 + c) + 2
*(2*a*b^3*cos(c) + (a^3*b - a*b^3)*d*x^2*sin(c))*cos(d*x^2) + 2*(((a^6 - a^
4*b^2)*cos(2*c)^2 + (a^6 - a^4*b^2)*sin(2*c)^2)*d*x^4*cos(2*d*x^2)^2 + 4*((
a^4*b^2 - a^2*b^4)*cos(c)^2 + (a^4*b^2 - a^2*b^4)*sin(c)^2)*d*x^4*cos(d*x^2
)^2 + ((a^6 - a^4*b^2)*cos(2*c)^2 + (a^6 - a^4*b^2)*sin(2*c)^2)*d*x^4*sin(2
*d*x^2)^2 + 4*(a^5*b - a^3*b^3)*d*x^4*cos(c)*sin(d*x^2) + 4*((a^4*b^2 - a^2
*b^4)*cos(c)^2 + (a^4*b^2 - a^2*b^4)*sin(c)^2)*d*x^4*sin(d*x^2)^2 + 4*(a^5*
b - a^3*b^3)*d*x^4*cos(d*x^2)*sin(c) + (a^6 - a^4*b^2)*d*x^4 + 2*(2*((a^5*b
 - a^3*b^3)*cos(c)*sin(2*c) - (a^5*b - a^3*b^3)*cos(2*c)*sin(c))*d*x^4*cos(
d*x^2) - (a^6 - a^4*b^2)*d*x^4*cos(2*c) - 2*((a^5*b - a^3*b^3)*cos(2*c)*cos
(c) + (a^5*b - a^3*b^3)*sin(2*c)*sin(c))*d*x^4*sin(d*x^2))*cos(2*d*x^2) + 2
*(2*((a^5*b - a^3*b^3)*cos(2*c)*cos(c) + (a^5*b - a^3*b^3)*sin(2*c)*sin(c))
*d*x^4*cos(d*x^2) + 2*((a^5*b - a^3*b^3)*cos(c)*sin(2*c) - (a^5*b - a^3*b^3
)*cos(2*c)*sin(c))*d*x^4*sin(d*x^2) + (a^6 - a^4*b^2)*d*x^4*sin(2*c))*sin(2
*d*x^2))*integrate(-2*(2*a^2*b^4*cos(2*c)*sin(2*d*x^2) + 2*a^2*b^4*cos(2*d*
```

```
x^2)*sin(2*c) - 4*(a^3*b^3 - a*b^5)*cos(d*x^2)*cos(c) + 4*(a^3*b^3 - a*b^5)
*sin(d*x^2)*sin(c) - (2*a^3*b^3*cos(d*x^2 + c) - (2*a^5*b - a^3*b^3)*d*x^2*
sin(d*x^2 + c))*cos(2*d*x^2 + 2*c) + (2*a^3*b^3 - 2*a*b^5 + (2*a*b^5*cos(2*
c) + (2*a^3*b^3 - a*b^5)*d*x^2*sin(2*c))*cos(2*d*x^2) - 2*((2*a^4*b^2 - 3*a
^2*b^4 + b^6)*d*x^2*cos(c) - 2*(a^2*b^4 - b^6)*sin(c))*cos(d*x^2) - (2*a*b^
5*sin(2*c) - (2*a^3*b^3 - a*b^5)*d*x^2*cos(2*c))*sin(2*d*x^2) + 2*((2*a^4*b
^2 - 3*a^2*b^4 + b^6)*d*x^2*sin(c) + 2*(a^2*b^4 - b^6)*cos(c))*sin(d*x^2))*
cos(d*x^2 + c) - (2*a^3*b^3*sin(d*x^2 + c) + 2*a^4*b^2 + (2*a^5*b - a^3*b^3
)*d*x^2*cos(d*x^2 + c))*sin(2*d*x^2 + 2*c) - ((2*a^5*b - 3*a^3*b^3 + a*b^5)
*d*x^2 - (2*a*b^5*sin(2*c) - (2*a^3*b^3 - a*b^5)*d*x^2*cos(2*c))*cos(2*d*x^
2) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*sin(c) + 2*(a^2*b^4 - b^6)*cos(
c))*cos(d*x^2) - (2*a*b^5*cos(2*c) + (2*a^3*b^3 - a*b^5)*d*x^2*sin(2*c))*si
n(2*d*x^2) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*x^2*cos(c) - 2*(a^2*b^4 - b
^6)*sin(c))*sin(d*x^2))*sin(d*x^2 + c))/(a^8*d*x^5*cos(2*d*x^2 + 2*c)^2 + a
^8*d*x^5*sin(2*d*x^2 + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d
*x^5*cos(2*d*x^2)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^
2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^5*cos(d*x^2)^2 + (a^4*b^4*cos(2*c)^2
 + a^4*b^4*sin(2*c)^2)*d*x^5*sin(2*d*x^2)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^
5)*d*x^5*cos(c)*sin(d*x^2) + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 +
(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*x^5*sin(d*x^2)^2 + 4*(a^7*b - 2
*a^5*b^3 + a^3*b^5)*d*x^5*cos(d*x^2)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d
*x^5 - 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(
2*c)*sin(c))*d*x^5*cos(d*x^2) - (a^6*b^2 - a^4*b^4)*d*x^5*cos(2*c) - 2*((a^
5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*x
^5*sin(d*x^2))*cos(2*d*x^2) - 2*(a^6*b^2*d*x^5*cos(2*d*x^2)*cos(2*c) - a^6*
b^2*d*x^5*sin(2*d*x^2)*sin(2*c) + 2*(a^7*b - a^5*b^3)*d*x^5*cos(c)*sin(d*x^
2) + 2*(a^7*b - a^5*b^3)*d*x^5*cos(d*x^2)*sin(c) + (a^8 - a^6*b^2)*d*x^5)*c
os(2*d*x^2 + 2*c) - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 -
a^3*b^5)*sin(2*c)*sin(c))*d*x^5*cos(d*x^2) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*
sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*x^5*sin(d*x^2) + (a^6*b^2
 - a^4*b^4)*d*x^5*sin(2*c))*sin(2*d*x^2) - 2*(a^6*b^2*d*x^5*cos(2*c)*sin(2*
d*x^2) + a^6*b^2*d*x^5*cos(2*d*x^2)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*x^5*co
s(d*x^2)*cos(c) + 2*(a^7*b - a^5*b^3)*d*x^5*sin(d*x^2)*sin(c))*sin(2*d*x^2
+ 2*c)), x) + (2*a^2*b^2*cos(2*c) + (a^4 - a^2*b^2)*d*x^2*sin(2*c))*sin(2*d
*x^2) + (2*(a^3*b - a*b^3)*d*x^2*cos(d*x^2)*cos(c) + (a^4 - a^2*b^2)*d*x^2*
cos(2*c)*sin(2*d*x^2) + (a^4 - a^2*b^2)*d*x^2*cos(2*d*x^2)*sin(2*c) - 2*(a^
3*b - a*b^3)*d*x^2*sin(d*x^2)*sin(c))*sin(2*d*x^2 + 2*c) + 2*((a^3*b - a*b^
3)*d*x^2 - ((a^3*b - a*b^3)*d*x^2*cos(2*c) - a*b^3*sin(2*c))*cos(2*d*x^2) +
 2*(b^4*cos(c) + (a^2*b^2 - b^4)*d*x^2*sin(c))*cos(d*x^2) + (a*b^3*cos(2*c)
 + (a^3*b - a*b^3)*d*x^2*sin(2*c))*sin(2*d*x^2) + 2*((a^2*b^2 - b^4)*d*x^2*
cos(c) - b^4*sin(c))*sin(d*x^2))*sin(d*x^2 + c) + 2*((a^3*b - a*b^3)*d*x^2*
cos(c) - 2*a*b^3*sin(c))*sin(d*x^2))/(((a^6 - a^4*b^2)*cos(2*c)^2 + (a^6 -
a^4*b^2)*sin(2*c)^2)*d*x^4*cos(2*d*x^2)^2 + 4*((a^4*b^2 - a^2*b^4)*cos(c)^2
 + (a^4*b^2 - a^2*b^4)*sin(c)^2)*d*x^4*cos(d*x^2)^2 + ((a^6 - a^4*b^2)*cos(
2*c)^2 + (a^6 - a^4*b^2)*sin(2*c)^2)*d*x^4*sin(2*d*x^2)^2 + 4*(a^5*b - a^3*
```

```
b^3)*d*x^4*cos(c)*sin(d*x^2) + 4*((a^4*b^2 - a^2*b^4)*cos(c)^2 + (a^4*b^2 -
 a^2*b^4)*sin(c)^2)*d*x^4*sin(d*x^2)^2 + 4*(a^5*b - a^3*b^3)*d*x^4*cos(d*x^
2)*sin(c) + (a^6 - a^4*b^2)*d*x^4 + 2*(2*((a^5*b - a^3*b^3)*cos(c)*sin(2*c)
 - (a^5*b - a^3*b^3)*cos(2*c)*sin(c))*d*x^4*cos(d*x^2) - (a^6 - a^4*b^2)*d*
x^4*cos(2*c) - 2*((a^5*b - a^3*b^3)*cos(2*c)*cos(c) + (a^5*b - a^3*b^3)*sin
(2*c)*sin(c))*d*x^4*sin(d*x^2))*cos(2*d*x^2) + 2*(2*((a^5*b - a^3*b^3)*cos(
2*c)*cos(c) + (a^5*b - a^3*b^3)*sin(2*c)*sin(c))*d*x^4*cos(d*x^2) + 2*((a^5
*b - a^3*b^3)*cos(c)*sin(2*c) - (a^5*b - a^3*b^3)*cos(2*c)*sin(c))*d*x^4*si
n(d*x^2) + (a^6 - a^4*b^2)*d*x^4*sin(2*c))*sin(2*d*x^2))
```

Giac [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^3 (a + b\csc(c + dx^2))^2} \, dx = \int \frac{1}{(b\csc(dx^2 + c) + a)^2 x^3} \, dx$$

```
[In] integrate(1/x^3/(a+b*csc(d*x^2+c))^2,x, algorithm="giac")
[Out] integrate(1/((b*csc(d*x^2 + c) + a)^2*x^3), x)
```

Mupad [N/A]

Not integrable

Time = 17.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{1}{x^3 (a + b\csc(c + dx^2))^2} \, dx = \int \frac{1}{x^3 \left(a + \frac{b}{\sin(d\,x^2+c)}\right)^2} \, dx$$

```
[In] int(1/(x^3*(a + b/sin(c + d*x^2))^2),x)
[Out] int(1/(x^3*(a + b/sin(c + d*x^2))^2), x)
```

3.31 $\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 432

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^4}{4}-\frac{4bx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{14ibx^3\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{14ibx^3\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{84bx^{5/2}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{84bx^{5/2}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}-\frac{420ibx^2\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{420ibx^2\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{1680bx^{3/2}\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5}+\frac{5040ibx\operatorname{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{5040ibx\operatorname{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,-e^{i(c+d\sqrt{x})}\right)}{d^7}+\frac{10080b\sqrt{x}\operatorname{PolyLog}\left(7,e^{i(c+d\sqrt{x})}\right)}{d^7}-\frac{10080ib\operatorname{PolyLog}\left(8,-e^{i(c+d\sqrt{x})}\right)}{d^8}+\frac{10080ib\operatorname{PolyLog}\left(8,e^{i(c+d\sqrt{x})}\right)}{d^8}$$

```
[Out] 1/4*a*x^4-4*b*x^(7/2)*arctanh(exp(I*(c+d*x^(1/2))))/d+5040*I*b*x*polylog(6,
-exp(I*(c+d*x^(1/2))))/d^6-14*I*b*x^3*polylog(2,exp(I*(c+d*x^(1/2))))/d^2-8
```

```
4*b*x^(5/2)*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+84*b*x^(5/2)*polylog(3,exp
(I*(c+d*x^(1/2))))/d^3-10080*I*b*polylog(8,-exp(I*(c+d*x^(1/2))))/d^8+10080
*I*b*polylog(8,exp(I*(c+d*x^(1/2))))/d^8+1680*b*x^(3/2)*polylog(5,-exp(I*(c
+d*x^(1/2))))/d^5-1680*b*x^(3/2)*polylog(5,exp(I*(c+d*x^(1/2))))/d^5-420*I*
b*x^2*polylog(4,-exp(I*(c+d*x^(1/2))))/d^4+14*I*b*x^3*polylog(2,-exp(I*(c+d
*x^(1/2))))/d^2-5040*I*b*x*polylog(6,exp(I*(c+d*x^(1/2))))/d^6+420*I*b*x^2*
polylog(4,exp(I*(c+d*x^(1/2))))/d^4-10080*b*polylog(7,-exp(I*(c+d*x^(1/2)))
)*x^(1/2)/d^7+10080*b*polylog(7,exp(I*(c+d*x^(1/2))))*x^(1/2)/d^7
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used

$= \{14, 4290, 4268, 2611, 6744, 2320, 6724\}$

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx = \frac{ax^4}{4} - \frac{4bx^{7/2}\text{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} - \frac{10080ib\,\text{PolyLog}\left(8,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^8} + \frac{10080ib\,\text{PolyLog}\left(8,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^8} - \frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^7} + \frac{10080b\sqrt{x}\,\text{PolyLog}\left(7,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^7} + \frac{5040ibx\,\text{PolyLog}\left(6,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6} - \frac{5040ibx\,\text{PolyLog}\left(6,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6} + \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5} - \frac{1680bx^{3/2}\,\text{PolyLog}\left(5,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5} - \frac{420ibx^2\,\text{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4} + \frac{420ibx^2\,\text{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4} - \frac{84bx^{5/2}\,\text{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{84bx^{5/2}\,\text{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3} + \frac{14ibx^3\,\text{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} - \frac{14ibx^3\,\text{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}$$

```
[In] Int[x^3*(a + b*Csc[c + d*Sqrt[x]]),x]
[Out] (a*x^4)/4 - (4*b*x^(7/2)*ArcTanh[E^(I*(c + d*Sqrt[x]))])/d + ((14*I)*b*x^3*
PolyLog[2, -E^(I*(c + d*Sqrt[x]))])/d^2 - ((14*I)*b*x^3*PolyLog[2, E^(I*(c
```

```
+ d*Sqrt[x]))])/d^2 - (84*b*x^(5/2)*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3
 + (84*b*x^(5/2)*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3 - ((420*I)*b*x^2*Po
lyLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((420*I)*b*x^2*PolyLog[4, E^(I*(c +
 d*Sqrt[x]))])/d^4 + (1680*b*x^(3/2)*PolyLog[5, -E^(I*(c + d*Sqrt[x]))])/d^
5 - (1680*b*x^(3/2)*PolyLog[5, E^(I*(c + d*Sqrt[x]))])/d^5 + ((5040*I)*b*x*
PolyLog[6, -E^(I*(c + d*Sqrt[x]))])/d^6 - ((5040*I)*b*x*PolyLog[6, E^(I*(c
+ d*Sqrt[x]))])/d^6 - (10080*b*Sqrt[x]*PolyLog[7, -E^(I*(c + d*Sqrt[x]))])/
d^7 + (10080*b*Sqrt[x]*PolyLog[7, E^(I*(c + d*Sqrt[x]))])/d^7 - ((10080*I)*
b*PolyLog[8, -E^(I*(c + d*Sqrt[x]))])/d^8 + ((10080*I)*b*PolyLog[8, E^(I*(c
 + d*Sqrt[x]))])/d^8
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
```

```
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^3 + bx^3 \csc\left(c + d\sqrt{x}\right)\right) dx \\
&= \frac{ax^4}{4} + b \int x^3 \csc\left(c + d\sqrt{x}\right) dx \\
&= \frac{ax^4}{4} + (2b)\text{Subst}\left(\int x^7 \csc(c + dx)\, dx, x, \sqrt{x}\right) \\
&= \frac{ax^4}{4} - \frac{4bx^{7/2}\text{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} - \frac{(14b)\text{Subst}\left(\int x^6 \log\left(1 - e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d} \\
&\quad + \frac{(14b)\text{Subst}\left(\int x^6 \log\left(1 + e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d} \\
&= \frac{ax^4}{4} - \frac{4bx^{7/2}\text{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} + \frac{14ibx^3 \text{PolyLog}\left(2, -e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{14ibx^3 \text{PolyLog}\left(2, e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} \\
&\quad - \frac{(84ib)\text{Subst}\left(\int x^5 \text{PolyLog}\left(2, -e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^2} \\
&\quad + \frac{(84ib)\text{Subst}\left(\int x^5 \text{PolyLog}\left(2, e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^2}
\end{aligned}$$

$$= \frac{ax^4}{4} - \frac{4bx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3 \operatorname{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3 \operatorname{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{(420b)\operatorname{Subst}\left(\int x^4 \operatorname{PolyLog}\left(3, -e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^3} - \frac{(420b)\operatorname{Subst}\left(\int x^4 \operatorname{PolyLog}\left(3, e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^3}$$

$$= \frac{ax^4}{4} - \frac{4bx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3 \operatorname{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3 \operatorname{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2 \operatorname{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2 \operatorname{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(1680ib)\operatorname{Subst}\left(\int x^3 \operatorname{PolyLog}\left(4, -e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^4} - \frac{(1680ib)\operatorname{Subst}\left(\int x^3 \operatorname{PolyLog}\left(4, e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^4}$$

$$= \frac{ax^4}{4} - \frac{4bx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3 \operatorname{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3 \operatorname{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2 \operatorname{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2 \operatorname{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, -e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{(5040b)\operatorname{Subst}\left(\int x^2 \operatorname{PolyLog}\left(5, -e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^5} + \frac{(5040b)\operatorname{Subst}\left(\int x^2 \operatorname{PolyLog}\left(5, e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^5}$$

$$= \frac{ax^4}{4} - \frac{4bx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3 \operatorname{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3 \operatorname{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2 \operatorname{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2 \operatorname{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, -e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, e^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{5040ibx \operatorname{PolyLog}\left(6, -e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{5040ibx \operatorname{PolyLog}\left(6, e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{(10080ib)\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(6, -e^{i(c+dx)}\right) \, dx, x, \sqrt{x}\right)}{d^6} + \frac{(10080ib)\operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(6, e^{i(c+dx)}\right) \, dx, x, \sqrt{x}\right)}{d^6}$$

$$= \frac{ax^4}{4} - \frac{4bx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3 \operatorname{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3 \operatorname{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2} \operatorname{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2 \operatorname{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2 \operatorname{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, -e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{1680bx^{3/2} \operatorname{PolyLog}\left(5, e^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{5040ibx \operatorname{PolyLog}\left(6, -e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{5040ibx \operatorname{PolyLog}\left(6, e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, -e^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{10080b\sqrt{x} \operatorname{PolyLog}\left(7, e^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{(10080b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(7, -e^{i(c+dx)}\right) \, dx, x, \sqrt{x}\right)}{d^7} - \frac{(10080b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(7, e^{i(c+dx)}\right) \, dx, x, \sqrt{x}\right)}{d^7}$$

$$= \frac{ax^4}{4} - \frac{4bx^{7/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3 \text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3 \text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2} \text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2} \text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2 \text{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2 \text{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{1680bx^{3/2} \text{PolyLog}\left(5, -e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{1680bx^{3/2} \text{PolyLog}\left(5, e^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{5040ibx \text{PolyLog}\left(6, -e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{5040ibx \text{PolyLog}\left(6, e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{10080b\sqrt{x} \text{PolyLog}\left(7, -e^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{10080b\sqrt{x} \text{PolyLog}\left(7, e^{i(c+d\sqrt{x})}\right)}{d^7} - \frac{(10080ib)\text{Subst}\left(\int \frac{\text{PolyLog}(7,-x)}{x} dx, x, e^{i(c+d\sqrt{x})}\right)}{d^8} + \frac{(10080ib)\text{Subst}\left(\int \frac{\text{PolyLog}(7,x)}{x} dx, x, e^{i(c+d\sqrt{x})}\right)}{d^8}$$

$$= \frac{ax^4}{4} - \frac{4bx^{7/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{14ibx^3 \text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{14ibx^3 \text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{84bx^{5/2} \text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{84bx^{5/2} \text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{420ibx^2 \text{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{420ibx^2 \text{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{1680bx^{3/2} \text{PolyLog}\left(5, -e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{1680bx^{3/2} \text{PolyLog}\left(5, e^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{5040ibx \text{PolyLog}\left(6, -e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{5040ibx \text{PolyLog}\left(6, e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{10080b\sqrt{x} \text{PolyLog}\left(7, -e^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{10080b\sqrt{x} \text{PolyLog}\left(7, e^{i(c+d\sqrt{x})}\right)}{d^7} - \frac{10080ib \text{PolyLog}\left(8, -e^{i(c+d\sqrt{x})}\right)}{d^8} + \frac{10080ib \text{PolyLog}\left(8, e^{i(c+d\sqrt{x})}\right)}{d^8}$$

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.03

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^4}{4}+\frac{2b\left(d^7x^{7/2}\log\left(1-e^{i(c+d\sqrt{x})}\right)-d^7x^{7/2}\log\left(1+e^{i(c+d\sqrt{x})}\right)+7id^6x^3\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)-7id^6x^3\operatorname{Po}\right.}{}$$

```
[In] Integrate[x^3*(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^4)/4 + (2*b*(d^7*x^(7/2)*Log[1 - E^(I*(c + d*Sqrt[x]))] - d^7*x^(7/2)*
Log[1 + E^(I*(c + d*Sqrt[x]))] + (7*I)*d^6*x^3*PolyLog[2, -E^(I*(c + d*Sqrt
[x]))] - (7*I)*d^6*x^3*PolyLog[2, E^(I*(c + d*Sqrt[x]))] - 42*d^5*x^(5/2)*P
olyLog[3, -E^(I*(c + d*Sqrt[x]))] + 42*d^5*x^(5/2)*PolyLog[3, E^(I*(c + d*S
qrt[x]))] - (210*I)*d^4*x^2*PolyLog[4, -E^(I*(c + d*Sqrt[x]))] + (210*I)*d^
4*x^2*PolyLog[4, E^(I*(c + d*Sqrt[x]))] + 840*d^3*x^(3/2)*PolyLog[5, -E^(I*
(c + d*Sqrt[x]))] - 840*d^3*x^(3/2)*PolyLog[5, E^(I*(c + d*Sqrt[x]))] + (25
20*I)*d^2*x*PolyLog[6, -E^(I*(c + d*Sqrt[x]))] - (2520*I)*d^2*x*PolyLog[6,
E^(I*(c + d*Sqrt[x]))] - 5040*d*Sqrt[x]*PolyLog[7, -E^(I*(c + d*Sqrt[x]))]
+ 5040*d*Sqrt[x]*PolyLog[7, E^(I*(c + d*Sqrt[x]))] - (5040*I)*PolyLog[8, -E
^(I*(c + d*Sqrt[x]))] + (5040*I)*PolyLog[8, E^(I*(c + d*Sqrt[x]))]))/d^8
```

Maple [F]

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)dx$$

```
[In] int(x^3*(a+b*csc(c+d*x^(1/2))),x)
```

```
[Out] int(x^3*(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^3\,dx$$

```
[In] integrate(x^3*(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral(b*x^3*csc(d*sqrt(x) + c) + a*x^3, x)
```

Sympy [F]

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x**3*(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(x**3*(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1498 vs. 2(338) = 676.

Time = 0.33 (sec) , antiderivative size = 1498, normalized size of antiderivative = 3.47

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\text{Too large to display}$$

```
[In] integrate(x^3*(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 1/4*((d*sqrt(x) + c)^8*a - 8*(d*sqrt(x) + c)^7*a*c + 28*(d*sqrt(x) + c)^6*a
*c^2 - 56*(d*sqrt(x) + c)^5*a*c^3 + 70*(d*sqrt(x) + c)^4*a*c^4 - 56*(d*sqrt
(x) + c)^3*a*c^5 + 28*(d*sqrt(x) + c)^2*a*c^6 - 8*(d*sqrt(x) + c)*a*c^7 + 8
*b*c^7*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) + 8*(-I*(d*sqrt(x) + c)
^7*b + 7*I*(d*sqrt(x) + c)^6*b*c - 21*I*(d*sqrt(x) + c)^5*b*c^2 + 35*I*(d*s
qrt(x) + c)^4*b*c^3 - 35*I*(d*sqrt(x) + c)^3*b*c^4 + 21*I*(d*sqrt(x) + c)^2
*b*c^5 - 7*I*(d*sqrt(x) + c)*b*c^6)*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(
x) + c) + 1) + 8*(-I*(d*sqrt(x) + c)^7*b + 7*I*(d*sqrt(x) + c)^6*b*c - 21*I
*(d*sqrt(x) + c)^5*b*c^2 + 35*I*(d*sqrt(x) + c)^4*b*c^3 - 35*I*(d*sqrt(x) +
 c)^3*b*c^4 + 21*I*(d*sqrt(x) + c)^2*b*c^5 - 7*I*(d*sqrt(x) + c)*b*c^6)*arc
tan2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c) + 1) + 56*(I*(d*sqrt(x) + c)^6
*b - 6*I*(d*sqrt(x) + c)^5*b*c + 15*I*(d*sqrt(x) + c)^4*b*c^2 - 20*I*(d*sqr
t(x) + c)^3*b*c^3 + 15*I*(d*sqrt(x) + c)^2*b*c^4 - 6*I*(d*sqrt(x) + c)*b*c^
5 + I*b*c^6)*dilog(-e^(I*d*sqrt(x) + I*c)) + 56*(-I*(d*sqrt(x) + c)^6*b + 6
*I*(d*sqrt(x) + c)^5*b*c - 15*I*(d*sqrt(x) + c)^4*b*c^2 + 20*I*(d*sqrt(x) +
 c)^3*b*c^3 - 15*I*(d*sqrt(x) + c)^2*b*c^4 + 6*I*(d*sqrt(x) + c)*b*c^5 - I*
b*c^6)*dilog(e^(I*d*sqrt(x) + I*c)) - 4*((d*sqrt(x) + c)^7*b - 7*(d*sqrt(x)
 + c)^6*b*c + 21*(d*sqrt(x) + c)^5*b*c^2 - 35*(d*sqrt(x) + c)^4*b*c^3 + 35*
(d*sqrt(x) + c)^3*b*c^4 - 21*(d*sqrt(x) + c)^2*b*c^5 + 7*(d*sqrt(x) + c)*b*
c^6)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*cos(d*sqrt(x) + c)
 + 1) + 4*((d*sqrt(x) + c)^7*b - 7*(d*sqrt(x) + c)^6*b*c + 21*(d*sqrt(x) +
c)^5*b*c^2 - 35*(d*sqrt(x) + c)^4*b*c^3 + 35*(d*sqrt(x) + c)^3*b*c^4 - 21*(
d*sqrt(x) + c)^2*b*c^5 + 7*(d*sqrt(x) + c)*b*c^6)*log(cos(d*sqrt(x) + c)^2
+ sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1) - 40320*I*b*polylog(8, -
```

```
e^(I*d*sqrt(x) + I*c)) + 40320*I*b*polylog(8, e^(I*d*sqrt(x) + I*c)) - 4032
0*((d*sqrt(x) + c)*b - b*c)*polylog(7, -e^(I*d*sqrt(x) + I*c)) + 40320*((d*
sqrt(x) + c)*b - b*c)*polylog(7, e^(I*d*sqrt(x) + I*c)) + 20160*(I*(d*sqrt(
x) + c)^2*b - 2*I*(d*sqrt(x) + c)*b*c + I*b*c^2)*polylog(6, -e^(I*d*sqrt(x)
 + I*c)) + 20160*(-I*(d*sqrt(x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b*c - I*b*c^
2)*polylog(6, e^(I*d*sqrt(x) + I*c)) + 6720*((d*sqrt(x) + c)^3*b - 3*(d*sqr
t(x) + c)^2*b*c + 3*(d*sqrt(x) + c)*b*c^2 - b*c^3)*polylog(5, -e^(I*d*sqrt(
x) + I*c)) - 6720*((d*sqrt(x) + c)^3*b - 3*(d*sqrt(x) + c)^2*b*c + 3*(d*sqr
t(x) + c)*b*c^2 - b*c^3)*polylog(5, e^(I*d*sqrt(x) + I*c)) + 1680*(-I*(d*sq
rt(x) + c)^4*b + 4*I*(d*sqrt(x) + c)^3*b*c - 6*I*(d*sqrt(x) + c)^2*b*c^2 +
4*I*(d*sqrt(x) + c)*b*c^3 - I*b*c^4)*polylog(4, -e^(I*d*sqrt(x) + I*c)) + 1
680*(I*(d*sqrt(x) + c)^4*b - 4*I*(d*sqrt(x) + c)^3*b*c + 6*I*(d*sqrt(x) + c
)^2*b*c^2 - 4*I*(d*sqrt(x) + c)*b*c^3 + I*b*c^4)*polylog(4, e^(I*d*sqrt(x)
+ I*c)) - 336*((d*sqrt(x) + c)^5*b - 5*(d*sqrt(x) + c)^4*b*c + 10*(d*sqrt(x
) + c)^3*b*c^2 - 10*(d*sqrt(x) + c)^2*b*c^3 + 5*(d*sqrt(x) + c)*b*c^4 - b*c
^5)*polylog(3, -e^(I*d*sqrt(x) + I*c)) + 336*((d*sqrt(x) + c)^5*b - 5*(d*sq
rt(x) + c)^4*b*c + 10*(d*sqrt(x) + c)^3*b*c^2 - 10*(d*sqrt(x) + c)^2*b*c^3
+ 5*(d*sqrt(x) + c)*b*c^4 - b*c^5)*polylog(3, e^(I*d*sqrt(x) + I*c)))/d^8
```

Giac [F]

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^3\,dx$$

```
[In] integrate(x^3*(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int x^3\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^3*(a + b/sin(c + d*x^(1/2))),x)
[Out] int(x^3*(a + b/sin(c + d*x^(1/2))), x)
```

3.32 $\int x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 316

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^3}{3}-\frac{4bx^{5/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{10ibx^2\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{10ibx^2\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{40bx^{3/2}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{40bx^{3/2}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}-\frac{120ibx\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{120ibx\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{240b\sqrt{x}\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5}+\frac{240ib\operatorname{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{240ib\operatorname{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6}$$

```
[Out] 1/3*a*x^3-4*b*x^(5/2)*arctanh(exp(I*(c+d*x^(1/2))))/d+10*I*b*x^2*polylog(2,
-exp(I*(c+d*x^(1/2))))/d^2-10*I*b*x^2*polylog(2,exp(I*(c+d*x^(1/2))))/d^2-4
0*b*x^(3/2)*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+40*b*x^(3/2)*polylog(3,exp
(I*(c+d*x^(1/2))))/d^3-120*I*b*x*polylog(4,-exp(I*(c+d*x^(1/2))))/d^4+120*I
*b*x*polylog(4,exp(I*(c+d*x^(1/2))))/d^4+240*I*b*polylog(6,-exp(I*(c+d*x^(1
/2))))/d^6-240*I*b*polylog(6,exp(I*(c+d*x^(1/2))))/d^6+240*b*polylog(5,-exp
(I*(c+d*x^(1/2))))*x^(1/2)/d^5-240*b*polylog(5,exp(I*(c+d*x^(1/2))))*x^(1/2
)/d^5
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {14, 4290, 4268, 2611, 6744, 2320, 6724}

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^3}{3}-\frac{4bx^{5/2}\operatorname{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}+\frac{240ib\,\text{PolyLog}\left(6,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}-\frac{240ib\,\text{PolyLog}\left(6,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}+\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{240b\sqrt{x}\,\text{PolyLog}\left(5,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{120ibx\,\text{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{120ibx\,\text{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{40bx^{3/2}\,\text{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{40bx^{3/2}\,\text{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{10ibx^2\,\text{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{10ibx^2\,\text{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}$$

```
[In] Int[x^2*(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^3)/3 - (4*b*x^(5/2)*ArcTanh[E^(I*(c + d*Sqrt[x]))])/d + ((10*I)*b*x^2*
PolyLog[2, -E^(I*(c + d*Sqrt[x]))])/d^2 - ((10*I)*b*x^2*PolyLog[2, E^(I*(c
+ d*Sqrt[x]))])/d^2 - (40*b*x^(3/2)*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3
 + (40*b*x^(3/2)*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3 - ((120*I)*b*x*Poly
Log[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((120*I)*b*x*PolyLog[4, E^(I*(c + d*S
qrt[x]))])/d^4 + (240*b*Sqrt[x]*PolyLog[5, -E^(I*(c + d*Sqrt[x]))])/d^5 - (
240*b*Sqrt[x]*PolyLog[5, E^(I*(c + d*Sqrt[x]))])/d^5 + ((240*I)*b*PolyLog[6
```

```
, -E^(I*(c + d*Sqrt[x]))])/d^6 - ((240*I)*b*PolyLog[6, E^(I*(c + d*Sqrt[x])
)])/d^6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^2 + bx^2 \csc\left(c + d\sqrt{x}\right)\right) dx \\
&= \frac{ax^3}{3} + b\int x^2 \csc\left(c + d\sqrt{x}\right) dx \\
&= \frac{ax^3}{3} + (2b)\text{Subst}\left(\int x^5 \csc(c+dx)\,dx, x, \sqrt{x}\right) \\
&= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{(10b)\text{Subst}\left(\int x^4 \log\left(1 - e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d} \\
&\quad + \frac{(10b)\text{Subst}\left(\int x^4 \log\left(1 + e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d} \\
&= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10ibx^2\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{10ibx^2\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{(40ib)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2, -e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^2} \\
&\quad + \frac{(40ib)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2, e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^2} \\
&= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10ibx^2\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{10ibx^2\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{40bx^{3/2}\,\text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&\quad + \frac{40bx^{3/2}\,\text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&\quad + \frac{(120b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3, -e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^3} \\
&\quad - \frac{(120b)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3, e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^3}
\end{aligned}$$

$$= \frac{ax^3}{3} - \frac{4bx^{5/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10ibx^2 \text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{10ibx^2 \text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{40bx^{3/2} \text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{40bx^{3/2} \text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{120ibx \text{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{120ibx \text{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(240ib)\text{Subst}\left(\int x \text{PolyLog}\left(4, -e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^4} - \frac{(240ib)\text{Subst}\left(\int x \text{PolyLog}\left(4, e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^4}$$

$$= \frac{ax^3}{3} - \frac{4bx^{5/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10ibx^2 \text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{10ibx^2 \text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{40bx^{3/2} \text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{40bx^{3/2} \text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{120ibx \text{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{120ibx \text{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{240b\sqrt{x} \text{PolyLog}\left(5, -e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{240b\sqrt{x} \text{PolyLog}\left(5, e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{(240b)\text{Subst}\left(\int \text{PolyLog}\left(5, -e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^5} + \frac{(240b)\text{Subst}\left(\int \text{PolyLog}\left(5, e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^5}$$

$$= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10ibx^2 \operatorname{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{10ibx^2 \operatorname{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{120ibx \operatorname{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{120ibx \operatorname{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, -e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, e^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{(240ib)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(5,-x)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{(240ib)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(5,x)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^6}$$

$$= \frac{ax^3}{3} - \frac{4bx^{5/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{10ibx^2 \operatorname{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{10ibx^2 \operatorname{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{40bx^{3/2} \operatorname{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{120ibx \operatorname{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{120ibx \operatorname{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, -e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{240b\sqrt{x} \operatorname{PolyLog}\left(5, e^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{240ib \operatorname{PolyLog}\left(6, -e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{240ib \operatorname{PolyLog}\left(6, e^{i(c+d\sqrt{x})}\right)}{d^6}$$

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.05

$$\int x^2\left(a + b\csc\left(c + d\sqrt{x}\right)\right)\,dx = \frac{ax^3}{3} + \frac{2b\left(d^5x^{5/2}\log\left(1 - e^{i(c+d\sqrt{x})}\right) - d^5x^{5/2}\log\left(1 + e^{i(c+d\sqrt{x})}\right) + 5id^4x^2 \operatorname{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right) - 5id^4x^2 \operatorname{I}\right.}{}$$

```
[In] Integrate[x^2*(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^3)/3 + (2*b*(d^5*x^(5/2)*Log[1 - E^(I*(c + d*Sqrt[x]))] - d^5*x^(5/2)*
Log[1 + E^(I*(c + d*Sqrt[x]))] + (5*I)*d^4*x^2*PolyLog[2, -E^(I*(c + d*Sqrt
```

```
[x]))] - (5*I)*d^4*x^2*PolyLog[2, E^(I*(c + d*Sqrt[x]))] - 20*d^3*x^(3/2)*P
olyLog[3, -E^(I*(c + d*Sqrt[x]))] + 20*d^3*x^(3/2)*PolyLog[3, E^(I*(c + d*S
qrt[x]))] - (60*I)*d^2*x*PolyLog[4, -E^(I*(c + d*Sqrt[x]))] + (60*I)*d^2*x*
PolyLog[4, E^(I*(c + d*Sqrt[x]))] + 120*d*Sqrt[x]*PolyLog[5, -E^(I*(c + d*S
qrt[x]))] - 120*d*Sqrt[x]*PolyLog[5, E^(I*(c + d*Sqrt[x]))] + (120*I)*PolyL
og[6, -E^(I*(c + d*Sqrt[x]))] - (120*I)*PolyLog[6, E^(I*(c + d*Sqrt[x]))]))
/d^6
```

Maple [F]

$$\int x^2 \left(a + b \csc \left(c + d\sqrt{x}\right)\right) dx$$

```
[In] int(x^2*(a+b*csc(c+d*x^(1/2))),x)
[Out] int(x^2*(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^2 \left(a + b \csc \left(c + d\sqrt{x}\right)\right)\, dx = \int \left(b \csc \left(d\sqrt{x} + c\right) + a\right) x^2\, dx$$

```
[In] integrate(x^2*(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x^2*csc(d*sqrt(x) + c) + a*x^2, x)
```

Sympy [F]

$$\int x^2 \left(a + b \csc \left(c + d\sqrt{x}\right)\right)\, dx = \int x^2 \left(a + b \csc \left(c + d\sqrt{x}\right)\right)\, dx$$

```
[In] integrate(x**2*(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(x**2*(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 956 vs. $2(246) = 492$.

Time = 0.29 (sec) , antiderivative size = 956, normalized size of antiderivative = 3.03

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] 1/3*((d*sqrt(x) + c)^6*a - 6*(d*sqrt(x) + c)^5*a*c + 15*(d*sqrt(x) + c)^4*a
*c^2 - 20*(d*sqrt(x) + c)^3*a*c^3 + 15*(d*sqrt(x) + c)^2*a*c^4 - 6*(d*sqrt(
x) + c)*a*c^5 + 6*b*c^5*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) + 6*(-
I*(d*sqrt(x) + c)^5*b + 5*I*(d*sqrt(x) + c)^4*b*c - 10*I*(d*sqrt(x) + c)^3*
b*c^2 + 10*I*(d*sqrt(x) + c)^2*b*c^3 - 5*I*(d*sqrt(x) + c)*b*c^4)*arctan2(s
in(d*sqrt(x) + c), cos(d*sqrt(x) + c) + 1) + 6*(-I*(d*sqrt(x) + c)^5*b + 5*
I*(d*sqrt(x) + c)^4*b*c - 10*I*(d*sqrt(x) + c)^3*b*c^2 + 10*I*(d*sqrt(x) +
c)^2*b*c^3 - 5*I*(d*sqrt(x) + c)*b*c^4)*arctan2(sin(d*sqrt(x) + c), -cos(d*
sqrt(x) + c) + 1) + 30*(I*(d*sqrt(x) + c)^4*b - 4*I*(d*sqrt(x) + c)^3*b*c +
 6*I*(d*sqrt(x) + c)^2*b*c^2 - 4*I*(d*sqrt(x) + c)*b*c^3 + I*b*c^4)*dilog(-
e^(I*d*sqrt(x) + I*c)) + 30*(-I*(d*sqrt(x) + c)^4*b + 4*I*(d*sqrt(x) + c)^3
*b*c - 6*I*(d*sqrt(x) + c)^2*b*c^2 + 4*I*(d*sqrt(x) + c)*b*c^3 - I*b*c^4)*d
ilog(e^(I*d*sqrt(x) + I*c)) - 3*((d*sqrt(x) + c)^5*b - 5*(d*sqrt(x) + c)^4*
b*c + 10*(d*sqrt(x) + c)^3*b*c^2 - 10*(d*sqrt(x) + c)^2*b*c^3 + 5*(d*sqrt(x
) + c)*b*c^4)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*cos(d*sqr
t(x) + c) + 1) + 3*((d*sqrt(x) + c)^5*b - 5*(d*sqrt(x) + c)^4*b*c + 10*(d*s
qrt(x) + c)^3*b*c^2 - 10*(d*sqrt(x) + c)^2*b*c^3 + 5*(d*sqrt(x) + c)*b*c^4)
*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1
) + 720*I*b*polylog(6, -e^(I*d*sqrt(x) + I*c)) - 720*I*b*polylog(6, e^(I*d*
sqrt(x) + I*c)) + 720*((d*sqrt(x) + c)*b - b*c)*polylog(5, -e^(I*d*sqrt(x)
+ I*c)) - 720*((d*sqrt(x) + c)*b - b*c)*polylog(5, e^(I*d*sqrt(x) + I*c)) +
 360*(-I*(d*sqrt(x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b*c - I*b*c^2)*polylog(4
, -e^(I*d*sqrt(x) + I*c)) + 360*(I*(d*sqrt(x) + c)^2*b - 2*I*(d*sqrt(x) + c
)*b*c + I*b*c^2)*polylog(4, e^(I*d*sqrt(x) + I*c)) - 120*((d*sqrt(x) + c)^3
*b - 3*(d*sqrt(x) + c)^2*b*c + 3*(d*sqrt(x) + c)*b*c^2 - b*c^3)*polylog(3,
-e^(I*d*sqrt(x) + I*c)) + 120*((d*sqrt(x) + c)^3*b - 3*(d*sqrt(x) + c)^2*b*
c + 3*(d*sqrt(x) + c)*b*c^2 - b*c^3)*polylog(3, e^(I*d*sqrt(x) + I*c)))/d^6
```

Giac [F]

$$\int x^2(a+b\csc(c+d\sqrt{x}))\,dx=\int(b\csc(d\sqrt{x}+c)+a)x^2\,dx$$

```
[In] integrate(x^2*(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\csc(c+d\sqrt{x}))\,dx=\int x^2\left(a+\frac{b}{\sin(c+d\sqrt{x})}\right)\,dx$$

```
[In] int(x^2*(a + b/sin(c + d*x^(1/2))),x)
[Out] int(x^2*(a + b/sin(c + d*x^(1/2))), x)
```

3.33 $\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 200

$$\begin{aligned}\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=&\frac{ax^2}{2}-\frac{4bx^{3/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}\\&+\frac{6ibx\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}\\&-\frac{6ibx\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}\\&-\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}\\&+\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}\\&-\frac{12ib\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}\\&+\frac{12ib\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}\end{aligned}$$

```
[Out] 1/2*a*x^2-4*b*x^(3/2)*arctanh(exp(I*(c+d*x^(1/2))))/d+6*I*b*x*polylog(2,-ex
p(I*(c+d*x^(1/2))))/d^2-6*I*b*x*polylog(2,exp(I*(c+d*x^(1/2))))/d^2-12*I*b*
polylog(4,-exp(I*(c+d*x^(1/2))))/d^4+12*I*b*polylog(4,exp(I*(c+d*x^(1/2))))
/d^4-12*b*polylog(3,-exp(I*(c+d*x^(1/2))))*x^(1/2)/d^3+12*b*polylog(3,exp(I
*(c+d*x^(1/2))))*x^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {14, 4290, 4268, 2611, 6744, 2320, 6724}

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{ax^2}{2}-\frac{4bx^{3/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{12ib\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{12ib\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}-\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{12b\sqrt{x}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{6ibx\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{6ibx\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}$$

```
[In] Int[x*(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^2)/2 - (4*b*x^(3/2)*ArcTanh[E^(I*(c + d*Sqrt[x]))])/d + ((6*I)*b*x*Pol
yLog[2, -E^(I*(c + d*Sqrt[x]))])/d^2 - ((6*I)*b*x*PolyLog[2, E^(I*(c + d*Sq
rt[x]))])/d^2 - (12*b*Sqrt[x]*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3 + (12
*b*Sqrt[x]*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3 - ((12*I)*b*PolyLog[4, -E
^(I*(c + d*Sqrt[x]))])/d^4 + ((12*I)*b*PolyLog[4, E^(I*(c + d*Sqrt[x]))])/d
^4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = \int \left(ax + bx\csc\left(c + d\sqrt{x}\right)\right)\,dx$$

$$= \frac{ax^2}{2} + b\int x\csc\left(c + d\sqrt{x}\right)\,dx$$

$$= \frac{ax^2}{2} + (2b)\text{Subst}\left(\int x^3 \csc(c+dx)\,dx, x, \sqrt{x}\right)$$

$$= \frac{ax^2}{2} - \frac{4bx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{(6b)\text{Subst}\left(\int x^2 \log\left(1-e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(6b)\text{Subst}\left(\int x^2 \log\left(1+e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}$$

$$= \frac{ax^2}{2} - \frac{4bx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6ibx\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(12ib)\text{Subst}\left(\int x\,\text{PolyLog}\left(2, -e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2} + \frac{(12ib)\text{Subst}\left(\int x\,\text{PolyLog}\left(2, e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= \frac{ax^2}{2} - \frac{4bx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6ibx\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{(12b)\text{Subst}\left(\int \text{PolyLog}\left(3, -e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3} - \frac{(12b)\text{Subst}\left(\int \text{PolyLog}\left(3, e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3}$$

$$= \frac{ax^2}{2} - \frac{4bx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6ibx\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{(12ib)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(12ib)\text{Subst}\left(\int \frac{\text{PolyLog}(3,x)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^4}$$

$$= \frac{ax^2}{2} - \frac{4bx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{6ibx\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ibx\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b\sqrt{x}\,\text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{12ib\,\text{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{12ib\,\text{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.30

$$\int x\left(a + b\csc\left(c + d\sqrt{x}\right)\right)\,dx = \frac{ax^2}{2} - \frac{2b\left(2d^3x^{3/2}\text{arctanh}\left(\cos\left(c + d\sqrt{x}\right) + i\sin\left(c + d\sqrt{x}\right)\right) - 3id^2x\,\text{PolyLog}\left(2, -\cos\left(c + d\sqrt{x}\right) - i\sin\left(c + \right.\right.\right.}{}$$

```
[In] Integrate[x*(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (a*x^2)/2 - (2*b*(2*d^3*x^(3/2)*ArcTanh[Cos[c + d*Sqrt[x]] + I*Sin[c + d*Sq
rt[x]]] - (3*I)*d^2*x*PolyLog[2, -Cos[c + d*Sqrt[x]] - I*Sin[c + d*Sqrt[x]]
] + (3*I)*d^2*x*PolyLog[2, Cos[c + d*Sqrt[x]] + I*Sin[c + d*Sqrt[x]]] + 6*d
*Sqrt[x]*PolyLog[3, -Cos[c + d*Sqrt[x]] - I*Sin[c + d*Sqrt[x]]] - 6*d*Sqrt[
x]*PolyLog[3, Cos[c + d*Sqrt[x]] + I*Sin[c + d*Sqrt[x]]] + (6*I)*PolyLog[4,
 -Cos[c + d*Sqrt[x]] - I*Sin[c + d*Sqrt[x]]] - (6*I)*PolyLog[4, Cos[c + d*S
qrt[x]] + I*Sin[c + d*Sqrt[x]]]))/d^4
```

Maple [F]

$$\int x\left(a + b\csc\left(c + d\sqrt{x}\right)\right)dx$$

```
[In] int(x*(a+b*csc(c+d*x^(1/2))),x)
```

```
[Out] int(x*(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\csc\left(d\sqrt{x}+c\right)+a\right)x\,dx$$

```
[In] integrate(x*(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x*csc(d*sqrt(x) + c) + a*x, x)
```

Sympy [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate(x*(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(x*(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(154) = 308.

Time = 0.27 (sec) , antiderivative size = 534, normalized size of antiderivative = 2.67

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$$
$$=\frac{\left(d\sqrt{x}+c\right)^4a-4\left(d\sqrt{x}+c\right)^3ac+6\left(d\sqrt{x}+c\right)^2ac^2-4\left(d\sqrt{x}+c\right)ac^3+4\,bc^3\log\left(\cot\left(d\sqrt{x}+c\right)+\csc\left(d\sqrt{}\right.\right.}{}$$

```
[In] integrate(x*(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 1/2*((d*sqrt(x) + c)^4*a - 4*(d*sqrt(x) + c)^3*a*c + 6*(d*sqrt(x) + c)^2*a*
c^2 - 4*(d*sqrt(x) + c)*a*c^3 + 4*b*c^3*log(cot(d*sqrt(x) + c) + csc(d*sqrt
(x) + c)) + 4*(-I*(d*sqrt(x) + c)^3*b + 3*I*(d*sqrt(x) + c)^2*b*c - 3*I*(d*
sqrt(x) + c)*b*c^2)*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) + 1) + 4
*(-I*(d*sqrt(x) + c)^3*b + 3*I*(d*sqrt(x) + c)^2*b*c - 3*I*(d*sqrt(x) + c)*
b*c^2)*arctan2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c) + 1) + 12*(I*(d*sqrt
(x) + c)^2*b - 2*I*(d*sqrt(x) + c)*b*c + I*b*c^2)*dilog(-e^(I*d*sqrt(x) + I
*c)) + 12*(-I*(d*sqrt(x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b*c - I*b*c^2)*dilo
g(e^(I*d*sqrt(x) + I*c)) - 2*((d*sqrt(x) + c)^3*b - 3*(d*sqrt(x) + c)^2*b*c
 + 3*(d*sqrt(x) + c)*b*c^2)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2
 + 2*cos(d*sqrt(x) + c) + 1) + 2*((d*sqrt(x) + c)^3*b - 3*(d*sqrt(x) + c)^2
```

```
*b*c + 3*(d*sqrt(x) + c)*b*c^2)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) +
c)^2 - 2*cos(d*sqrt(x) + c) + 1) - 24*I*b*polylog(4, -e^(I*d*sqrt(x) + I*c)
) + 24*I*b*polylog(4, e^(I*d*sqrt(x) + I*c)) - 24*((d*sqrt(x) + c)*b - b*c)
*polylog(3, -e^(I*d*sqrt(x) + I*c)) + 24*((d*sqrt(x) + c)*b - b*c)*polylog(
3, e^(I*d*sqrt(x) + I*c)))/d^4
```

Giac [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x\,dx$$

```
[In] integrate(x*(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int x\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x*(a + b/sin(c + d*x^(1/2))),x)
[Out] int(x*(a + b/sin(c + d*x^(1/2))), x)
```

3.34 $\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx=a\log(x)+b\text{Int}\left(\frac{\csc\left(c+d\sqrt{x}\right)}{x},x\right)$$

```
[Out] a*ln(x)+b*Unintegrable(csc(c+d*x^(1/2))/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx$$

```
[In] Int[(a + b*Csc[c + d*Sqrt[x]])/x,x]
```

```
[Out] a*Log[x] + b*Defer[Int][Csc[c + d*Sqrt[x]]/x, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x}+\frac{b\csc\left(c+d\sqrt{x}\right)}{x}\right)\,dx\\&=a\log(x)+b\int \frac{\csc\left(c+d\sqrt{x}\right)}{x}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 11.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*Sqrt[x]])/x,x]
[Out] Integrate[(a + b*Csc[c + d*Sqrt[x]])/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}dx$$

```
[In] int((a+b*csc(c+d*x^(1/2)))/x,x)
[Out] int((a+b*csc(c+d*x^(1/2)))/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x,x, algorithm="fricas")
[Out] integral((b*csc(d*sqrt(x) + c) + a)/x, x)
```

Sympy [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))/x,x)
[Out] Integral((a + b*csc(c + d*sqrt(x)))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.83

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x,x, algorithm="maxima")
```

```
[Out] b*integrate(sin(d*sqrt(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^
2 + 2*cos(d*sqrt(x) + c) + 1)*x), x) + b*integrate(sin(d*sqrt(x) + c)/((cos
(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1)*x), x)
 + a*log(x)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x,x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*sqrt(x) + c) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 18.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x}\,dx=\int \frac{a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}}{x}\,dx$$

```
[In] int((a + b/sin(c + d*x^(1/2)))/x,x)
```

```
[Out] int((a + b/sin(c + d*x^(1/2)))/x, x)
```

3.35 $\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx = -\frac{a}{x}+b\text{Int}\left(\frac{\csc\left(c+d\sqrt{x}\right)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(csc(c+d*x^(1/2))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx = \int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Int[(a + b*Csc[c + d*Sqrt[x]])/x^2,x]
```

```
[Out] -(a/x) + b*Defer[Int][Csc[c + d*Sqrt[x]]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{a}{x^2}+\frac{b\csc\left(c+d\sqrt{x}\right)}{x^2}\right)dx \\ &= -\frac{a}{x}+b\int \frac{\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 16.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^2,x]
[Out] Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}dx$$

```
[In] int((a+b*csc(c+d*x^(1/2)))/x^2,x)
[Out] int((a+b*csc(c+d*x^(1/2)))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="fricas")
[Out] integral((b*csc(d*sqrt(x) + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))/x**2,x)
[Out] Integral((a + b*csc(c + d*sqrt(x)))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 111, normalized size of antiderivative = 6.17

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

[In] integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="maxima")

[Out] ((b*integrate(sin(d*sqrt(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c
)^2 + 2*cos(d*sqrt(x) + c) + 1)*x^2), x) + b*integrate(sin(d*sqrt(x) + c)/(
(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1)*x^
2), x))*x - a)/x

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

[In] integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="giac")

[Out] integrate((b*csc(d*sqrt(x) + c) + a)/x^2, x)

Mupad [N/A]

Not integrable

Time = 18.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}}{x^2}\,dx$$

[In] int((a + b/sin(c + d*x^(1/2)))/x^2,x)

[Out] int((a + b/sin(c + d*x^(1/2)))/x^2, x)

3.36 $\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx$

Optimal result

Integrand size = 20, antiderivative size = 695

$$\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=-\frac{2ib^2x^{7/2}}{d}+\frac{a^2x^4}{4}-\frac{8abx^{7/2}\text{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}-\frac{2b^2x^{7/2}\cot\left(c+d\sqrt{x}\right)}{d}+\frac{14b^2x^3\log\left(1-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2}+\frac{28iabx^3\,\text{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{28iabx^3\,\text{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{42ib^2x^{5/2}\,\text{PolyLog}\left(2,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3}-\frac{168abx^{5/2}\,\text{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{168abx^{5/2}\,\text{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{105b^2x^2\,\text{PolyLog}\left(3,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{840iabx^2\,\text{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{840iabx^2\,\text{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{210ib^2x^{3/2}\,\text{PolyLog}\left(4,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^5}+\frac{3360abx^{3/2}\,\text{PolyLog}\left(5,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{3360abx^{3/2}\,\text{PolyLog}\left(5,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{315b^2x\,\text{PolyLog}\left(5,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^6}+\frac{10080iabx\,\text{PolyLog}\left(6,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}-\frac{10080iabx\,\text{PolyLog}\left(6,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}-\frac{315ib^2\sqrt{x}\,\text{PolyLog}\left(6,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^7}-\frac{20160ab\sqrt{x}\,\text{PolyLog}\left(7,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^7}$$

```
[Out] 28*I*a*b*x^3*polylog(2,-exp(I*(c+d*x^(1/2))))/d^2+840*I*a*b*x^2*polylog(4,e
xp(I*(c+d*x^(1/2))))/d^4+10080*I*a*b*x*polylog(6,-exp(I*(c+d*x^(1/2))))/d^6
+210*I*b^2*x^(3/2)*polylog(4,exp(2*I*(c+d*x^(1/2))))/d^5+20160*I*a*b*polylo
g(8,exp(I*(c+d*x^(1/2))))/d^8-8*a*b*x^(7/2)*arctanh(exp(I*(c+d*x^(1/2))))/d
-168*a*b*x^(5/2)*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+168*a*b*x^(5/2)*polyl
og(3,exp(I*(c+d*x^(1/2))))/d^3+3360*a*b*x^(3/2)*polylog(5,-exp(I*(c+d*x^(1/
2))))/d^5-3360*a*b*x^(3/2)*polylog(5,exp(I*(c+d*x^(1/2))))/d^5-20160*a*b*po
lylog(7,-exp(I*(c+d*x^(1/2))))*x^(1/2)/d^7+20160*a*b*polylog(7,exp(I*(c+d*x
^(1/2))))*x^(1/2)/d^7-42*I*b^2*x^(5/2)*polylog(2,exp(2*I*(c+d*x^(1/2))))/d^
3-20160*I*a*b*polylog(8,-exp(I*(c+d*x^(1/2))))/d^8-315*I*b^2*polylog(6,exp(
2*I*(c+d*x^(1/2))))*x^(1/2)/d^7+1/4*a^2*x^4-28*I*a*b*x^3*polylog(2,exp(I*(c
+d*x^(1/2))))/d^2-840*I*a*b*x^2*polylog(4,-exp(I*(c+d*x^(1/2))))/d^4-10080*
I*a*b*x*polylog(6,exp(I*(c+d*x^(1/2))))/d^6-2*b^2*x^(7/2)*cot(c+d*x^(1/2))/
d+14*b^2*x^3*ln(1-exp(2*I*(c+d*x^(1/2))))/d^2+105*b^2*x^2*polylog(3,exp(2*I
*(c+d*x^(1/2))))/d^4-315*b^2*x*polylog(5,exp(2*I*(c+d*x^(1/2))))/d^6-2*I*b^
2*x^(7/2)/d+315/2*b^2*polylog(7,exp(2*I*(c+d*x^(1/2))))/d^8
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.00, number of steps used = 30, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {4290, 4275, 4268, 2611, 6744, 2320, 6724, 4269, 3798, 2221}

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^4}{4}-\frac{8abx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{20160iab\,\text{PolyLog}\left(8,-e^{i(c+d\sqrt{x})}\right)}{d^8}+\frac{20160iab\,\text{PolyLog}\left(8,e^{i(c+d\sqrt{x})}\right)}{d^8}-\frac{20160ab\sqrt{x}\,\text{PolyLog}\left(7,-e^{i(c+d\sqrt{x})}\right)}{d^7}+\frac{20160ab\sqrt{x}\,\text{PolyLog}\left(7,e^{i(c+d\sqrt{x})}\right)}{d^7}+\frac{10080iabx\,\text{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6}-\frac{10080iabx\,\text{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6}+\frac{3360abx^{3/2}\,\text{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{3360abx^{3/2}\,\text{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{840iabx^2\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{840iabx^2\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}-\frac{168abx^{5/2}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{168abx^{5/2}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{28iabx^3\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{28iabx^3\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}+\frac{315b^2\,\text{PolyLog}\left(7,e^{2i(c+d\sqrt{x})}\right)}{2d^8}-\frac{315ib^2\sqrt{x}\,\text{PolyLog}\left(6,e^{2i(c+d\sqrt{x})}\right)}{d^7}-\frac{315b^2x\,\text{PolyLog}\left(5,e^{2i(c+d\sqrt{x})}\right)}{d^6}$$

$$210ib^2x^{3/2}\,\text{PolyLog}\left(4,e^{2i(c+d\sqrt{x})}\right)$$

```
[In] Int[x^3*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x^(7/2))/d + (a^2*x^4)/4 - (8*a*b*x^(7/2)*ArcTanh[E^(I*(c + d*S
qrt[x]))])/d - (2*b^2*x^(7/2)*Cot[c + d*Sqrt[x]])/d + (14*b^2*x^3*Log[1 - E
^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((28*I)*a*b*x^3*PolyLog[2, -E^(I*(c + d*Sq
rt[x]))])/d^2 - ((28*I)*a*b*x^3*PolyLog[2, E^(I*(c + d*Sqrt[x]))])/d^2 - ((
42*I)*b^2*x^(5/2)*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (168*a*b*x^(
5/2)*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3 + (168*a*b*x^(5/2)*PolyLog[3,
E^(I*(c + d*Sqrt[x]))])/d^3 + (105*b^2*x^2*PolyLog[3, E^((2*I)*(c + d*Sqrt[
x]))])/d^4 - ((840*I)*a*b*x^2*PolyLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((8
40*I)*a*b*x^2*PolyLog[4, E^(I*(c + d*Sqrt[x]))])/d^4 + ((210*I)*b^2*x^(3/2)
*PolyLog[4, E^((2*I)*(c + d*Sqrt[x]))])/d^5 + (3360*a*b*x^(3/2)*PolyLog[5,
-E^(I*(c + d*Sqrt[x]))])/d^5 - (3360*a*b*x^(3/2)*PolyLog[5, E^(I*(c + d*Sqr
t[x]))])/d^5 - (315*b^2*x*PolyLog[5, E^((2*I)*(c + d*Sqrt[x]))])/d^6 + ((10
080*I)*a*b*x*PolyLog[6, -E^(I*(c + d*Sqrt[x]))])/d^6 - ((10080*I)*a*b*x*Pol
yLog[6, E^(I*(c + d*Sqrt[x]))])/d^6 - ((315*I)*b^2*Sqrt[x]*PolyLog[6, E^((2
*I)*(c + d*Sqrt[x]))])/d^7 - (20160*a*b*Sqrt[x]*PolyLog[7, -E^(I*(c + d*Sqr
t[x]))])/d^7 + (20160*a*b*Sqrt[x]*PolyLog[7, E^(I*(c + d*Sqrt[x]))])/d^7 +
(315*b^2*PolyLog[7, E^((2*I)*(c + d*Sqrt[x]))])/(2*d^8) - ((20160*I)*a*b*Po
lyLog[8, -E^(I*(c + d*Sqrt[x]))])/d^8 + ((20160*I)*a*b*PolyLog[8, E^(I*(c +
 d*Sqrt[x]))])/d^8
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x^7 (a+b\csc(c+dx))^2\,dx, x, \sqrt{x}\right)$$
$$= 2\text{Subst}\left(\int \left(a^2x^7+2abx^7\csc(c+dx)+b^2x^7\csc^2(c+dx)\right)\,dx, x, \sqrt{x}\right)$$
$$= \frac{a^2x^4}{4}+(4ab)\text{Subst}\left(\int x^7\csc(c+dx)\,dx, x, \sqrt{x}\right)+\left(2b^2\right)\text{Subst}\left(\int x^7\csc^2(c+dx)\,dx, x, \sqrt{x}\right)$$
$$= \frac{a^2x^4}{4}-\frac{8abx^{7/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{2b^2x^{7/2}\cot\left(c+d\sqrt{x}\right)}{d} - \frac{(28ab)\text{Subst}\left(\int x^6\log\left(1-e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(28ab)\text{Subst}\left(\int x^6\log\left(1+e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{\left(14b^2\right)\text{Subst}\left(\int x^6\cot(c+dx)\,dx, x, \sqrt{x}\right)}{d}$$
$$= -\frac{2ib^2x^{7/2}}{d}+\frac{a^2x^4}{4}-\frac{8abx^{7/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{2b^2x^{7/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{28iabx^3\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{28iabx^3\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(168iab)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2} + \frac{(168iab)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{\left(28ib^2\right)\text{Subst}\left(\int \frac{e^{2i(c+dx)}x^6}{1-e^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} \\
&\quad - \frac{2b^2x^{7/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{14b^2x^3\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad + \frac{28iabx^3\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{28iabx^3\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{168abx^{5/2}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&\quad + \frac{(840ab)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3} \\
&\quad - \frac{(840ab)\text{Subst}\left(\int x^4\,\text{PolyLog}\left(3,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3} \\
&\quad - \frac{(84b^2)\,\text{Subst}\left(\int x^5\log\left(1-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{7/2}\cot\left(c+d\sqrt{x}\right)}{d} \\
&\quad + \frac{14b^2x^3\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{28iabx^3\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{28iabx^3\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{42ib^2x^{5/2}\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} \\
&\quad - \frac{168abx^{5/2}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{168abx^{5/2}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&\quad - \frac{840iabx^2\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{840iabx^2\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} \\
&\quad + \frac{(3360iab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(4,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} \\
&\quad - \frac{(3360iab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(4,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} \\
&\quad + \frac{(210ib^2)\,\text{Subst}\left(\int x^4\,\text{PolyLog}\left(2,e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}
\end{aligned}
$$

$$
\begin{aligned}
= & -\frac{2ib^2x^{7/2}}{d}+\frac{a^2x^4}{4}-\frac{8abx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} \\
& -\frac{2b^2x^{7/2}\cot\left(c+d\sqrt{x}\right)}{d}+\frac{14b^2x^3\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} \\
& +\frac{28iabx^3\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{28iabx^3\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} \\
& -\frac{42ib^2x^{5/2}\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3}-\frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} \\
& +\frac{168abx^{5/2}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{105b^2x^2\operatorname{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} \\
& -\frac{840iabx^2\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{840iabx^2\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} \\
& +\frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} \\
& -\frac{(10080ab)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(5,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5} \\
& +\frac{(10080ab)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(5,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5} \\
& -\frac{(420b^2)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(3,e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{7/2}\cot\left(c+d\sqrt{x}\right)}{d} \\
&+ \frac{14b^2x^3\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{28iabx^3\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&- \frac{28iabx^3\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{42ib^2x^{5/2}\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} \\
&- \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&+ \frac{105b^2x^2\operatorname{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{840iabx^2\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} \\
&+ \frac{840iabx^2\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{210ib^2x^{3/2}\operatorname{PolyLog}\left(4,e^{2i(c+d\sqrt{x})}\right)}{d^5} \\
&+ \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} \\
&+ \frac{10080iabx\operatorname{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{10080iabx\operatorname{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6} \\
&- \frac{(20160iab)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^6} \\
&+ \frac{(20160iab)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(6,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^6} \\
&- \frac{(630ib^2)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(4,e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} \\
&- \frac{2b^2x^{7/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{14b^2x^3\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} \\
&+ \frac{28iabx^3\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{28iabx^3\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&- \frac{42ib^2x^{5/2}\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&+ \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{105b^2x^2\operatorname{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} \\
&- \frac{840iabx^2\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{840iabx^2\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} \\
&+ \frac{210ib^2x^{3/2}\operatorname{PolyLog}\left(4,e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5} \\
&- \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{315b^2x\operatorname{PolyLog}\left(5,e^{2i(c+d\sqrt{x})}\right)}{d^6} \\
&+ \frac{10080iabx\operatorname{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{10080iabx\operatorname{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6} \\
&- \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,-e^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,e^{i(c+d\sqrt{x})}\right)}{d^7} \\
&+ \frac{(20160ab)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(7,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^7} \\
&- \frac{(20160ab)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(7,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^7} \\
&+ \frac{(630b^2)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(5,e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^6}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{7/2}\cot\left(c+d\sqrt{x}\right)}{d} \\
&+ \frac{14b^2x^3\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{28iabx^3\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&- \frac{28iabx^3\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{42ib^2x^{5/2}\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} \\
&- \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&+ \frac{105b^2x^2\operatorname{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{840iabx^2\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} \\
&+ \frac{840iabx^2\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{210ib^2x^{3/2}\operatorname{PolyLog}\left(4,e^{2i(c+d\sqrt{x})}\right)}{d^5} \\
&+ \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} \\
&- \frac{315b^2x\operatorname{PolyLog}\left(5,e^{2i(c+d\sqrt{x})}\right)}{d^6} + \frac{10080iabx\operatorname{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6} \\
&- \frac{10080iabx\operatorname{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{315ib^2\sqrt{x}\operatorname{PolyLog}\left(6,e^{2i(c+d\sqrt{x})}\right)}{d^7} \\
&- \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,-e^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,e^{i(c+d\sqrt{x})}\right)}{d^7} \\
&- \frac{(20160iab)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(7,-x)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^8} \\
&+ \frac{(20160iab)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(7,x)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^8} \\
&+ \frac{(315ib^2)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(6,e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^7}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{7/2}\cot\left(c+d\sqrt{x}\right)}{d} \\
&+ \frac{14b^2x^3\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{28iabx^3\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&- \frac{28iabx^3\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{42ib^2x^{5/2}\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} \\
&- \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&+ \frac{105b^2x^2\operatorname{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{840iabx^2\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} \\
&+ \frac{840iabx^2\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{210ib^2x^{3/2}\operatorname{PolyLog}\left(4,e^{2i(c+d\sqrt{x})}\right)}{d^5} \\
&+ \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} \\
&- \frac{315b^2x\operatorname{PolyLog}\left(5,e^{2i(c+d\sqrt{x})}\right)}{d^6} + \frac{10080iabx\operatorname{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6} \\
&- \frac{10080iabx\operatorname{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{315ib^2\sqrt{x}\operatorname{PolyLog}\left(6,e^{2i(c+d\sqrt{x})}\right)}{d^7} \\
&- \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,-e^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,e^{i(c+d\sqrt{x})}\right)}{d^7} \\
&- \frac{20160iab\operatorname{PolyLog}\left(8,-e^{i(c+d\sqrt{x})}\right)}{d^8} + \frac{20160iab\operatorname{PolyLog}\left(8,e^{i(c+d\sqrt{x})}\right)}{d^8} \\
&+ \frac{(315b^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(6,x)}{x}\,dx,x,e^{2i(c+d\sqrt{x})}\right)}{2d^8}
\end{aligned}
$$

$$= -\frac{2ib^2x^{7/2}}{d} + \frac{a^2x^4}{4} - \frac{8abx^{7/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}$$
$$- \frac{2b^2x^{7/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{14b^2x^3\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{28iabx^3\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{28iabx^3\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{42ib^2x^{5/2}\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{168abx^{5/2}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{105b^2x^2\operatorname{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4}$$
$$- \frac{840iabx^2\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{840iabx^2\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{210ib^2x^{3/2}\operatorname{PolyLog}\left(4,e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$- \frac{3360abx^{3/2}\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{315b^2x\operatorname{PolyLog}\left(5,e^{2i(c+d\sqrt{x})}\right)}{d^6}$$
$$+ \frac{10080iabx\operatorname{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{10080iabx\operatorname{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6}$$
$$- \frac{315ib^2\sqrt{x}\operatorname{PolyLog}\left(6,e^{2i(c+d\sqrt{x})}\right)}{d^7} - \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,-e^{i(c+d\sqrt{x})}\right)}{d^7}$$
$$+ \frac{20160ab\sqrt{x}\operatorname{PolyLog}\left(7,e^{i(c+d\sqrt{x})}\right)}{d^7} + \frac{315b^2\operatorname{PolyLog}\left(7,e^{2i(c+d\sqrt{x})}\right)}{2d^8}$$
$$- \frac{20160iab\operatorname{PolyLog}\left(8,-e^{i(c+d\sqrt{x})}\right)}{d^8} + \frac{20160iab\operatorname{PolyLog}\left(8,e^{i(c+d\sqrt{x})}\right)}{d^8}$$

Mathematica [A] (verified)

Time = 21.47 (sec) , antiderivative size = 1323, normalized size of antiderivative = 1.90

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^4\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\sin^2\left(c+d\sqrt{x}\right)}{4\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}$$
$$-\frac{b^2e^{ic}\csc(c)\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\left(2d^7e^{-2ic}x^{7/2}+7id^6(1-e^{-2ic})\,x^3\log\left(1-e^{-i(c+d\sqrt{x})}\right)+7id^6(1-e^{-\cdots}\right.}{\cdots}$$
$$+\frac{4ab\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\left(d^7x^{7/2}\log\left(1-e^{i(c+d\sqrt{x})}\right)-d^7x^{7/2}\log\left(1+e^{i(c+d\sqrt{x})}\right)+7id^6x^3\operatorname{PolyLog}\left(2,\cdots\right.\right.}{\cdots}$$
$$+\frac{b^2x^{7/2}\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\sin^2\left(c+d\sqrt{x}\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}$$
$$+\frac{b^2x^{7/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\sin^2\left(c+d\sqrt{x}\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}$$

```
[In] Integrate[x^3*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] (a^2*x^4*(a + b*Csc[c + d*Sqrt[x]])^2*Sin[c + d*Sqrt[x]]^2)/(4*(b + a*Sin[c
 + d*Sqrt[x]])^2) - (b^2*E^(I*c)*Csc[c]*(a + b*Csc[c + d*Sqrt[x]])^2*((2*d^
7*x^(7/2))/E^((2*I)*c) + (7*I)*d^6*(1 - E^((-2*I)*c))*x^3*Log[1 - E^((-I)*(
c + d*Sqrt[x]))] + (7*I)*d^6*(1 - E^((-2*I)*c))*x^3*Log[1 + E^((-I)*(c + d*
Sqrt[x]))] - 42*d^5*(1 - E^((-2*I)*c))*x^(5/2)*PolyLog[2, -E^((-I)*(c + d*S
qrt[x]))] - 42*d^5*(1 - E^((-2*I)*c))*x^(5/2)*PolyLog[2, E^((-I)*(c + d*Sqr
t[x]))] + (210*I)*d^4*(1 - E^((-2*I)*c))*x^2*PolyLog[3, -E^((-I)*(c + d*Sqr
t[x]))] + (210*I)*d^4*(1 - E^((-2*I)*c))*x^2*PolyLog[3, E^((-I)*(c + d*Sqrt
[x]))] + 840*d^3*(1 - E^((-2*I)*c))*x^(3/2)*PolyLog[4, -E^((-I)*(c + d*Sqrt
[x]))] + 840*d^3*(1 - E^((-2*I)*c))*x^(3/2)*PolyLog[4, E^((-I)*(c + d*Sqrt[
x]))] - (2520*I)*d^2*(1 - E^((-2*I)*c))*x*PolyLog[5, -E^((-I)*(c + d*Sqrt[x
]))] - (2520*I)*d^2*(1 - E^((-2*I)*c))*x*PolyLog[5, E^((-I)*(c + d*Sqrt[x])
)] - 5040*d*(1 - E^((-2*I)*c))*Sqrt[x]*PolyLog[6, -E^((-I)*(c + d*Sqrt[x]))
] - 5040*d*(1 - E^((-2*I)*c))*Sqrt[x]*PolyLog[6, E^((-I)*(c + d*Sqrt[x]))]
+ (5040*I)*(1 - E^((-2*I)*c))*PolyLog[7, -E^((-I)*(c + d*Sqrt[x]))] + (5040
*I)*(1 - E^((-2*I)*c))*PolyLog[7, E^((-I)*(c + d*Sqrt[x]))])*Sin[c + d*Sqrt
[x]]^2)/(d^8*(b + a*Sin[c + d*Sqrt[x]])^2) + (4*a*b*(a + b*Csc[c + d*Sqrt[x
]])^2*(d^7*x^(7/2)*Log[1 - E^(I*(c + d*Sqrt[x]))] - d^7*x^(7/2)*Log[1 + E^(
I*(c + d*Sqrt[x]))] + (7*I)*d^6*x^3*PolyLog[2, -E^(I*(c + d*Sqrt[x]))] - (7
*I)*d^6*x^3*PolyLog[2, E^(I*(c + d*Sqrt[x]))] - 42*d^5*x^(5/2)*PolyLog[3, -
E^(I*(c + d*Sqrt[x]))] + 42*d^5*x^(5/2)*PolyLog[3, E^(I*(c + d*Sqrt[x]))] -
 (210*I)*d^4*x^2*PolyLog[4, -E^(I*(c + d*Sqrt[x]))] + (210*I)*d^4*x^2*PolyL
og[4, E^(I*(c + d*Sqrt[x]))] + 840*d^3*x^(3/2)*PolyLog[5, -E^(I*(c + d*Sqrt
```

```
[x]))] - 840*d^3*x^(3/2)*PolyLog[5, E^(I*(c + d*Sqrt[x]))] + (2520*I)*d^2*x
*PolyLog[6, -E^(I*(c + d*Sqrt[x]))] - (2520*I)*d^2*x*PolyLog[6, E^(I*(c + d
*Sqrt[x]))] - 5040*d*Sqrt[x]*PolyLog[7, -E^(I*(c + d*Sqrt[x]))] + 5040*d*Sq
rt[x]*PolyLog[7, E^(I*(c + d*Sqrt[x]))] - (5040*I)*PolyLog[8, -E^(I*(c + d*
Sqrt[x]))] + (5040*I)*PolyLog[8, E^(I*(c + d*Sqrt[x]))])*Sin[c + d*Sqrt[x]]
^2)/(d^8*(b + a*Sin[c + d*Sqrt[x]])^2) + (b^2*x^(7/2)*Csc[c/2]*Csc[c/2 + (d
*Sqrt[x])/2]*(a + b*Csc[c + d*Sqrt[x]])^2*Sin[c + d*Sqrt[x]]^2*Sin[(d*Sqrt[
x])/2])/(d*(b + a*Sin[c + d*Sqrt[x]])^2) + (b^2*x^(7/2)*(a + b*Csc[c + d*Sq
rt[x]])^2*Sec[c/2]*Sec[c/2 + (d*Sqrt[x])/2]*Sin[c + d*Sqrt[x]]^2*Sin[(d*Sqr
t[x])/2])/(d*(b + a*Sin[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx$$

```
[In] int(x^3*(a+b*csc(c+d*x^(1/2)))^2,x)
[Out] int(x^3*(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 \, dx = \int \left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(b^2*x^3*csc(d*sqrt(x) + c)^2 + 2*a*b*x^3*csc(d*sqrt(x) + c) + a^2*
x^3, x)
```

Sympy [F]

$$\int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 \, dx = \int x^3 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 \, dx$$

```
[In] integrate(x**3*(a+b*csc(c+d*x**(1/2)))**2,x)
[Out] Integral(x**3*(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 6462 vs. 2(550) = 1100.

Time = 0.67 (sec) , antiderivative size = 6462, normalized size of antiderivative = 9.30

$$\int x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(x^3*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/4*((d*sqrt(x) + c)^8*a^2 - 8*(d*sqrt(x) + c)^7*a^2*c + 28*(d*sqrt(x) + c)
^6*a^2*c^2 - 56*(d*sqrt(x) + c)^5*a^2*c^3 + 70*(d*sqrt(x) + c)^4*a^2*c^4 -
56*(d*sqrt(x) + c)^3*a^2*c^5 + 28*(d*sqrt(x) + c)^2*a^2*c^6 - 8*(d*sqrt(x)
+ c)*a^2*c^7 + 16*a*b*c^7*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) + 8*
(4*b^2*c^7 + 2*(2*(d*sqrt(x) + c)^7*a*b - 7*b^2*c^6 - 7*(2*a*b*c + b^2)*(d*
sqrt(x) + c)^6 + 42*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^5 - 35*(2*a*b*c^3 + 3
*b^2*c^2)*(d*sqrt(x) + c)^4 + 70*(a*b*c^4 + 2*b^2*c^3)*(d*sqrt(x) + c)^3 -
21*(2*a*b*c^5 + 5*b^2*c^4)*(d*sqrt(x) + c)^2 + 14*(a*b*c^6 + 3*b^2*c^5)*(d*
sqrt(x) + c) - (2*(d*sqrt(x) + c)^7*a*b - 7*b^2*c^6 - 7*(2*a*b*c + b^2)*(d*
sqrt(x) + c)^6 + 42*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^5 - 35*(2*a*b*c^3 + 3
*b^2*c^2)*(d*sqrt(x) + c)^4 + 70*(a*b*c^4 + 2*b^2*c^3)*(d*sqrt(x) + c)^3 -
21*(2*a*b*c^5 + 5*b^2*c^4)*(d*sqrt(x) + c)^2 + 14*(a*b*c^6 + 3*b^2*c^5)*(d*
sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-2*I*(d*sqrt(x) + c)^7*a*b + 7*I*b^
2*c^6 + 7*(2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^6 + 42*(-I*a*b*c^2 - I*b^2*c)
*(d*sqrt(x) + c)^5 + 35*(2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) + c)^4 + 70*
(-I*a*b*c^4 - 2*I*b^2*c^3)*(d*sqrt(x) + c)^3 + 21*(2*I*a*b*c^5 + 5*I*b^2*c^
4)*(d*sqrt(x) + c)^2 + 14*(-I*a*b*c^6 - 3*I*b^2*c^5)*(d*sqrt(x) + c))*sin(2
*d*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) + 1) + 14
*(b^2*c^6*cos(2*d*sqrt(x) + 2*c) + I*b^2*c^6*sin(2*d*sqrt(x) + 2*c) - b^2*c
^6)*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) - 1) + 2*(2*(d*sqrt(x) +
 c)^7*a*b - 7*(2*a*b*c - b^2)*(d*sqrt(x) + c)^6 + 42*(a*b*c^2 - b^2*c)*(d*s
qrt(x) + c)^5 - 35*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c)^4 + 70*(a*b*c^4
- 2*b^2*c^3)*(d*sqrt(x) + c)^3 - 21*(2*a*b*c^5 - 5*b^2*c^4)*(d*sqrt(x) + c)
^2 + 14*(a*b*c^6 - 3*b^2*c^5)*(d*sqrt(x) + c) - (2*(d*sqrt(x) + c)^7*a*b -
7*(2*a*b*c - b^2)*(d*sqrt(x) + c)^6 + 42*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c)^
5 - 35*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c)^4 + 70*(a*b*c^4 - 2*b^2*c^3)
*(d*sqrt(x) + c)^3 - 21*(2*a*b*c^5 - 5*b^2*c^4)*(d*sqrt(x) + c)^2 + 14*(a*b
*c^6 - 3*b^2*c^5)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-2*I*(d*sqrt(x
) + c)^7*a*b + 7*(2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^6 + 42*(-I*a*b*c^2 + I
*b^2*c)*(d*sqrt(x) + c)^5 + 35*(2*I*a*b*c^3 - 3*I*b^2*c^2)*(d*sqrt(x) + c)^
4 + 70*(-I*a*b*c^4 + 2*I*b^2*c^3)*(d*sqrt(x) + c)^3 + 21*(2*I*a*b*c^5 - 5*I
*b^2*c^4)*(d*sqrt(x) + c)^2 + 14*(-I*a*b*c^6 + 3*I*b^2*c^5)*(d*sqrt(x) + c)
)*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c) +
 1) - 4*((d*sqrt(x) + c)^7*b^2 - 7*(d*sqrt(x) + c)^6*b^2*c + 21*(d*sqrt(x)
```

```
+ c)^5*b^2*c^2 - 35*(d*sqrt(x) + c)^4*b^2*c^3 + 35*(d*sqrt(x) + c)^3*b^2*c^
4 - 21*(d*sqrt(x) + c)^2*b^2*c^5 + 7*(d*sqrt(x) + c)*b^2*c^6)*cos(2*d*sqrt(
x) + 2*c) - 28*((d*sqrt(x) + c)^6*a*b + a*b*c^6 + 3*b^2*c^5 - 3*(2*a*b*c +
b^2)*(d*sqrt(x) + c)^5 + 15*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^4 - 10*(2*a*b
*c^3 + 3*b^2*c^2)*(d*sqrt(x) + c)^3 + 15*(a*b*c^4 + 2*b^2*c^3)*(d*sqrt(x) +
 c)^2 - 3*(2*a*b*c^5 + 5*b^2*c^4)*(d*sqrt(x) + c) - ((d*sqrt(x) + c)^6*a*b
+ a*b*c^6 + 3*b^2*c^5 - 3*(2*a*b*c + b^2)*(d*sqrt(x) + c)^5 + 15*(a*b*c^2 +
 b^2*c)*(d*sqrt(x) + c)^4 - 10*(2*a*b*c^3 + 3*b^2*c^2)*(d*sqrt(x) + c)^3 +
15*(a*b*c^4 + 2*b^2*c^3)*(d*sqrt(x) + c)^2 - 3*(2*a*b*c^5 + 5*b^2*c^4)*(d*s
qrt(x) + c))*cos(2*d*sqrt(x) + 2*c) - (I*(d*sqrt(x) + c)^6*a*b + I*a*b*c^6
+ 3*I*b^2*c^5 + 3*(-2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^5 + 15*(I*a*b*c^2 +
I*b^2*c)*(d*sqrt(x) + c)^4 + 10*(-2*I*a*b*c^3 - 3*I*b^2*c^2)*(d*sqrt(x) + c
)^3 + 15*(I*a*b*c^4 + 2*I*b^2*c^3)*(d*sqrt(x) + c)^2 + 3*(-2*I*a*b*c^5 - 5*
I*b^2*c^4)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*dilog(-e^(I*d*sqrt(x) +
 I*c)) + 28*((d*sqrt(x) + c)^6*a*b + a*b*c^6 - 3*b^2*c^5 - 3*(2*a*b*c - b^2
)*(d*sqrt(x) + c)^5 + 15*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c)^4 - 10*(2*a*b*c^
3 - 3*b^2*c^2)*(d*sqrt(x) + c)^3 + 15*(a*b*c^4 - 2*b^2*c^3)*(d*sqrt(x) + c)
^2 - 3*(2*a*b*c^5 - 5*b^2*c^4)*(d*sqrt(x) + c) - ((d*sqrt(x) + c)^6*a*b + a
*b*c^6 - 3*b^2*c^5 - 3*(2*a*b*c - b^2)*(d*sqrt(x) + c)^5 + 15*(a*b*c^2 - b^
2*c)*(d*sqrt(x) + c)^4 - 10*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c)^3 + 15*
(a*b*c^4 - 2*b^2*c^3)*(d*sqrt(x) + c)^2 - 3*(2*a*b*c^5 - 5*b^2*c^4)*(d*sqrt
(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-I*(d*sqrt(x) + c)^6*a*b - I*a*b*c^6 +
3*I*b^2*c^5 + 3*(2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^5 + 15*(-I*a*b*c^2 + I*
b^2*c)*(d*sqrt(x) + c)^4 + 10*(2*I*a*b*c^3 - 3*I*b^2*c^2)*(d*sqrt(x) + c)^3
 + 15*(-I*a*b*c^4 + 2*I*b^2*c^3)*(d*sqrt(x) + c)^2 + 3*(2*I*a*b*c^5 - 5*I*b
^2*c^4)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*dilog(e^(I*d*sqrt(x) + I*c
)) - (2*I*(d*sqrt(x) + c)^7*a*b - 7*I*b^2*c^6 - 7*(2*I*a*b*c + I*b^2)*(d*sq
rt(x) + c)^6 - 42*(-I*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c)^5 - 35*(2*I*a*b*c^
3 + 3*I*b^2*c^2)*(d*sqrt(x) + c)^4 - 70*(-I*a*b*c^4 - 2*I*b^2*c^3)*(d*sqrt(
x) + c)^3 - 21*(2*I*a*b*c^5 + 5*I*b^2*c^4)*(d*sqrt(x) + c)^2 - 14*(-I*a*b*c
^6 - 3*I*b^2*c^5)*(d*sqrt(x) + c) + (-2*I*(d*sqrt(x) + c)^7*a*b + 7*I*b^2*c
^6 - 7*(-2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^6 - 42*(I*a*b*c^2 + I*b^2*c)*(d
*sqrt(x) + c)^5 - 35*(-2*I*a*b*c^3 - 3*I*b^2*c^2)*(d*sqrt(x) + c)^4 - 70*(I
*a*b*c^4 + 2*I*b^2*c^3)*(d*sqrt(x) + c)^3 - 21*(-2*I*a*b*c^5 - 5*I*b^2*c^4)
*(d*sqrt(x) + c)^2 - 14*(I*a*b*c^6 + 3*I*b^2*c^5)*(d*sqrt(x) + c))*cos(2*d*
sqrt(x) + 2*c) + (2*(d*sqrt(x) + c)^7*a*b - 7*b^2*c^6 - 7*(2*a*b*c + b^2)*(
d*sqrt(x) + c)^6 + 42*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^5 - 35*(2*a*b*c^3 +
 3*b^2*c^2)*(d*sqrt(x) + c)^4 + 70*(a*b*c^4 + 2*b^2*c^3)*(d*sqrt(x) + c)^3
- 21*(2*a*b*c^5 + 5*b^2*c^4)*(d*sqrt(x) + c)^2 + 14*(a*b*c^6 + 3*b^2*c^5)*(
d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2 + sin(d*sq
rt(x) + c)^2 + 2*cos(d*sqrt(x) + c) + 1) - (-2*I*(d*sqrt(x) + c)^7*a*b - 7*
I*b^2*c^6 - 7*(-2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^6 - 42*(I*a*b*c^2 - I*b^
2*c)*(d*sqrt(x) + c)^5 - 35*(-2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) + c)^4
- 70*(I*a*b*c^4 - 2*I*b^2*c^3)*(d*sqrt(x) + c)^3 - 21*(-2*I*a*b*c^5 + 5*I*b
^2*c^4)*(d*sqrt(x) + c)^2 - 14*(I*a*b*c^6 - 3*I*b^2*c^5)*(d*sqrt(x) + c) +
```

```
(2*I*(d*sqrt(x) + c)^7*a*b + 7*I*b^2*c^6 - 7*(2*I*a*b*c - I*b^2)*(d*sqrt(x)
 + c)^6 - 42*(-I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c)^5 - 35*(2*I*a*b*c^3 - 3
*I*b^2*c^2)*(d*sqrt(x) + c)^4 - 70*(-I*a*b*c^4 + 2*I*b^2*c^3)*(d*sqrt(x) +
c)^3 - 21*(2*I*a*b*c^5 - 5*I*b^2*c^4)*(d*sqrt(x) + c)^2 - 14*(-I*a*b*c^6 +
3*I*b^2*c^5)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) - (2*(d*sqrt(x) + c)^7
*a*b + 7*b^2*c^6 - 7*(2*a*b*c - b^2)*(d*sqrt(x) + c)^6 + 42*(a*b*c^2 - b^2*
c)*(d*sqrt(x) + c)^5 - 35*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c)^4 + 70*(a
*b*c^4 - 2*b^2*c^3)*(d*sqrt(x) + c)^3 - 21*(2*a*b*c^5 - 5*b^2*c^4)*(d*sqrt(
x) + c)^2 + 14*(a*b*c^6 - 3*b^2*c^5)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c
))*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) +
 1) - 20160*(a*b*cos(2*d*sqrt(x) + 2*c) + I*a*b*sin(2*d*sqrt(x) + 2*c) - a*
b)*polylog(8, -e^(I*d*sqrt(x) + I*c)) + 20160*(a*b*cos(2*d*sqrt(x) + 2*c) +
 I*a*b*sin(2*d*sqrt(x) + 2*c) - a*b)*polylog(8, e^(I*d*sqrt(x) + I*c)) + 10
080*(-2*I*(d*sqrt(x) + c)*a*b + 2*I*a*b*c + I*b^2 + (2*I*(d*sqrt(x) + c)*a*
b - 2*I*a*b*c - I*b^2)*cos(2*d*sqrt(x) + 2*c) - (2*(d*sqrt(x) + c)*a*b - 2*
a*b*c - b^2)*sin(2*d*sqrt(x) + 2*c))*polylog(7, -e^(I*d*sqrt(x) + I*c)) + 1
0080*(2*I*(d*sqrt(x) + c)*a*b - 2*I*a*b*c + I*b^2 + (-2*I*(d*sqrt(x) + c)*a
*b + 2*I*a*b*c - I*b^2)*cos(2*d*sqrt(x) + 2*c) + (2*(d*sqrt(x) + c)*a*b - 2
*a*b*c + b^2)*sin(2*d*sqrt(x) + 2*c))*polylog(7, e^(I*d*sqrt(x) + I*c)) - 1
0080*((d*sqrt(x) + c)^2*a*b + a*b*c^2 + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x)
+ c) - ((d*sqrt(x) + c)^2*a*b + a*b*c^2 + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x
) + c))*cos(2*d*sqrt(x) + 2*c) - (I*(d*sqrt(x) + c)^2*a*b + I*a*b*c^2 + I*b
^2*c + (-2*I*a*b*c - I*b^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*polylo
g(6, -e^(I*d*sqrt(x) + I*c)) + 10080*((d*sqrt(x) + c)^2*a*b + a*b*c^2 - b^2
*c - (2*a*b*c - b^2)*(d*sqrt(x) + c) - ((d*sqrt(x) + c)^2*a*b + a*b*c^2 - b
^2*c - (2*a*b*c - b^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-I*(d*sqr
t(x) + c)^2*a*b - I*a*b*c^2 + I*b^2*c + (2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)
)*sin(2*d*sqrt(x) + 2*c))*polylog(6, e^(I*d*sqrt(x) + I*c)) + 1680*(2*I*(d*
sqrt(x) + c)^3*a*b - 2*I*a*b*c^3 - 3*I*b^2*c^2 + 3*(-2*I*a*b*c - I*b^2)*(d*
sqrt(x) + c)^2 + 6*(I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c) + (-2*I*(d*sqrt(x)
 + c)^3*a*b + 2*I*a*b*c^3 + 3*I*b^2*c^2 + 3*(2*I*a*b*c + I*b^2)*(d*sqrt(x)
+ c)^2 + 6*(-I*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) +
 (2*(d*sqrt(x) + c)^3*a*b - 2*a*b*c^3 - 3*b^2*c^2 - 3*(2*a*b*c + b^2)*(d*sq
rt(x) + c)^2 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))
*polylog(5, -e^(I*d*sqrt(x) + I*c)) + 1680*(-2*I*(d*sqrt(x) + c)^3*a*b + 2*
I*a*b*c^3 - 3*I*b^2*c^2 + 3*(2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^2 + 6*(-I*a
*b*c^2 + I*b^2*c)*(d*sqrt(x) + c) + (2*I*(d*sqrt(x) + c)^3*a*b - 2*I*a*b*c^
3 + 3*I*b^2*c^2 + 3*(-2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^2 + 6*(I*a*b*c^2 -
 I*b^2*c)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) - (2*(d*sqrt(x) + c)^3*a*
b - 2*a*b*c^3 + 3*b^2*c^2 - 3*(2*a*b*c - b^2)*(d*sqrt(x) + c)^2 + 6*(a*b*c^
2 - b^2*c)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*polylog(5, e^(I*d*sqrt(
x) + I*c)) + 840*((d*sqrt(x) + c)^4*a*b + a*b*c^4 + 2*b^2*c^3 - 2*(2*a*b*c
+ b^2)*(d*sqrt(x) + c)^3 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^2 - 2*(2*a*b
*c^3 + 3*b^2*c^2)*(d*sqrt(x) + c) - ((d*sqrt(x) + c)^4*a*b + a*b*c^4 + 2*b^
2*c^3 - 2*(2*a*b*c + b^2)*(d*sqrt(x) + c)^3 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x
```

```
) + c)^2 - 2*(2*a*b*c^3 + 3*b^2*c^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c
) + (-I*(d*sqrt(x) + c)^4*a*b - I*a*b*c^4 - 2*I*b^2*c^3 + 2*(2*I*a*b*c + I*
b^2)*(d*sqrt(x) + c)^3 + 6*(-I*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c)^2 + 2*(2*
I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*polylog(4
, -e^(I*d*sqrt(x) + I*c)) - 840*((d*sqrt(x) + c)^4*a*b + a*b*c^4 - 2*b^2*c^
3 - 2*(2*a*b*c - b^2)*(d*sqrt(x) + c)^3 + 6*(a*b*c^2 - b^2*c)*(d*sqrt(x) +
c)^2 - 2*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c) - ((d*sqrt(x) + c)^4*a*b +
 a*b*c^4 - 2*b^2*c^3 - 2*(2*a*b*c - b^2)*(d*sqrt(x) + c)^3 + 6*(a*b*c^2 - b
^2*c)*(d*sqrt(x) + c)^2 - 2*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c))*cos(2*
d*sqrt(x) + 2*c) - (I*(d*sqrt(x) + c)^4*a*b + I*a*b*c^4 - 2*I*b^2*c^3 + 2*(
-2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^3 + 6*(I*a*b*c^2 - I*b^2*c)*(d*sqrt(x)
+ c)^2 + 2*(-2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) +
2*c))*polylog(4, e^(I*d*sqrt(x) + I*c)) + 84*(-2*I*(d*sqrt(x) + c)^5*a*b +
2*I*a*b*c^5 + 5*I*b^2*c^4 + 5*(2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^4 + 20*(-
I*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c)^3 + 10*(2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*
sqrt(x) + c)^2 + 10*(-I*a*b*c^4 - 2*I*b^2*c^3)*(d*sqrt(x) + c) + (2*I*(d*sq
rt(x) + c)^5*a*b - 2*I*a*b*c^5 - 5*I*b^2*c^4 + 5*(-2*I*a*b*c - I*b^2)*(d*sq
rt(x) + c)^4 + 20*(I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c)^3 + 10*(-2*I*a*b*c^
3 - 3*I*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(I*a*b*c^4 + 2*I*b^2*c^3)*(d*sqrt(x
) + c))*cos(2*d*sqrt(x) + 2*c) - (2*(d*sqrt(x) + c)^5*a*b - 2*a*b*c^5 - 5*b
^2*c^4 - 5*(2*a*b*c + b^2)*(d*sqrt(x) + c)^4 + 20*(a*b*c^2 + b^2*c)*(d*sqrt
(x) + c)^3 - 10*(2*a*b*c^3 + 3*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(a*b*c^4 + 2
*b^2*c^3)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*polylog(3, -e^(I*d*sqrt(
x) + I*c)) + 84*(2*I*(d*sqrt(x) + c)^5*a*b - 2*I*a*b*c^5 + 5*I*b^2*c^4 + 5*
(-2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^4 + 20*(I*a*b*c^2 - I*b^2*c)*(d*sqrt(x
) + c)^3 + 10*(-2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(I*a*b*c^
4 - 2*I*b^2*c^3)*(d*sqrt(x) + c) + (-2*I*(d*sqrt(x) + c)^5*a*b + 2*I*a*b*c^
5 - 5*I*b^2*c^4 + 5*(2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^4 + 20*(-I*a*b*c^2
+ I*b^2*c)*(d*sqrt(x) + c)^3 + 10*(2*I*a*b*c^3 - 3*I*b^2*c^2)*(d*sqrt(x) +
c)^2 + 10*(-I*a*b*c^4 + 2*I*b^2*c^3)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c
) + (2*(d*sqrt(x) + c)^5*a*b - 2*a*b*c^5 + 5*b^2*c^4 - 5*(2*a*b*c - b^2)*(d
*sqrt(x) + c)^4 + 20*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c)^3 - 10*(2*a*b*c^3 -
3*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(a*b*c^4 - 2*b^2*c^3)*(d*sqrt(x) + c))*si
n(2*d*sqrt(x) + 2*c))*polylog(3, e^(I*d*sqrt(x) + I*c)) + 4*(-I*(d*sqrt(x)
+ c)^7*b^2 + 7*I*(d*sqrt(x) + c)^6*b^2*c - 21*I*(d*sqrt(x) + c)^5*b^2*c^2 +
 35*I*(d*sqrt(x) + c)^4*b^2*c^3 - 35*I*(d*sqrt(x) + c)^3*b^2*c^4 + 21*I*(d*
sqrt(x) + c)^2*b^2*c^5 - 7*I*(d*sqrt(x) + c)*b^2*c^6)*sin(2*d*sqrt(x) + 2*c
))/(-2*I*cos(2*d*sqrt(x) + 2*c) + 2*sin(2*d*sqrt(x) + 2*c) + 2*I))/d^8
```

Giac [F]

$$\int x^3 \left(a + b \csc \left(c + d\sqrt{x}\right)\right)^2 \, dx = \int \left(b \csc \left(d\sqrt{x} + c\right) + a\right)^2 x^3 \, dx$$

```
[In] integrate(x^3*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)^2*x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b \csc \left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^3 \left(a + \frac{b}{\sin \left(c + d\sqrt{x}\right)}\right)^2 \, dx$$

```
[In] int(x^3*(a + b/sin(c + d*x^(1/2)))^2,x)
[Out] int(x^3*(a + b/sin(c + d*x^(1/2)))^2, x)
```

3.37 $\int x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 513

$$\int x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx = -\frac{2ib^2x^{5/2}}{d}+\frac{a^2x^3}{3}-\frac{8abx^{5/2}\operatorname{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}-\frac{2b^2x^{5/2}\cot\left(c+d\sqrt{x}\right)}{d}+\frac{10b^2x^2\log\left(1-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2}+\frac{20iabx^2\operatorname{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{20iabx^2\operatorname{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{20ib^2x^{3/2}\operatorname{PolyLog}\left(2,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3}-\frac{80abx^{3/2}\operatorname{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{80abx^{3/2}\operatorname{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{30b^2x\operatorname{PolyLog}\left(3,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{240iabx\operatorname{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{240iabx\operatorname{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{30ib^2\sqrt{x}\operatorname{PolyLog}\left(4,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^5}+\frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{15b^2\operatorname{PolyLog}\left(5,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^6}+\frac{480iab\operatorname{PolyLog}\left(6,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}-\frac{480iab\operatorname{PolyLog}\left(6,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}$$

```
[Out] -240*I*a*b*x*polylog(4,-exp(I*(c+d*x^(1/2))))/d^4+1/3*a^2*x^3-8*a*b*x^(5/2)
*arctanh(exp(I*(c+d*x^(1/2))))/d-2*b^2*x^(5/2)*cot(c+d*x^(1/2))/d+10*b^2*x^
2*ln(1-exp(2*I*(c+d*x^(1/2))))/d^2-20*I*a*b*x^2*polylog(2,exp(I*(c+d*x^(1/2
))))/d^2+30*I*b^2*polylog(4,exp(2*I*(c+d*x^(1/2))))*x^(1/2)/d^5-2*I*b^2*x^(
5/2)/d-80*a*b*x^(3/2)*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+80*a*b*x^(3/2)*p
olylog(3,exp(I*(c+d*x^(1/2))))/d^3+30*b^2*x*polylog(3,exp(2*I*(c+d*x^(1/2))
))/d^4+20*I*a*b*x^2*polylog(2,-exp(I*(c+d*x^(1/2))))/d^2+480*I*a*b*polylog(
6,-exp(I*(c+d*x^(1/2))))/d^6-15*b^2*polylog(5,exp(2*I*(c+d*x^(1/2))))/d^6-4
80*I*a*b*polylog(6,exp(I*(c+d*x^(1/2))))/d^6-20*I*b^2*x^(3/2)*polylog(2,exp
(2*I*(c+d*x^(1/2))))/d^3+240*I*a*b*x*polylog(4,exp(I*(c+d*x^(1/2))))/d^4+48
0*a*b*polylog(5,-exp(I*(c+d*x^(1/2))))*x^(1/2)/d^5-480*a*b*polylog(5,exp(I*
(c+d*x^(1/2))))*x^(1/2)/d^5
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {4290, 4275, 4268, 2611, 6744, 2320, 6724, 4269, 3798, 2221}

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^3}{3}-\frac{8abx^{5/2}\text{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}+\frac{480iab\,\text{PolyLog}\left(6,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}-\frac{480iab\,\text{PolyLog}\left(6,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^6}+\frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{240iabx\,\text{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{240iabx\,\text{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{80abx^{3/2}\,\text{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{80abx^{3/2}\,\text{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{20iabx^2\,\text{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{20iabx^2\,\text{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{15b^2\,\text{PolyLog}\left(5,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^6}+\frac{30ib^2\sqrt{x}\,\text{PolyLog}\left(4,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^5}+\frac{30b^2x\,\text{PolyLog}\left(3,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{20ib^2x^{3/2}\,\text{PolyLog}\left(2,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{10b^2x^2\log\left(1-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{2b^2x^{5/2}\cot\left(c+d\sqrt{x}\right)}{d}-\frac{2ib^2x^{5/2}}{d}$$

```
[In] Int[x^2*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x^(5/2))/d + (a^2*x^3)/3 - (8*a*b*x^(5/2)*ArcTanh[E^(I*(c + d*S
qrt[x]))])/d - (2*b^2*x^(5/2)*Cot[c + d*Sqrt[x]])/d + (10*b^2*x^2*Log[1 - E
^((2*I)*(c + d*Sqrt[x]))])/d^2 + ((20*I)*a*b*x^2*PolyLog[2, -E^(I*(c + d*Sq
rt[x]))])/d^2 - ((20*I)*a*b*x^2*PolyLog[2, E^(I*(c + d*Sqrt[x]))])/d^2 - ((
20*I)*b^2*x^(3/2)*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (80*a*b*x^(3
/2)*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3 + (80*a*b*x^(3/2)*PolyLog[3, E^
(I*(c + d*Sqrt[x]))])/d^3 + (30*b^2*x*PolyLog[3, E^((2*I)*(c + d*Sqrt[x]))]
)/d^4 - ((240*I)*a*b*x*PolyLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((240*I)*a
*b*x*PolyLog[4, E^(I*(c + d*Sqrt[x]))])/d^4 + ((30*I)*b^2*Sqrt[x]*PolyLog[4
, E^((2*I)*(c + d*Sqrt[x]))])/d^5 + (480*a*b*Sqrt[x]*PolyLog[5, -E^(I*(c +
d*Sqrt[x]))])/d^5 - (480*a*b*Sqrt[x]*PolyLog[5, E^(I*(c + d*Sqrt[x]))])/d^5
 - (15*b^2*PolyLog[5, E^((2*I)*(c + d*Sqrt[x]))])/d^6 + ((480*I)*a*b*PolyLo
g[6, -E^(I*(c + d*Sqrt[x]))])/d^6 - ((480*I)*a*b*PolyLog[6, E^(I*(c + d*Sqr
t[x]))])/d^6
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int x^5(a+b\csc(c+dx))^2\,dx, x, \sqrt{x}\right)$$

$$= 2\text{Subst}\left(\int \left(a^2x^5 + 2abx^5\csc(c+dx) + b^2x^5\csc^2(c+dx)\right)\,dx, x, \sqrt{x}\right)$$

$$= \frac{a^2x^3}{3} + (4ab)\text{Subst}\left(\int x^5\csc(c+dx)\,dx, x, \sqrt{x}\right) + \left(2b^2\right)\text{Subst}\left(\int x^5\csc^2(c+dx)\,dx, x, \sqrt{x}\right)$$

$$= \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{5/2}\cot\left(c+d\sqrt{x}\right)}{d} - \frac{(20ab)\text{Subst}\left(\int x^4\log\left(1-e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{(20ab)\text{Subst}\left(\int x^4\log\left(1+e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d} + \frac{\left(10b^2\right)\text{Subst}\left(\int x^4\cot(c+dx)\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{5/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{20iabx^2\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(80iab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2, -e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2} + \frac{(80iab)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2, e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2} - \frac{\left(20ib^2\right)\text{Subst}\left(\int \frac{e^{2i(c+dx)}x^4}{1-e^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{5/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{10b^2x^2\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{80abx^{3/2}\,\text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{80abx^{3/2}\,\text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{(240ab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3, -e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3} - \frac{(240ab)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3, e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^3} - \frac{\left(40b^2\right)\text{Subst}\left(\int x^3\log\left(1-e^{2i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d^2}$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{5/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{10b^2x^2\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20ib^2x^{3/2}\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{80abx^{3/2}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{80abx^{3/2}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{240iabx\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{240iabx\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(480iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(4,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(480iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(4,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} + \frac{(60ib^2)\,\text{Subst}\left(\int x^2\,\text{PolyLog}\left(2,e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{5/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{10b^2x^2\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20ib^2x^{3/2}\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{80abx^{3/2}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{80abx^{3/2}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{30b^2x\,\text{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{240iabx\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{240iabx\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{(480ab)\text{Subst}\left(\int \text{PolyLog}\left(5,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5} + \frac{(480ab)\text{Subst}\left(\int \text{PolyLog}\left(5,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5} - \frac{(60b^2)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(3,e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{5/2}\cot\left(c+d\sqrt{x}\right)}{d} \\
&+ \frac{10b^2x^2\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{20iabx^2\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&- \frac{20iabx^2\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20ib^2x^{3/2}\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} \\
&- \frac{80abx^{3/2}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{80abx^{3/2}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&+ \frac{30b^2x\,\text{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{240iabx\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} \\
&+ \frac{240iabx\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{30ib^2\sqrt{x}\,\text{PolyLog}\left(4,e^{2i(c+d\sqrt{x})}\right)}{d^5} \\
&+ \frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} \\
&+ \frac{(480iab)\text{Subst}\left(\int\frac{\text{PolyLog}(5,-x)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^6} \\
&- \frac{(480iab)\text{Subst}\left(\int\frac{\text{PolyLog}(5,x)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^6} \\
&- \frac{(30ib^2)\,\text{Subst}\left(\int\text{PolyLog}\left(4,e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^5}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} \\
&- \frac{2b^2x^{5/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{10b^2x^2\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} \\
&+ \frac{20iabx^2\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&- \frac{20ib^2x^{3/2}\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&+ \frac{80abx^{3/2}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{30b^2x\operatorname{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} \\
&- \frac{240iabx\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{240iabx\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} \\
&+ \frac{30ib^2\sqrt{x}\operatorname{PolyLog}\left(4,e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5} \\
&- \frac{480ab\sqrt{x}\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{480iab\operatorname{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6} \\
&- \frac{480iab\operatorname{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{(15b^2)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(4,x)}{x}\,dx, x, e^{2i(c+d\sqrt{x})}\right)}{d^6}
\end{aligned}
$$

$$= -\frac{2ib^2x^{5/2}}{d} + \frac{a^2x^3}{3} - \frac{8abx^{5/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}$$
$$- \frac{2b^2x^{5/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{10b^2x^2\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2}$$
$$+ \frac{20iabx^2\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{20iabx^2\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}$$
$$- \frac{20ib^2x^{3/2}\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{80abx^{3/2}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}$$
$$+ \frac{80abx^{3/2}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{30b^2x\,\text{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4}$$
$$- \frac{240iabx\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{240iabx\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}$$
$$+ \frac{30ib^2\sqrt{x}\,\text{PolyLog}\left(4,e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5}$$
$$- \frac{480ab\sqrt{x}\,\text{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{15b^2\,\text{PolyLog}\left(5,e^{2i(c+d\sqrt{x})}\right)}{d^6}$$
$$+ \frac{480iab\,\text{PolyLog}\left(6,-e^{i(c+d\sqrt{x})}\right)}{d^6} - \frac{480iab\,\text{PolyLog}\left(6,e^{i(c+d\sqrt{x})}\right)}{d^6}$$

Mathematica [A] (verified)

Time = 14.59 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.52

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx = \frac{a^2x^3\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\sin^2\left(c+d\sqrt{x}\right)}{3\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}$$
$$- \frac{ib\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\left(\frac{4bd^5e^{2ic}x^{5/2}}{-1+e^{2ic}} - 20ad^4x^2\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right) + 20ad^4x^2\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x}}\right.\right.}{}$$
$$+ \frac{b^2x^{5/2}\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\sin^2\left(c+d\sqrt{x}\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}$$
$$+ \frac{b^2x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\sin^2\left(c+d\sqrt{x}\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}$$

```
[In] Integrate[x^2*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] (a^2*x^3*(a + b*Csc[c + d*Sqrt[x]])^2*Sin[c + d*Sqrt[x]]^2)/(3*(b + a*Sin[c
 + d*Sqrt[x]])^2) - (I*b*(a + b*Csc[c + d*Sqrt[x]])^2*((4*b*d^5*E^((2*I)*c)
```

```
*x^(5/2))/(-1 + E^((2*I)*c)) - 20*a*d^4*x^2*PolyLog[2, -E^(I*(c + d*Sqrt[x]
))] + 20*a*d^4*x^2*PolyLog[2, E^(I*(c + d*Sqrt[x]))] + I*(4*a*d^5*x^(5/2)*L
og[1 - E^(I*(c + d*Sqrt[x]))] - 4*a*d^5*x^(5/2)*Log[1 + E^(I*(c + d*Sqrt[x]
))] + 10*b*d^4*x^2*Log[1 - E^((2*I)*(c + d*Sqrt[x]))] - (20*I)*b*d^3*x^(3/2
)*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))] - 80*a*d^3*x^(3/2)*PolyLog[3, -E^(I
*(c + d*Sqrt[x]))] + 80*a*d^3*x^(3/2)*PolyLog[3, E^(I*(c + d*Sqrt[x]))] + 3
0*b*d^2*x*PolyLog[3, E^((2*I)*(c + d*Sqrt[x]))] - (240*I)*a*d^2*x*PolyLog[4
, -E^(I*(c + d*Sqrt[x]))] + (240*I)*a*d^2*x*PolyLog[4, E^(I*(c + d*Sqrt[x])
)] + (30*I)*b*d*Sqrt[x]*PolyLog[4, E^((2*I)*(c + d*Sqrt[x]))] + 480*a*d*Sqr
t[x]*PolyLog[5, -E^(I*(c + d*Sqrt[x]))] - 480*a*d*Sqrt[x]*PolyLog[5, E^(I*(
c + d*Sqrt[x]))] - 15*b*PolyLog[5, E^((2*I)*(c + d*Sqrt[x]))] + (480*I)*a*P
olyLog[6, -E^(I*(c + d*Sqrt[x]))] - (480*I)*a*PolyLog[6, E^(I*(c + d*Sqrt[x
]))]))*Sin[c + d*Sqrt[x]]^2)/(d^6*(b + a*Sin[c + d*Sqrt[x]])^2) + (b^2*x^(5
/2)*Csc[c/2]*Csc[c/2 + (d*Sqrt[x])/2]*(a + b*Csc[c + d*Sqrt[x]])^2*Sin[c +
d*Sqrt[x]]^2*Sin[(d*Sqrt[x])/2])/(d*(b + a*Sin[c + d*Sqrt[x]])^2) + (b^2*x^
(5/2)*(a + b*Csc[c + d*Sqrt[x]])^2*Sec[c/2]*Sec[c/2 + (d*Sqrt[x])/2]*Sin[c
+ d*Sqrt[x]]^2*Sin[(d*Sqrt[x])/2])/(d*(b + a*Sin[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int x^2\big(a+b\csc\big(c+d\sqrt{x}\big)\big)^2\,dx$$

```
[In] int(x^2*(a+b*csc(c+d*x^(1/2)))^2,x)
[Out] int(x^2*(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x^2\big(a+b\csc\big(c+d\sqrt{x}\big)\big)^2\,dx=\int\big(b\csc\big(d\sqrt{x}+c\big)+a\big)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*csc(d*sqrt(x) + c)^2 + 2*a*b*x^2*csc(d*sqrt(x) + c) + a^2*
x^2, x)
```

Sympy [F]

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$$

```
[In] integrate(x**2*(a+b*csc(c+d*x**(1/2)))**2,x)
```

```
[Out] Integral(x**2*(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3885 vs. 2(406) = 812.

Time = 0.45 (sec) , antiderivative size = 3885, normalized size of antiderivative = 7.57

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\text{Too large to display}$$

```
[In] integrate(x^2*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((d*sqrt(x) + c)^6*a^2 - 6*(d*sqrt(x) + c)^5*a^2*c + 15*(d*sqrt(x) + c)
^4*a^2*c^2 - 20*(d*sqrt(x) + c)^3*a^2*c^3 + 15*(d*sqrt(x) + c)^2*a^2*c^4 -
6*(d*sqrt(x) + c)*a^2*c^5 + 12*a*b*c^5*log(cot(d*sqrt(x) + c) + csc(d*sqrt(
x) + c)) + 6*(4*b^2*c^5 + 2*(2*(d*sqrt(x) + c)^5*a*b - 5*b^2*c^4 - 5*(2*a*b
*c + b^2)*(d*sqrt(x) + c)^4 + 20*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^3 - 10*(
2*a*b*c^3 + 3*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(a*b*c^4 + 2*b^2*c^3)*(d*sqrt
(x) + c) - (2*(d*sqrt(x) + c)^5*a*b - 5*b^2*c^4 - 5*(2*a*b*c + b^2)*(d*sqrt
(x) + c)^4 + 20*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^3 - 10*(2*a*b*c^3 + 3*b^2
*c^2)*(d*sqrt(x) + c)^2 + 10*(a*b*c^4 + 2*b^2*c^3)*(d*sqrt(x) + c))*cos(2*d
*sqrt(x) + 2*c) + (-2*I*(d*sqrt(x) + c)^5*a*b + 5*I*b^2*c^4 + 5*(2*I*a*b*c
+ I*b^2)*(d*sqrt(x) + c)^4 + 20*(-I*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c)^3 +
10*(2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(-I*a*b*c^4 - 2*I*b^2
*c^3)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c),
cos(d*sqrt(x) + c) + 1) + 10*(b^2*c^4*cos(2*d*sqrt(x) + 2*c) + I*b^2*c^4*si
n(2*d*sqrt(x) + 2*c) - b^2*c^4)*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) +
 c) - 1) + 2*(2*(d*sqrt(x) + c)^5*a*b - 5*(2*a*b*c - b^2)*(d*sqrt(x) + c)^4
 + 20*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c)^3 - 10*(2*a*b*c^3 - 3*b^2*c^2)*(d*s
qrt(x) + c)^2 + 10*(a*b*c^4 - 2*b^2*c^3)*(d*sqrt(x) + c) - (2*(d*sqrt(x) +
c)^5*a*b - 5*(2*a*b*c - b^2)*(d*sqrt(x) + c)^4 + 20*(a*b*c^2 - b^2*c)*(d*sq
rt(x) + c)^3 - 10*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(a*b*c^4 -
 2*b^2*c^3)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-2*I*(d*sqrt(x) + c)
^5*a*b + 5*(2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^4 + 20*(-I*a*b*c^2 + I*b^2*c
)*(d*sqrt(x) + c)^3 + 10*(2*I*a*b*c^3 - 3*I*b^2*c^2)*(d*sqrt(x) + c)^2 + 10
*(-I*a*b*c^4 + 2*I*b^2*c^3)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*arctan
```

```
2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c) + 1) - 4*((d*sqrt(x) + c)^5*b^2 -
 5*(d*sqrt(x) + c)^4*b^2*c + 10*(d*sqrt(x) + c)^3*b^2*c^2 - 10*(d*sqrt(x) +
 c)^2*b^2*c^3 + 5*(d*sqrt(x) + c)*b^2*c^4)*cos(2*d*sqrt(x) + 2*c) - 20*((d*
sqrt(x) + c)^4*a*b + a*b*c^4 + 2*b^2*c^3 - 2*(2*a*b*c + b^2)*(d*sqrt(x) + c
)^3 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^2 - 2*(2*a*b*c^3 + 3*b^2*c^2)*(d*
sqrt(x) + c) - ((d*sqrt(x) + c)^4*a*b + a*b*c^4 + 2*b^2*c^3 - 2*(2*a*b*c +
b^2)*(d*sqrt(x) + c)^3 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^2 - 2*(2*a*b*c
^3 + 3*b^2*c^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) - (I*(d*sqrt(x) + c
)^4*a*b + I*a*b*c^4 + 2*I*b^2*c^3 + 2*(-2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^
3 + 6*(I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c)^2 + 2*(-2*I*a*b*c^3 - 3*I*b^2*c
^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*dilog(-e^(I*d*sqrt(x) + I*c))
+ 20*((d*sqrt(x) + c)^4*a*b + a*b*c^4 - 2*b^2*c^3 - 2*(2*a*b*c - b^2)*(d*sq
rt(x) + c)^3 + 6*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c)^2 - 2*(2*a*b*c^3 - 3*b^2
*c^2)*(d*sqrt(x) + c) - ((d*sqrt(x) + c)^4*a*b + a*b*c^4 - 2*b^2*c^3 - 2*(2
*a*b*c - b^2)*(d*sqrt(x) + c)^3 + 6*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c)^2 - 2
*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-I*(d*s
qrt(x) + c)^4*a*b - I*a*b*c^4 + 2*I*b^2*c^3 + 2*(2*I*a*b*c - I*b^2)*(d*sqrt
(x) + c)^3 + 6*(-I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c)^2 + 2*(2*I*a*b*c^3 -
3*I*b^2*c^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*dilog(e^(I*d*sqrt(x)
+ I*c)) - (2*I*(d*sqrt(x) + c)^5*a*b - 5*I*b^2*c^4 - 5*(2*I*a*b*c + I*b^2)*
(d*sqrt(x) + c)^4 - 20*(-I*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c)^3 - 10*(2*I*a
*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) + c)^2 - 10*(-I*a*b*c^4 - 2*I*b^2*c^3)*(d*
sqrt(x) + c) + (-2*I*(d*sqrt(x) + c)^5*a*b + 5*I*b^2*c^4 - 5*(-2*I*a*b*c -
I*b^2)*(d*sqrt(x) + c)^4 - 20*(I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c)^3 - 10*
(-2*I*a*b*c^3 - 3*I*b^2*c^2)*(d*sqrt(x) + c)^2 - 10*(I*a*b*c^4 + 2*I*b^2*c^
3)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (2*(d*sqrt(x) + c)^5*a*b - 5*b
^2*c^4 - 5*(2*a*b*c + b^2)*(d*sqrt(x) + c)^4 + 20*(a*b*c^2 + b^2*c)*(d*sqrt
(x) + c)^3 - 10*(2*a*b*c^3 + 3*b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(a*b*c^4 + 2
*b^2*c^3)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2
 + sin(d*sqrt(x) + c)^2 + 2*cos(d*sqrt(x) + c) + 1) - (-2*I*(d*sqrt(x) + c)
^5*a*b - 5*I*b^2*c^4 - 5*(-2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^4 - 20*(I*a*b
*c^2 - I*b^2*c)*(d*sqrt(x) + c)^3 - 10*(-2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt
(x) + c)^2 - 10*(I*a*b*c^4 - 2*I*b^2*c^3)*(d*sqrt(x) + c) + (2*I*(d*sqrt(x)
 + c)^5*a*b + 5*I*b^2*c^4 - 5*(2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^4 - 20*(-
I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c)^3 - 10*(2*I*a*b*c^3 - 3*I*b^2*c^2)*(d*
sqrt(x) + c)^2 - 10*(-I*a*b*c^4 + 2*I*b^2*c^3)*(d*sqrt(x) + c))*cos(2*d*sqr
t(x) + 2*c) - (2*(d*sqrt(x) + c)^5*a*b + 5*b^2*c^4 - 5*(2*a*b*c - b^2)*(d*s
qrt(x) + c)^4 + 20*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c)^3 - 10*(2*a*b*c^3 - 3*
b^2*c^2)*(d*sqrt(x) + c)^2 + 10*(a*b*c^4 - 2*b^2*c^3)*(d*sqrt(x) + c))*sin(
2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos
(d*sqrt(x) + c) + 1) + 480*(a*b*cos(2*d*sqrt(x) + 2*c) + I*a*b*sin(2*d*sqrt
(x) + 2*c) - a*b)*polylog(6, -e^(I*d*sqrt(x) + I*c)) - 480*(a*b*cos(2*d*sqr
t(x) + 2*c) + I*a*b*sin(2*d*sqrt(x) + 2*c) - a*b)*polylog(6, e^(I*d*sqrt(x)
 + I*c)) + 240*(2*I*(d*sqrt(x) + c)*a*b - 2*I*a*b*c - I*b^2 + (-2*I*(d*sqrt
(x) + c)*a*b + 2*I*a*b*c + I*b^2)*cos(2*d*sqrt(x) + 2*c) + (2*(d*sqrt(x) +
```

```
c)*a*b - 2*a*b*c - b^2)*sin(2*d*sqrt(x) + 2*c))*polylog(5, -e^(I*d*sqrt(x)
+ I*c)) + 240*(-2*I*(d*sqrt(x) + c)*a*b + 2*I*a*b*c - I*b^2 + (2*I*(d*sqrt(
x) + c)*a*b - 2*I*a*b*c + I*b^2)*cos(2*d*sqrt(x) + 2*c) - (2*(d*sqrt(x) + c
)*a*b - 2*a*b*c + b^2)*sin(2*d*sqrt(x) + 2*c))*polylog(5, e^(I*d*sqrt(x) +
I*c)) + 240*((d*sqrt(x) + c)^2*a*b + a*b*c^2 + b^2*c - (2*a*b*c + b^2)*(d*s
qrt(x) + c) - ((d*sqrt(x) + c)^2*a*b + a*b*c^2 + b^2*c - (2*a*b*c + b^2)*(d
*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-I*(d*sqrt(x) + c)^2*a*b - I*a*b*c
^2 - I*b^2*c + (2*I*a*b*c + I*b^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))
*polylog(4, -e^(I*d*sqrt(x) + I*c)) - 240*((d*sqrt(x) + c)^2*a*b + a*b*c^2
- b^2*c - (2*a*b*c - b^2)*(d*sqrt(x) + c) - ((d*sqrt(x) + c)^2*a*b + a*b*c^
2 - b^2*c - (2*a*b*c - b^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) - (I*(d
*sqrt(x) + c)^2*a*b + I*a*b*c^2 - I*b^2*c + (-2*I*a*b*c + I*b^2)*(d*sqrt(x)
 + c))*sin(2*d*sqrt(x) + 2*c))*polylog(4, e^(I*d*sqrt(x) + I*c)) + 40*(-2*I
*(d*sqrt(x) + c)^3*a*b + 2*I*a*b*c^3 + 3*I*b^2*c^2 + 3*(2*I*a*b*c + I*b^2)*
(d*sqrt(x) + c)^2 + 6*(-I*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c) + (2*I*(d*sqrt
(x) + c)^3*a*b - 2*I*a*b*c^3 - 3*I*b^2*c^2 + 3*(-2*I*a*b*c - I*b^2)*(d*sqrt
(x) + c)^2 + 6*(I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c
) - (2*(d*sqrt(x) + c)^3*a*b - 2*a*b*c^3 - 3*b^2*c^2 - 3*(2*a*b*c + b^2)*(d
*sqrt(x) + c)^2 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*
c))*polylog(3, -e^(I*d*sqrt(x) + I*c)) + 40*(2*I*(d*sqrt(x) + c)^3*a*b - 2*
I*a*b*c^3 + 3*I*b^2*c^2 + 3*(-2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^2 + 6*(I*a
*b*c^2 - I*b^2*c)*(d*sqrt(x) + c) + (-2*I*(d*sqrt(x) + c)^3*a*b + 2*I*a*b*c
^3 - 3*I*b^2*c^2 + 3*(2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^2 + 6*(-I*a*b*c^2
+ I*b^2*c)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (2*(d*sqrt(x) + c)^3*a
*b - 2*a*b*c^3 + 3*b^2*c^2 - 3*(2*a*b*c - b^2)*(d*sqrt(x) + c)^2 + 6*(a*b*c
^2 - b^2*c)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*polylog(3, e^(I*d*sqrt
(x) + I*c)) + 4*(-I*(d*sqrt(x) + c)^5*b^2 + 5*I*(d*sqrt(x) + c)^4*b^2*c - 1
0*I*(d*sqrt(x) + c)^3*b^2*c^2 + 10*I*(d*sqrt(x) + c)^2*b^2*c^3 - 5*I*(d*sqr
t(x) + c)*b^2*c^4)*sin(2*d*sqrt(x) + 2*c))/(-2*I*cos(2*d*sqrt(x) + 2*c) + 2
*sin(2*d*sqrt(x) + 2*c) + 2*I))/d^6
```

Giac [F]

$$\int x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x^2\,dx$$

```
[In] integrate(x^2*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)^2*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + b \csc \left(c + d\sqrt{x}\right)\right)^2 \, dx = \int x^2 \left(a + \frac{b}{\sin \left(c + d\,\sqrt{x}\right)}\right)^2 \, dx$$

```
[In] int(x^2*(a + b/sin(c + d*x^(1/2)))^2,x)
[Out] int(x^2*(a + b/sin(c + d*x^(1/2)))^2, x)
```

3.38 $\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 333

$$\begin{aligned}\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx &= -\frac{2ib^2x^{3/2}}{d}+\frac{a^2x^2}{2}-\frac{8abx^{3/2}\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}\\
&\quad-\frac{2b^2x^{3/2}\cot\left(c+d\sqrt{x}\right)}{d}+\frac{6b^2x\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2}\\
&\quad+\frac{12iabx\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}\\
&\quad-\frac{12iabx\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}\\
&\quad-\frac{6ib^2\sqrt{x}\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3}\\
&\quad-\frac{24ab\sqrt{x}\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}\\
&\quad+\frac{24ab\sqrt{x}\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}\\
&\quad+\frac{3b^2\operatorname{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4}\\
&\quad-\frac{24iab\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}\\
&\quad+\frac{24iab\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}\end{aligned}$$

```
[Out] -2*I*b^2*x^(3/2)/d+1/2*a^2*x^2-8*a*b*x^(3/2)*arctanh(exp(I*(c+d*x^(1/2))))/
d-2*b^2*x^(3/2)*cot(c+d*x^(1/2))/d+6*b^2*x*ln(1-exp(2*I*(c+d*x^(1/2))))/d^2
+12*I*a*b*x*polylog(2,-exp(I*(c+d*x^(1/2))))/d^2-12*I*a*b*x*polylog(2,exp(I
*(c+d*x^(1/2))))/d^2+3*b^2*polylog(3,exp(2*I*(c+d*x^(1/2))))/d^4-24*I*a*b*p
olylog(4,-exp(I*(c+d*x^(1/2))))/d^4+24*I*a*b*polylog(4,exp(I*(c+d*x^(1/2)))
)/d^4-6*I*b^2*polylog(2,exp(2*I*(c+d*x^(1/2))))*x^(1/2)/d^3-24*a*b*polylog(
3,-exp(I*(c+d*x^(1/2))))*x^(1/2)/d^3+24*a*b*polylog(3,exp(I*(c+d*x^(1/2))))
*x^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4290, 4275, 4268, 2611, 6744, 2320, 6724, 4269, 3798, 2221}

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{a^2x^2}{2}-\frac{8abx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{24iab\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{24iab\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}-\frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{12iabx\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{12iabx\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}+\frac{3b^2\,\text{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4}-\frac{6ib^2\sqrt{x}\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3}+\frac{6b^2x\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2}-\frac{2b^2x^{3/2}\cot\left(c+d\sqrt{x}\right)}{d}-\frac{2ib^2x^{3/2}}{d}$$

```
[In] Int[x*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x^(3/2))/d + (a^2*x^2)/2 - (8*a*b*x^(3/2)*ArcTanh[E^(I*(c + d*S
qrt[x]))])/d - (2*b^2*x^(3/2)*Cot[c + d*Sqrt[x]])/d + (6*b^2*x*Log[1 - E^((
2*I)*(c + d*Sqrt[x]))])/d^2 + ((12*I)*a*b*x*PolyLog[2, -E^(I*(c + d*Sqrt[x]
))])/d^2 - ((12*I)*a*b*x*PolyLog[2, E^(I*(c + d*Sqrt[x]))])/d^2 - ((6*I)*b^
2*Sqrt[x]*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (24*a*b*Sqrt[x]*Poly
Log[3, -E^(I*(c + d*Sqrt[x]))])/d^3 + (24*a*b*Sqrt[x]*PolyLog[3, E^(I*(c +
d*Sqrt[x]))])/d^3 + (3*b^2*PolyLog[3, E^((2*I)*(c + d*Sqrt[x]))])/d^4 - ((2
4*I)*a*b*PolyLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((24*I)*a*b*PolyLog[4, E
^(I*(c + d*Sqrt[x]))])/d^4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
```

```
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^3(a+b\csc(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^3+2abx^3\csc(c+dx)+b^2x^3\csc^2(c+dx)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{a^2x^2}{2}+(4ab)\text{Subst}\left(\int x^3\csc(c+dx)\,dx, x, \sqrt{x}\right)+\left(2b^2\right)\text{Subst}\left(\int x^3\csc^2(c+dx)\,dx, x, \sqrt{x}\right)
\end{aligned}
$$

$$= \frac{a^2x^2}{2} - \frac{8abx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{3/2}\cot\left(c+d\sqrt{x}\right)}{d} - \frac{(12ab)\text{Subst}\left(\int x^2\log\left(1-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d} + \frac{(12ab)\text{Subst}\left(\int x^2\log\left(1+e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d} + \frac{(6b^2)\,\text{Subst}\left(\int x^2\cot(c+dx)\,dx,x,\sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8abx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{3/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{12iabx\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12iabx\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(24iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2} + \frac{(24iab)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2} - \frac{(12ib^2)\,\text{Subst}\left(\int \frac{e^{2i(c+dx)}x^2}{1-e^{2i(c+dx)}}\,dx,x,\sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8abx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{3/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{6b^2x\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{12iabx\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12iabx\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{(24ab)\text{Subst}\left(\int \text{PolyLog}\left(3,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3} - \frac{(24ab)\text{Subst}\left(\int \text{PolyLog}\left(3,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3} - \frac{(12b^2)\,\text{Subst}\left(\int x\log\left(1-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8abx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{3/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{6b^2x\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{12iabx\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12iabx\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ib^2\sqrt{x}\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{(24iab)\text{Subst}\left(\int\frac{\text{PolyLog}(3,-x)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(24iab)\text{Subst}\left(\int\frac{\text{PolyLog}(3,x)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(6ib^2)\,\text{Subst}\left(\int\text{PolyLog}\left(2,e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8abx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{3/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{6b^2x\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{12iabx\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12iabx\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ib^2\sqrt{x}\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{24iab\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{24iab\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(3b^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{2i(c+d\sqrt{x})}\right)}{d^4}$$

$$= -\frac{2ib^2x^{3/2}}{d} + \frac{a^2x^2}{2} - \frac{8abx^{3/2}\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^{3/2}\cot\left(c+d\sqrt{x}\right)}{d} + \frac{6b^2x\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{12iabx\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12iabx\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{6ib^2\sqrt{x}\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24ab\sqrt{x}\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{3b^2\,\text{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{24iab\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{24iab\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}$$

Mathematica [A] (verified)

Time = 10.31 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.35

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx = \frac{a^2x^2}{2}$$

$$- 2ib\left(\frac{2bd^3e^{2ic}x^{3/2}}{-1+e^{2ic}} + \left(6bd\sqrt{x}-6ad^2x\right)\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right) + i\left(3bd^2x\log\left(1-e^{i(c+d\sqrt{x})}\right) + 2ad^3x^{3/2}\right.\right.$$

$$+ \frac{b^2x^{3/2}\csc\left(\frac{c}{2}\right)\csc\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d} + \frac{b^2x^{3/2}\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d}$$

```
[In] Integrate[x*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] (a^2*x^2)/2 - ((2*I)*b*((2*b*d^3*E^((2*I)*c)*x^(3/2))/(-1 + E^((2*I)*c)) +
(6*b*d*Sqrt[x] - 6*a*d^2*x)*PolyLog[2, -E^(I*(c + d*Sqrt[x]))] + I*(3*b*d^2
*x*Log[1 - E^(I*(c + d*Sqrt[x]))] + 2*a*d^3*x^(3/2)*Log[1 - E^(I*(c + d*Sqr
t[x]))] + 3*b*d^2*x*Log[1 + E^(I*(c + d*Sqrt[x]))] - 2*a*d^3*x^(3/2)*Log[1
+ E^(I*(c + d*Sqrt[x]))] - (6*I)*(b*d*Sqrt[x] + a*d^2*x)*PolyLog[2, E^(I*(c
 + d*Sqrt[x]))] + 6*(b - 2*a*d*Sqrt[x])*PolyLog[3, -E^(I*(c + d*Sqrt[x]))]
+ 6*b*PolyLog[3, E^(I*(c + d*Sqrt[x]))] + 12*a*d*Sqrt[x]*PolyLog[3, E^(I*(c
 + d*Sqrt[x]))] - (12*I)*a*PolyLog[4, -E^(I*(c + d*Sqrt[x]))] + (12*I)*a*Po
lyLog[4, E^(I*(c + d*Sqrt[x]))])))/d^4 + (b^2*x^(3/2)*Csc[c/2]*Csc[(c + d*S
qrt[x])/2]*Sin[(d*Sqrt[x])/2])/d + (b^2*x^(3/2)*Sec[c/2]*Sec[(c + d*Sqrt[x]
)/2]*Sin[(d*Sqrt[x])/2])/d
```

Maple [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx$$

```
[In] int(x*(a+b*csc(c+d*x^(1/2)))^2,x)
[Out] int(x*(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx=\int \left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2 x\,dx$$

```
[In] integrate(x*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(b^2*x*csc(d*sqrt(x) + c)^2 + 2*a*b*x*csc(d*sqrt(x) + c) + a^2*x, x
)
```

Sympy [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx=\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx$$

```
[In] integrate(x*(a+b*csc(c+d*x**(1/2)))**2,x)
[Out] Integral(x*(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1950 vs. $2(262) = 524$.

Time = 0.32 (sec) , antiderivative size = 1950, normalized size of antiderivative = 5.86

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2 dx=\text{Too large to display}$$

```
[In] integrate(x*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] 1/2*((d*sqrt(x) + c)^4*a^2 - 4*(d*sqrt(x) + c)^3*a^2*c + 6*(d*sqrt(x) + c)^
2*a^2*c^2 - 4*(d*sqrt(x) + c)*a^2*c^3 + 8*a*b*c^3*log(cot(d*sqrt(x) + c) +
csc(d*sqrt(x) + c)) + 4*(4*b^2*c^3 + 2*(2*(d*sqrt(x) + c)^3*a*b - 3*b^2*c^2
 - 3*(2*a*b*c + b^2)*(d*sqrt(x) + c)^2 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c
) - (2*(d*sqrt(x) + c)^3*a*b - 3*b^2*c^2 - 3*(2*a*b*c + b^2)*(d*sqrt(x) + c
```

```
)^2 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-2*I*(
d*sqrt(x) + c)^3*a*b + 3*I*b^2*c^2 + 3*(2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^
2 + 6*(-I*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*arcta
n2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) + 1) + 6*(b^2*c^2*cos(2*d*sqrt(x)
 + 2*c) + I*b^2*c^2*sin(2*d*sqrt(x) + 2*c) - b^2*c^2)*arctan2(sin(d*sqrt(x)
 + c), cos(d*sqrt(x) + c) - 1) + 2*(2*(d*sqrt(x) + c)^3*a*b - 3*(2*a*b*c -
b^2)*(d*sqrt(x) + c)^2 + 6*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c) - (2*(d*sqrt(x
) + c)^3*a*b - 3*(2*a*b*c - b^2)*(d*sqrt(x) + c)^2 + 6*(a*b*c^2 - b^2*c)*(d
*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-2*I*(d*sqrt(x) + c)^3*a*b + 3*(2*
I*a*b*c - I*b^2)*(d*sqrt(x) + c)^2 + 6*(-I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) +
c))*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c)
 + 1) - 4*((d*sqrt(x) + c)^3*b^2 - 3*(d*sqrt(x) + c)^2*b^2*c + 3*(d*sqrt(x)
 + c)*b^2*c^2)*cos(2*d*sqrt(x) + 2*c) - 12*((d*sqrt(x) + c)^2*a*b + a*b*c^2
 + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x) + c) - ((d*sqrt(x) + c)^2*a*b + a*b*c
^2 + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) - (I*(
d*sqrt(x) + c)^2*a*b + I*a*b*c^2 + I*b^2*c + (-2*I*a*b*c - I*b^2)*(d*sqrt(x
) + c))*sin(2*d*sqrt(x) + 2*c))*dilog(-e^(I*d*sqrt(x) + I*c)) + 12*((d*sqrt
(x) + c)^2*a*b + a*b*c^2 - b^2*c - (2*a*b*c - b^2)*(d*sqrt(x) + c) - ((d*sq
rt(x) + c)^2*a*b + a*b*c^2 - b^2*c - (2*a*b*c - b^2)*(d*sqrt(x) + c))*cos(2
*d*sqrt(x) + 2*c) + (-I*(d*sqrt(x) + c)^2*a*b - I*a*b*c^2 + I*b^2*c + (2*I*
a*b*c - I*b^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*dilog(e^(I*d*sqrt(x
) + I*c)) - (2*I*(d*sqrt(x) + c)^3*a*b - 3*I*b^2*c^2 - 3*(2*I*a*b*c + I*b^2
)*(d*sqrt(x) + c)^2 - 6*(-I*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c) + (-2*I*(d*s
qrt(x) + c)^3*a*b + 3*I*b^2*c^2 - 3*(-2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^2
 - 6*(I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (2*(d*s
qrt(x) + c)^3*a*b - 3*b^2*c^2 - 3*(2*a*b*c + b^2)*(d*sqrt(x) + c)^2 + 6*(a*
b*c^2 + b^2*c)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) +
 c)^2 + sin(d*sqrt(x) + c)^2 + 2*cos(d*sqrt(x) + c) + 1) - (-2*I*(d*sqrt(x)
 + c)^3*a*b - 3*I*b^2*c^2 - 3*(-2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^2 - 6*(I
*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c) + (2*I*(d*sqrt(x) + c)^3*a*b + 3*I*b^2*
c^2 - 3*(2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^2 - 6*(-I*a*b*c^2 + I*b^2*c)*(d
*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) - (2*(d*sqrt(x) + c)^3*a*b + 3*b^2*c^
2 - 3*(2*a*b*c - b^2)*(d*sqrt(x) + c)^2 + 6*(a*b*c^2 - b^2*c)*(d*sqrt(x) +
c))*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2
 - 2*cos(d*sqrt(x) + c) + 1) - 24*(a*b*cos(2*d*sqrt(x) + 2*c) + I*a*b*sin(2
*d*sqrt(x) + 2*c) - a*b)*polylog(4, -e^(I*d*sqrt(x) + I*c)) + 24*(a*b*cos(2
*d*sqrt(x) + 2*c) + I*a*b*sin(2*d*sqrt(x) + 2*c) - a*b)*polylog(4, e^(I*d*s
qrt(x) + I*c)) + 12*(-2*I*(d*sqrt(x) + c)*a*b + 2*I*a*b*c + I*b^2 + (2*I*(d
*sqrt(x) + c)*a*b - 2*I*a*b*c - I*b^2)*cos(2*d*sqrt(x) + 2*c) - (2*(d*sqrt(
x) + c)*a*b - 2*a*b*c - b^2)*sin(2*d*sqrt(x) + 2*c))*polylog(3, -e^(I*d*sqr
t(x) + I*c)) + 12*(2*I*(d*sqrt(x) + c)*a*b - 2*I*a*b*c + I*b^2 + (-2*I*(d*s
qrt(x) + c)*a*b + 2*I*a*b*c - I*b^2)*cos(2*d*sqrt(x) + 2*c) + (2*(d*sqrt(x)
 + c)*a*b - 2*a*b*c + b^2)*sin(2*d*sqrt(x) + 2*c))*polylog(3, e^(I*d*sqrt(x
) + I*c)) + 4*(-I*(d*sqrt(x) + c)^3*b^2 + 3*I*(d*sqrt(x) + c)^2*b^2*c - 3*I
*(d*sqrt(x) + c)*b^2*c^2)*sin(2*d*sqrt(x) + 2*c))/(-2*I*cos(2*d*sqrt(x) + 2
```

```
*c) + 2*sin(2*d*sqrt(x) + 2*c) + 2*I))/d^4
```

Giac [F]

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x\,dx$$

```
[In] integrate(x*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)^2*x, x)
```

Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

```
[In] int(x*(a + b/sin(c + d*x^(1/2)))^2,x)
[Out] int(x*(a + b/sin(c + d*x^(1/2)))^2, x)
```

3.39 $\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\operatorname{Int}\left(\frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x},x\right)$$

```
[Out] Unintegrable((a+b*csc(c+d*x^(1/2)))^2/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

```
[In] Int[(a + b*Csc[c + d*Sqrt[x]])^2/x,x]
[Out] Defer[Int][(a + b*Csc[c + d*Sqrt[x]])^2/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 137.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x,x]
[Out] Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}dx$$

```
[In] int((a+b*csc(c+d*x^(1/2)))^2/x,x)
[Out] int((a+b*csc(c+d*x^(1/2)))^2/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x,x, algorithm="fricas")
[Out] integral((b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2)/x, x)
```

Sympy [N/A]

Not integrable

Time = 8.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))**2/x,x)
[Out] Integral((a + b*csc(c + d*sqrt(x)))**2/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 386, normalized size of antiderivative = 19.30

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x,x, algorithm="maxima")
```

```
[Out] -(4*b^2*sqrt(x)*sin(2*d*sqrt(x) + 2*c) - (d*cos(2*d*sqrt(x) + 2*c)^2 + d*si
n(2*d*sqrt(x) + 2*c)^2 - 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x*integrate((2*a*b
*d*x*sin(d*sqrt(x) + c) + b^2*sqrt(x)*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x)
 + c)^2 + d*sin(d*sqrt(x) + c)^2 + 2*d*cos(d*sqrt(x) + c) + d)*x^2), x) + (
d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 - 2*d*cos(2*d*sqrt(
x) + 2*c) + d)*x*integrate(-(2*a*b*d*x*sin(d*sqrt(x) + c) - b^2*sqrt(x)*sin
(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 - 2*d*co
s(d*sqrt(x) + c) + d)*x^2), x) - (a^2*d*cos(2*d*sqrt(x) + 2*c)^2 + a^2*d*si
n(2*d*sqrt(x) + 2*c)^2 - 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a^2*d)*x*log(x))/
((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 - 2*d*cos(2*d*sqr
t(x) + 2*c) + d)*x)
```

Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*sqrt(x) + c) + a)^2/x, x)
```

Mupad [N/A]

Not integrable

Time = 18.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}{x}\,dx$$

```
[In] int((a + b/sin(c + d*x^(1/2)))^2/x,x)
```

```
[Out] int((a + b/sin(c + d*x^(1/2)))^2/x, x)
```

3.40 $\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^2}\,dx = \text{Int}\left(\frac{(a+b\csc(c+d\sqrt{x}))^2}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*csc(c+d*x^(1/2)))^2/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^2}\,dx = \int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^2}\,dx$$

```
[In] Int[(a + b*Csc[c + d*Sqrt[x]])^2/x^2,x]
```

```
[Out] Defer[Int][(a + b*Csc[c + d*Sqrt[x]])^2/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 85.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^2,x]
[Out] Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}dx$$

```
[In] int((a+b*csc(c+d*x^(1/2)))^2/x^2,x)
[Out] int((a+b*csc(c+d*x^(1/2)))^2/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2)/x^2, x
)
```

Sympy [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))**2/x**2,x)
[Out] Integral((a + b*csc(c + d*sqrt(x)))**2/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 388, normalized size of antiderivative = 19.40

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^2,x, algorithm="maxima")
[Out] ((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 - 2*d*cos(2*d*sqr
t(x) + 2*c) + d)*x^2*integrate((2*a*b*d*x*sin(d*sqrt(x) + c) + 3*b^2*sqrt(x
)*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 + 2
*d*cos(d*sqrt(x) + c) + d)*x^3), x) - (d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2
*d*sqrt(x) + 2*c)^2 - 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x^2*integrate(-(2*a*b
*d*x*sin(d*sqrt(x) + c) - 3*b^2*sqrt(x)*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(
x) + c)^2 + d*sin(d*sqrt(x) + c)^2 - 2*d*cos(d*sqrt(x) + c) + d)*x^3), x) -
 4*b^2*sqrt(x)*sin(2*d*sqrt(x) + 2*c) - (a^2*d*cos(2*d*sqrt(x) + 2*c)^2 + a
^2*d*sin(2*d*sqrt(x) + 2*c)^2 - 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a^2*d)*x)/
((d*cos(2*d*sqrt(x) + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 - 2*d*cos(2*d*sqr
t(x) + 2*c) + d)*x^2)
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^2}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^2,x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)^2/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 18.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)^2}{x^2}\,dx$$

```
[In] int((a + b/sin(c + d*x^(1/2)))^2/x^2,x)
[Out] int((a + b/sin(c + d*x^(1/2)))^2/x^2, x)
```

3.41 $\int \frac{x^3}{a+b\csc(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 1075

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{14bx^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{420bx^2\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} - \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{5040bx\operatorname{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{5040bx\operatorname{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} + \frac{10080ib\sqrt{x}\operatorname{PolyLog}\left(7,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} - \frac{10080ib\sqrt{x}\operatorname{PolyLog}\left(7,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} - \frac{10080b\operatorname{PolyLog}\left(8,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8} + \frac{10080b\operatorname{PolyLog}\left(8,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8}$$

```
[Out] 1/4*x^4/a-84*I*b*x^(5/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(
1/2)))/a/d^3/(-a^2+b^2)^(1/2)-1680*I*b*x^(3/2)*polylog(5,I*a*exp(I*(c+d*x^(
1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^5/(-a^2+b^2)^(1/2)+14*b*x^3*polylog(2,I*a*
exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)-14*b*x^3*
polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(
1/2)+10080*I*b*polylog(7,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(
1/2)/a/d^7/(-a^2+b^2)^(1/2)+2*I*b*x^(7/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-
(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2)-420*b*x^2*polylog(4,I*a*exp(I*(c+d*
x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+420*b*x^2*polylog(4,
I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)-2*I*b
*x^(7/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d/(-a^2+b^2)
^(1/2)+1680*I*b*x^(3/2)*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1
/2)))/a/d^5/(-a^2+b^2)^(1/2)+5040*b*x*polylog(6,I*a*exp(I*(c+d*x^(1/2)))/(b
-(-a^2+b^2)^(1/2)))/a/d^6/(-a^2+b^2)^(1/2)-5040*b*x*polylog(6,I*a*exp(I*(c+
d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^6/(-a^2+b^2)^(1/2)-10080*b*polylog(8,
I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^8/(-a^2+b^2)^(1/2)+10080
*b*polylog(8,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^8/(-a^2+b^2
)^(1/2)-10080*I*b*polylog(7,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*
x^(1/2)/a/d^7/(-a^2+b^2)^(1/2)+84*I*b*x^(5/2)*polylog(3,I*a*exp(I*(c+d*x^(1
/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 1075, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used

$= \{4290, 4276, 3404, 2296, 2221, 2611, 6744, 2320, 6724\}$

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \frac{x^4}{4a}+\frac{2ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{7/2}}{a\sqrt{b^2-a^2}d}$$
$$-\frac{2ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{7/2}}{a\sqrt{b^2-a^2}d}+\frac{14b\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^3}{a\sqrt{b^2-a^2}d^2}$$
$$-\frac{14b\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^3}{a\sqrt{b^2-a^2}d^2}$$
$$+\frac{84ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a\sqrt{b^2-a^2}d^3}$$
$$-\frac{84ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a\sqrt{b^2-a^2}d^3}$$
$$-\frac{420b\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^2}{a\sqrt{b^2-a^2}d^4}$$
$$+\frac{420b\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^2}{a\sqrt{b^2-a^2}d^4}$$
$$-\frac{1680ib\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{3/2}}{a\sqrt{b^2-a^2}d^5}$$
$$+\frac{1680ib\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{3/2}}{a\sqrt{b^2-a^2}d^5}$$
$$+\frac{5040b\,\text{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x}{a\sqrt{b^2-a^2}d^6}$$
$$-\frac{5040b\,\text{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x}{a\sqrt{b^2-a^2}d^6}$$
$$+\frac{10080ib\,\text{PolyLog}\left(7,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)\sqrt{x}}{a\sqrt{b^2-a^2}d^7}$$
$$-\frac{10080ib\,\text{PolyLog}\left(7,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)\sqrt{x}}{a\sqrt{b^2-a^2}d^7}$$
$$-\frac{10080b\,\text{PolyLog}\left(8,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{a\sqrt{b^2-a^2}d^8}$$
$$+\frac{10080b\,\text{PolyLog}\left(8,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{a\sqrt{b^2-a^2}d^8}$$

```
[In] Int[x^3/(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] x^4/(4*a) + ((2*I)*b*x^(7/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((2*I)*b*x^(7/2)*Log[1 - (I*a*E^(I*
(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (14*b*x
^3*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[
-a^2 + b^2]*d^2) - (14*b*x^3*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sq
rt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((84*I)*b*x^(5/2)*PolyLog[3, (
I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3
) - ((84*I)*b*x^(5/2)*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2
 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3) - (420*b*x^2*PolyLog[4, (I*a*E^(I*(c +
d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^4) + (420*b*x^2
*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a
^2 + b^2]*d^4) - ((1680*I)*b*x^(3/2)*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))
/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^5) + ((1680*I)*b*x^(3/2)*Po
lyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2
+ b^2]*d^5) + (5040*b*x*PolyLog[6, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a
^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^6) - (5040*b*x*PolyLog[6, (I*a*E^(I*(c +
 d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^6) + ((10080*I
)*b*Sqrt[x]*PolyLog[7, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])
/(a*Sqrt[-a^2 + b^2]*d^7) - ((10080*I)*b*Sqrt[x]*PolyLog[7, (I*a*E^(I*(c +
d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^7) - (10080*b*P
olyLog[8, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2
 + b^2]*d^8) + (10080*b*PolyLog[8, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a
^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^8)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^7}{a+b\csc(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^7}{a}-\frac{bx^7}{a(b+a\sin(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{x^4}{4a}-\frac{(2b)\text{Subst}\left(\int \frac{x^7}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{x^4}{4a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{x^4}{4a}+\frac{(4ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&\quad-\frac{(4ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&= \frac{x^4}{4a}+\frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad-\frac{(14ib)\text{Subst}\left(\int x^6\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad+\frac{(14ib)\text{Subst}\left(\int x^6\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}\\
&= \frac{x^4}{4a}+\frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad+\frac{14bx^3\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{14bx^3\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}\\
&\quad-\frac{(84b)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}\\
&\quad+\frac{(84b)\text{Subst}\left(\int x^5\,\text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}
\end{aligned}
$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{14bx^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{(420ib)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(3,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$+ \frac{(420ib)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(3,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{14bx^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{420bx^2\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+ \frac{(1680b)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(4,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{(1680b)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(4,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{14bx^3\,\mathrm{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\,\mathrm{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{84ibx^{5/2}\,\mathrm{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\,\mathrm{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{420bx^2\,\mathrm{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\,\mathrm{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} - \frac{1680ibx^{3/2}\,\mathrm{PolyLog}\left(5, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{1680ibx^{3/2}\,\mathrm{PolyLog}\left(5, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{(5040ib)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(5, \frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5} - \frac{(5040ib)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(5, \frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5}$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{14bx^3\,\mathrm{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\,\mathrm{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{84ibx^{5/2}\,\mathrm{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\,\mathrm{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{420bx^2\,\mathrm{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\,\mathrm{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} - \frac{1680ibx^{3/2}\,\mathrm{PolyLog}\left(5, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{1680ibx^{3/2}\,\mathrm{PolyLog}\left(5, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{5040bx\,\mathrm{PolyLog}\left(6, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{5040bx\,\mathrm{PolyLog}\left(6, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{(10080b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(6, \frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^6} + \frac{(10080b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(6, \frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^6}$$

$$
\begin{aligned}
&= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
&+ \frac{14bx^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
&+ \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
&- \frac{420bx^2\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
&- \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{1680ibx^{3/2}\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
&+ \frac{5040bx\operatorname{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{5040bx\operatorname{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} \\
&+ \frac{10080ib\sqrt{x}\operatorname{PolyLog}\left(7,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} - \frac{10080ib\sqrt{x}\operatorname{PolyLog}\left(7,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} \\
&- \frac{(10080ib)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(7,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^7} \\
&+ \frac{(10080ib)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(7,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^7}
\end{aligned}
$$

$$= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{14bx^3\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{84ibx^{5/2}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{420bx^2\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{1680ibx^{3/2}\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{1680ibx^{3/2}\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$+ \frac{5040bx\,\text{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{5040bx\,\text{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$+ \frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} - \frac{10080ib\sqrt{x}\,\text{PolyLog}\left(7,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7}$$
$$- \frac{(10080b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(7,\frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^8}$$
$$+ \frac{(10080b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(7,\frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^8}$$

$$
\begin{aligned}
&= \frac{x^4}{4a} + \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\
&+ \frac{14bx^3\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{14bx^3\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\
&+ \frac{84ibx^{5/2}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{84ibx^{5/2}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} \\
&- \frac{420bx^2\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{420bx^2\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} \\
&- \frac{1680ibx^{3/2}\,\mathrm{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{1680ibx^{3/2}\,\mathrm{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} \\
&+ \frac{5040bx\,\mathrm{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{5040bx\,\mathrm{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} \\
&+ \frac{10080ib\sqrt{x}\,\mathrm{PolyLog}\left(7,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} - \frac{10080ib\sqrt{x}\,\mathrm{PolyLog}\left(7,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^7} \\
&- \frac{10080b\,\mathrm{PolyLog}\left(8,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8} + \frac{10080b\,\mathrm{PolyLog}\left(8,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^8}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 2.08 (sec) , antiderivative size = 850, normalized size of antiderivative = 0.79

$$
\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx
= \frac{\sqrt{a^2-b^2}d^8x^4 - 8bd^7x^{7/2}\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-ib+\sqrt{a^2-b^2}}\right) + 8bd^7x^{7/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right) + 56ibd^6x^3\,\mathrm{PolyLog}\left(2,\frac{ae}{-ib}\right.}{}
$$

```
[In] Integrate[x^3/(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[a^2 - b^2]*d^8*x^4 - 8*b*d^7*x^(7/2)*Log[1 - (a*E^(I*(c + d*Sqrt[x]))
)/((-I)*b + Sqrt[a^2 - b^2])] + 8*b*d^7*x^(7/2)*Log[1 + (a*E^(I*(c + d*Sqrt
[x])))/(I*b + Sqrt[a^2 - b^2])] + (56*I)*b*d^6*x^3*PolyLog[2, (a*E^(I*(c +
d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (56*I)*b*d^6*x^3*PolyLog[2, -((a
*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - 336*b*d^5*x^(5/2)*PolyL
og[3, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + 336*b*d^5*x^(
5/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - (16
80*I)*b*d^4*x^2*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b
^2])] + (1680*I)*b*d^4*x^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sq
rt[a^2 - b^2]))] + 6720*b*d^3*x^(3/2)*PolyLog[5, (a*E^(I*(c + d*Sqrt[x])))/
```

```
((-I)*b + Sqrt[a^2 - b^2])] - 6720*b*d^3*x^(3/2)*PolyLog[5, -((a*E^(I*(c +
d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] + (20160*I)*b*d^2*x*PolyLog[6, (a*E^
(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (20160*I)*b*d^2*x*PolyLo
g[6, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - 40320*b*d*Sqrt
[x]*PolyLog[7, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + 4032
0*b*d*Sqrt[x]*PolyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]
))] - (40320*I)*b*PolyLog[8, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 -
 b^2])] + (40320*I)*b*PolyLog[8, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^
2 - b^2]))])/(4*a*Sqrt[a^2 - b^2]*d^8)
```

Maple [F]

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x^3/(a+b*csc(c+d*x^(1/2))),x)
[Out] int(x^3/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^3}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^3/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(x^3/(b*csc(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^3}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x**3/(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(x**3/(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{a + b\csc\left(c + d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^3}{a + b\csc\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^3}{b\csc\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x^3/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(x^3/(b*csc(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a + b\csc\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x^3}{a + \frac{b}{\sin\left(c + d\sqrt{x}\right)}}\,dx$$

```
[In] int(x^3/(a + b/sin(c + d*x^(1/2))),x)
```

```
[Out] int(x^3/(a + b/sin(c + d*x^(1/2))), x)
```

3.42 $\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 807

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\frac{x^3}{3a}+\frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{10bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{10bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{40ibx^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{40ibx^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{120bx\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}+\frac{120bx\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}-\frac{240ib\sqrt{x}\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}+\frac{240ib\sqrt{x}\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}+\frac{240b\operatorname{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}-\frac{240b\operatorname{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$

```
[Out] 1/3*x^3/a+2*I*b*x^(5/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))
/a/d/(-a^2+b^2)^(1/2)-2*I*b*x^(5/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+
b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2)+10*b*x^2*polylog(2,I*a*exp(I*(c+d*x^(1/2)
))/(b-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)-10*b*x^2*polylog(2,I*a*exp(
I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+40*I*b*x^(3/2
)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)
^(1/2)-40*I*b*x^(3/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2
)))/a/d^3/(-a^2+b^2)^(1/2)-120*b*x*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b-(-
a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+120*b*x*polylog(4,I*a*exp(I*(c+d*x^
(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+240*b*polylog(6,I*a*ex
p(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^6/(-a^2+b^2)^(1/2)-240*b*polyl
og(6,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^6/(-a^2+b^2)^(1/2)-
```

```
240*I*b*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a/
d^5/(-a^2+b^2)^(1/2)+240*I*b*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^
2)^(1/2)))*x^(1/2)/a/d^5/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 807, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {4290, 4276, 3404, 2296, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}dx = &\frac{x^3}{3a}+\frac{2ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a\sqrt{b^2-a^2}d}\\ &-\frac{2ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a\sqrt{b^2-a^2}d}+\frac{10b\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^2}{a\sqrt{b^2-a^2}d^2}\\ &-\frac{10b\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^2}{a\sqrt{b^2-a^2}d^2}\\ &+\frac{40ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{3/2}}{a\sqrt{b^2-a^2}d^3}\\ &-\frac{40ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{3/2}}{a\sqrt{b^2-a^2}d^3}\\ &-\frac{120b\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x}{a\sqrt{b^2-a^2}d^4}\\ &+\frac{120b\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x}{a\sqrt{b^2-a^2}d^4}\\ &-\frac{240ib\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)\sqrt{x}}{a\sqrt{b^2-a^2}d^5}\\ &+\frac{240ib\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)\sqrt{x}}{a\sqrt{b^2-a^2}d^5}\\ &+\frac{240b\,\text{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{a\sqrt{b^2-a^2}d^6}-\frac{240b\,\text{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{a\sqrt{b^2-a^2}d^6}\end{aligned}$$

```
[In] Int[x^2/(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] x^3/(3*a) + ((2*I)*b*x^(5/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((2*I)*b*x^(5/2)*Log[1 - (I*a*E^(I*
(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (10*b*x
^2*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[
```

```
-a^2 + b^2]*d^2) - (10*b*x^2*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sq
rt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((40*I)*b*x^(3/2)*PolyLog[3, (
I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3
) - ((40*I)*b*x^(3/2)*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2
 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3) - (120*b*x*PolyLog[4, (I*a*E^(I*(c + d*
Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^4) + (120*b*x*Pol
yLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 +
 b^2]*d^4) - ((240*I)*b*Sqrt[x]*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b -
 Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^5) + ((240*I)*b*Sqrt[x]*PolyLog[
5, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]
*d^5) + (240*b*PolyLog[6, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2]
)])/(a*Sqrt[-a^2 + b^2]*d^6) - (240*b*PolyLog[6, (I*a*E^(I*(c + d*Sqrt[x]))
)/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^6)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^5}{a+b\csc(c+dx)}\,dx, x, \sqrt{x}\right) \\
&= 2\text{Subst}\left(\int \left(\frac{x^5}{a} - \frac{bx^5}{a(b+a\sin(c+dx))}\right)\,dx, x, \sqrt{x}\right) \\
&= \frac{x^3}{3a} - \frac{(2b)\text{Subst}\left(\int \frac{x^5}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a} \\
&= \frac{x^3}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}
\end{aligned}$$

$$= \frac{x^3}{3a} + \frac{(4ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}} - \frac{(4ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}$$

$$= \frac{x^3}{3a} + \frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{(10ib)\text{Subst}\left(\int x^4\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d} + \frac{(10ib)\text{Subst}\left(\int x^4\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}$$

$$= \frac{x^3}{3a} + \frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{10bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{10bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{(40b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(40b)\text{Subst}\left(\int x^3\,\text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{x^3}{3a} + \frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{10bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{10bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{(120ib)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3} + \frac{(120ib)\text{Subst}\left(\int x^2\,\text{PolyLog}\left(3,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$=\frac{x^3}{3a}+\frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{10bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{10bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{40ibx^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{40ibx^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{120bx\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}+\frac{120bx\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}+\frac{(240b)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(4,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}-\frac{(240b)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(4,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}$$

$$=\frac{x^3}{3a}+\frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{10bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{10bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{40ibx^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{40ibx^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{120bx\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}+\frac{120bx\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}-\frac{240ib\sqrt{x}\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}+\frac{240ib\sqrt{x}\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}+\frac{(240ib)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(5,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5}-\frac{(240ib)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(5,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^5}$$

$$= \frac{x^3}{3a} + \frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{10bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{10bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{120bx\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{120bx\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$+ \frac{(240b)\text{Subst}\left(\int\frac{\text{PolyLog}\left(5,\frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^6}$$
$$- \frac{(240b)\text{Subst}\left(\int\frac{\text{PolyLog}\left(5,\frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^6}$$

$$= \frac{x^3}{3a} + \frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{10bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{10bx^2\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{40ibx^{3/2}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{120bx\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{120bx\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$- \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{240ib\sqrt{x}\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$
$$+ \frac{240b\,\text{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6} - \frac{240b\,\text{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^6}$$

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 644, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$
$$=\frac{\sqrt{a^2-b^2}d^6x^3-6bd^5x^{5/2}\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-ib+\sqrt{a^2-b^2}}\right)+6bd^5x^{5/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)+30ibd^4x^2\operatorname{PolyLog}\left(2,\frac{ae^{i(c}}{-ib+}\right.}{}$$

```
[In] Integrate[x^2/(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[a^2 - b^2]*d^6*x^3 - 6*b*d^5*x^(5/2)*Log[1 - (a*E^(I*(c + d*Sqrt[x]))
)/((-I)*b + Sqrt[a^2 - b^2])] + 6*b*d^5*x^(5/2)*Log[1 + (a*E^(I*(c + d*Sqrt
[x])))/(I*b + Sqrt[a^2 - b^2])] + (30*I)*b*d^4*x^2*PolyLog[2, (a*E^(I*(c +
d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (30*I)*b*d^4*x^2*PolyLog[2, -((a
*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - 120*b*d^3*x^(3/2)*PolyL
og[3, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + 120*b*d^3*x^(
3/2)*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - (36
0*I)*b*d^2*x*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2]
)] + (360*I)*b*d^2*x*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2
 - b^2]))] + 720*b*d*Sqrt[x]*PolyLog[5, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b +
 Sqrt[a^2 - b^2])] - 720*b*d*Sqrt[x]*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))
/(I*b + Sqrt[a^2 - b^2]))] + (720*I)*b*PolyLog[6, (a*E^(I*(c + d*Sqrt[x])))
/((-I)*b + Sqrt[a^2 - b^2])] - (720*I)*b*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x
])))/(I*b + Sqrt[a^2 - b^2]))])/(3*a*Sqrt[a^2 - b^2]*d^6)
```

Maple [F]

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x^2/(a+b*csc(c+d*x^(1/2))),x)
```

```
[Out] int(x^2/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^2}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^2/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
```

```
[Out] integral(x^2/(b*csc(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x**2/(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(x**2/(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^2}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^2/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(x^2/(b*csc(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^2}{a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}}\,dx$$

```
[In] int(x^2/(a + b/sin(c + d*x^(1/2))),x)
[Out] int(x^2/(a + b/sin(c + d*x^(1/2))), x)
```

3.43 $\int \frac{x}{a+b\csc(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 539

$$\begin{aligned}\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx &= \frac{x^2}{2a}+\frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\ &+\frac{6bx\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}-\frac{6bx\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}\\ &+\frac{12ib\sqrt{x}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}\\ &-\frac{12ib\sqrt{x}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}\\ &-\frac{12b\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}+\frac{12b\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}\end{aligned}$$

```
[Out] 1/2*x^2/a+2*I*b*x^(3/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))
/a/d/(-a^2+b^2)^(1/2)-2*I*b*x^(3/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+
b^2)^(1/2)))/a/d/(-a^2+b^2)^(1/2)+6*b*x*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/
(b-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)-6*b*x*polylog(2,I*a*exp(I*(c+d
*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)-12*b*polylog(4,I*a*
exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)+12*b*poly
log(4,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^4/(-a^2+b^2)^(1/2)
+12*I*b*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a/
d^3/(-a^2+b^2)^(1/2)-12*I*b*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2
)^(1/2)))*x^(1/2)/a/d^3/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4290, 4276, 3404, 2296, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = -\frac{12b\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}}+\frac{12b\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} + \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}+\frac{6bx\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{6bx\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}+\frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}} - \frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad\sqrt{b^2-a^2}}+\frac{x^2}{2a}$$

```
[In] Int[x/(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] x^2/(2*a) + ((2*I)*b*x^(3/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((2*I)*b*x^(3/2)*Log[1 - (I*a*E^(I*
(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (6*b*x*
PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^
2 + b^2]*d^2) - (6*b*x*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^
2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((12*I)*b*Sqrt[x]*PolyLog[3, (I*a*E^
(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3) - ((
12*I)*b*Sqrt[x]*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2
])])/(a*Sqrt[-a^2 + b^2]*d^3) - (12*b*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x]))
)/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^4) + (12*b*PolyLog[4, (I*a
*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^3}{a+b\csc(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^3}{a}-\frac{bx^3}{a(b+a\sin(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{x^2}{2a}-\frac{(2b)\text{Subst}\left(\int \frac{x^3}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{x^2}{2a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{x^2}{2a}+\frac{(4ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&\quad-\frac{(4ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&= \frac{x^2}{2a}+\frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}-\frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad-\frac{(6ib)\text{Subst}\left(\int x^2\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}\\
&\quad+\frac{(6ib)\text{Subst}\left(\int x^2\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}
\end{aligned}$$

$$= \frac{x^2}{2a} + \frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{6bx\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{6bx\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$- \frac{(12b)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(12b)\text{Subst}\left(\int x\,\text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{x^2}{2a} + \frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{6bx\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{6bx\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{(12ib)\text{Subst}\left(\int \text{PolyLog}\left(3,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$+ \frac{(12ib)\text{Subst}\left(\int \text{PolyLog}\left(3,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{x^2}{2a} + \frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{6bx\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{6bx\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{(12b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(3,\frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^4}$$
$$+ \frac{(12b)\text{Subst}\left(\int \frac{\text{PolyLog}\left(3,\frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^4}$$

$$= \frac{x^2}{2a} + \frac{2ibx^{3/2}\log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^{3/2}\log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}$$
$$+ \frac{6bx\,\text{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{6bx\,\text{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{12ib\sqrt{x}\,\text{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$
$$- \frac{12b\,\text{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{12b\,\text{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}$$

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.81

$$\int \frac{x}{a + b\csc\left(c + d\sqrt{x}\right)}\,dx$$
$$= \frac{\sqrt{a^2-b^2}d^4x^2 - 4bd^3x^{3/2}\log\left(1 - \frac{ae^{i(c+d\sqrt{x})}}{-ib+\sqrt{a^2-b^2}}\right) + 4bd^3x^{3/2}\log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right) + 12ibd^2x\,\text{PolyLog}\left(2, \frac{ae^{i(}}{-ib+}\right.}{}$$

```
[In] Integrate[x/(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (Sqrt[a^2 - b^2]*d^4*x^2 - 4*b*d^3*x^(3/2)*Log[1 - (a*E^(I*(c + d*Sqrt[x]))
)/((-I)*b + Sqrt[a^2 - b^2])] + 4*b*d^3*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt
[x])))/(I*b + Sqrt[a^2 - b^2])] + (12*I)*b*d^2*x*PolyLog[2, (a*E^(I*(c + d*
Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (12*I)*b*d^2*x*PolyLog[2, -((a*E^(
I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - 24*b*d*Sqrt[x]*PolyLog[3, (
a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + 24*b*d*Sqrt[x]*PolyL
og[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - (24*I)*b*Poly
Log[4, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + (24*I)*b*Pol
yLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(2*a*Sqrt[a^
2 - b^2]*d^4)
```

Maple [F]

$$\int \frac{x}{a + b\csc\left(c + d\sqrt{x}\right)}\,dx$$

```
[In] int(x/(a+b*csc(c+d*x^(1/2))),x)
```

```
[Out] int(x/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x}{a + b\csc\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x}{b\csc\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(x/(b*csc(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x}{a + b\csc\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x}{a + b\csc\left(c + d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x/(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(x/(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{a + b\csc\left(c + d\sqrt{x}\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x}{a + b\csc\left(c + d\sqrt{x}\right)}\,dx = \int \frac{x}{b\csc\left(d\sqrt{x} + c\right) + a}\,dx$$

```
[In] integrate(x/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(x/(b*csc(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x}{a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}}\,dx$$

```
[In] int(x/(a + b/sin(c + d*x^(1/2))),x)
[Out] int(x/(a + b/sin(c + d*x^(1/2))), x)
```

3.44 $\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Int}\left(\frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*csc(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x*(a + b*Csc[c + d*Sqrt[x]])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*Csc[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 4.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Integrate[1/(x*(a + b*Csc[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x*(a + b*Csc[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}dx$$

```
[In] int(1/x/(a+b*csc(c+d*x^(1/2))),x)
[Out] int(1/x/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(1/(b*x*csc(d*sqrt(x) + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(1/(x*(a + b*csc(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 242, normalized size of antiderivative = 12.10

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] -(2*a*b*integrate((2*b*cos(d*sqrt(x) + c)^2 + a*cos(d*sqrt(x) + c)*sin(2*d*
sqrt(x) + 2*c) - a*cos(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 2*b*sin(d*sq
rt(x) + c)^2 + a*sin(d*sqrt(x) + c))/((a^3*cos(2*d*sqrt(x) + 2*c)^2 + 4*a*b
^2*cos(d*sqrt(x) + c)^2 + 4*a^2*b*cos(d*sqrt(x) + c)*sin(2*d*sqrt(x) + 2*c)
 + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a*b^2*sin(d*sqrt(x) + c)^2 + 4*a^2*b*si
n(d*sqrt(x) + c) + a^3 - 2*(2*a^2*b*sin(d*sqrt(x) + c) + a^3)*cos(2*d*sqrt(
x) + 2*c))*x), x) - log(x))/a
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*csc(d*sqrt(x) + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 18.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)}\,dx$$

```
[In] int(1/(x*(a + b/sin(c + d*x^(1/2)))),x)
```

```
[Out] int(1/(x*(a + b/sin(c + d*x^(1/2)))), x)
```

3.45 $\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=-\frac{a}{x}+b\text{Int}\left(\frac{\csc\left(c+d\sqrt{x}\right)}{x^2},x\right)$$

```
[Out] -a/x+b*Unintegrable(csc(c+d*x^(1/2))/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Int[(a + b*Csc[c + d*Sqrt[x]])/x^2,x]
```

```
[Out] -(a/x) + b*Defer[Int][Csc[c + d*Sqrt[x]]/x^2, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^2}+\frac{b\csc\left(c+d\sqrt{x}\right)}{x^2}\right)dx\\&=-\frac{a}{x}+b\int \frac{\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^2,x]
[Out] Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}dx$$

```
[In] int((a+b*csc(c+d*x^(1/2)))/x^2,x)
[Out] int((a+b*csc(c+d*x^(1/2)))/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="fricas")
[Out] integral((b*csc(d*sqrt(x) + c) + a)/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))/x**2,x)
[Out] Integral((a + b*csc(c + d*sqrt(x)))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 111, normalized size of antiderivative = 6.17

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="maxima")
```

```
[Out] ((b*integrate(sin(d*sqrt(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c
)^2 + 2*cos(d*sqrt(x) + c) + 1)*x^2), x) + b*integrate(sin(d*sqrt(x) + c)/(
(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1)*x^
2), x))*x - a)/x
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^2}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*sqrt(x) + c) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^2}\,dx=\int \frac{a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}}{x^2}\,dx$$

```
[In] int((a + b/sin(c + d*x^(1/2)))/x^2,x)
```

```
[Out] int((a + b/sin(c + d*x^(1/2)))/x^2, x)
```

3.46 $\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 3205

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=-\frac{2ib^2x^{7/2}}{a^2(a^2-b^2)d}+\frac{x^4}{4a^2}+\frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+\frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}-\frac{2ib^3x^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+\frac{4ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+\frac{2ib^3x^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$-\frac{4ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$-\frac{84ib^2x^{5/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$-\frac{84ib^2x^{5/2}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$-\frac{14b^3x^3\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+\frac{28bx^3\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+\frac{14b^3x^3\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$-\frac{28bx^3\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+\frac{420b^2x^2\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^4}$$
$$+\frac{420b^2x^2\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^4}$$
$$-\frac{84ib^3x^{5/2}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$+\frac{168ibx^{5/2}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+\frac{84ib^3x^{5/2}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$

```
[Out] 10080*I*b^3*polylog(7,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2
)/a^2/(-a^2+b^2)^(3/2)/d^7+20160*I*b*polylog(7,I*a*exp(I*(c+d*x^(1/2)))/(b-
(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^7/(-a^2+b^2)^(1/2)+2*I*b^3*x^(7/2)*ln(1-I*
a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+84*I*b^
3*x^(5/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^
2+b^2)^(3/2)/d^3+1680*I*b^2*x^(3/2)*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(I*b-
(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^5+1680*I*b^2*x^(3/2)*polylog(4,-a*exp(I*(
c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^5+1680*I*b^3*x^(3/2)*p
olylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2
)/d^5+4*I*b*x^(7/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2
/d/(-a^2+b^2)^(1/2)+168*I*b*x^(5/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(
-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)+3360*I*b*x^(3/2)*polylog(5,I*a*e
xp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)-2*b^2*x^
(7/2)*cos(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*sin(c+d*x^(1/2)))-2*I*b^3*x^(7/2)
*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d
-84*I*b^2*x^(5/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/
a^2/(a^2-b^2)/d^3-84*I*b^2*x^(5/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b+(
a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-84*I*b^3*x^(5/2)*polylog(3,I*a*exp(I*(c+
d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-1680*I*b^3*x^(3/
2)*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^
(3/2)/d^5-4*I*b*x^(7/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))
/a^2/d/(-a^2+b^2)^(1/2)-168*I*b*x^(5/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/
(b+(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-3360*I*b*x^(3/2)*polylog(5,I
*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)-1008
0*I*b^2*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))*x^(1/2)/a^
2/(a^2-b^2)/d^7-10080*I*b^2*polylog(6,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2
)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^7-10080*I*b^3*polylog(7,I*a*exp(I*(c+d*x^
(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^7-20160*I*b*po
lylog(7,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^7/(-a^
2+b^2)^(1/2)+1/4*x^4/a^2+840*b*x^2*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b+(-
a^2+b^2)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1/2)+10080*b*x*polylog(6,I*a*exp(I*(c+
d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)-10080*b*x*polylo
g(6,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)
-2*I*b^2*x^(7/2)/a^2/(a^2-b^2)/d+14*b^2*x^3*ln(1+a*exp(I*(c+d*x^(1/2)))/(I*
b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+14*b^2*x^3*ln(1+a*exp(I*(c+d*x^(1/2))
)/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2-14*b^3*x^3*polylog(2,I*a*exp(I*(
c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+14*b^3*x^3*pol
ylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/
d^2+420*b^2*x^2*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a^
2/(a^2-b^2)/d^4+420*b^2*x^2*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2
)^(1/2)))/a^2/(a^2-b^2)/d^4+420*b^3*x^2*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/
(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4-420*b^3*x^2*polylog(4,I*a*ex
p(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4-5040*b^2*
x*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^
6-5040*b^2*x*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/(
```

```
a^2-b^2)/d^6-5040*b^3*x*polylog(6,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1
/2)))/a^2/(-a^2+b^2)^(3/2)/d^6+5040*b^3*x*polylog(6,I*a*exp(I*(c+d*x^(1/2))
)/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^6+28*b*x^3*polylog(2,I*a*exp
(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-28*b*x^3*p
olylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^
(1/2)-840*b*x^2*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^
2/d^4/(-a^2+b^2)^(1/2)+10080*b^3*polylog(8,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^
2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^8-10080*b^3*polylog(8,I*a*exp(I*(c+d*
x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^8+10080*b^2*polylog(
7,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^8+10080*b^
2*polylog(7,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^
8-20160*b*polylog(8,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^8/
(-a^2+b^2)^(1/2)+20160*b*polylog(8,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(
1/2)))/a^2/d^8/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 5.18 (sec) , antiderivative size = 3205, normalized size of antiderivative = 1.00, number of steps used = 61, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules

used = {4290, 4276, 3405, 3404, 2296, 2221, 2611, 6744, 2320, 6724, 4617}

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \frac{x^4}{4a^2}+\frac{4ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{7/2}}{a^2\sqrt{b^2-a^2}d}$$

$$-\frac{2ib^3\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{7/2}}{a^2\left(b^2-a^2\right)^{3/2}d}$$

$$-\frac{4ib\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{7/2}}{a^2\sqrt{b^2-a^2}d}$$

$$+\frac{2ib^3\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{7/2}}{a^2\left(b^2-a^2\right)^{3/2}d}-\frac{2ib^2x^{7/2}}{a^2\left(a^2-b^2\right)d}$$

$$-\frac{2b^2\cos\left(c+d\sqrt{x}\right)x^{7/2}}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}$$

$$+\frac{14b^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{ib-\sqrt{a^2-b^2}}+1\right)x^3}{a^2\left(a^2-b^2\right)d^2}+\frac{14b^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{ib+\sqrt{a^2-b^2}}+1\right)x^3}{a^2\left(a^2-b^2\right)d^2}$$

$$+\frac{28b\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^3}{a^2\sqrt{b^2-a^2}d^2}$$

$$-\frac{14b^3\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$

$$-\frac{28b\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^3}{a^2\sqrt{b^2-a^2}d^2}$$

$$+\frac{14b^3\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^3}{a^2\left(b^2-a^2\right)^{3/2}d^2}$$

$$-\frac{84ib^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)x^{5/2}}{a^2\left(a^2-b^2\right)d^3}$$

$$-\frac{84ib^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)x^{5/2}}{a^2\left(a^2-b^2\right)d^3}$$

$$+\frac{168ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\sqrt{b^2-a^2}d^3}$$

$$-\frac{84ib^3\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$

$$-\frac{168ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\sqrt{b^2-a^2}d^3}$$

$$+\frac{84ib^3\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)x^{5/2}}{a^2\left(b^2-a^2\right)^{3/2}d^3}$$

$$+\frac{420b^2\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)x^2}{a^2\left(a^2-b^2\right)d^4}$$

```
[In] Int[x^3/(a + b*Csc[c + d*Sqrt[x]])^2,x]

[Out] ((-2*I)*b^2*x^(7/2))/(a^2*(a^2 - b^2)*d) + x^4/(4*a^2) + (14*b^2*x^3*Log[1
+ (a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2)
 + (14*b^2*x^3*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2])])/
(a^2*(a^2 - b^2)*d^2) - ((2*I)*b^3*x^(7/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x]
)))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((4*I)*b*x^(7/2)*
Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2
 + b^2]*d) + ((2*I)*b^3*x^(7/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sq
rt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((4*I)*b*x^(7/2)*Log[1 - (I*
a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d)
- ((84*I)*b^2*x^(5/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^
2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - ((84*I)*b^2*x^(5/2)*PolyLog[2, -((a*E^
(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (14
*b^3*x^3*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a
^2*(-a^2 + b^2)^(3/2)*d^2) + (28*b*x^3*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (14*b^3*x^3*PolyLo
g[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)
^(3/2)*d^2) - (28*b*x^3*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a
^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (420*b^2*x^2*PolyLog[3, -((a*E^(I
*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) + (420*
b^2*x^2*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(
a^2*(a^2 - b^2)*d^4) - ((84*I)*b^3*x^(5/2)*PolyLog[3, (I*a*E^(I*(c + d*Sqrt
[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((168*I)*b*x
^(5/2)*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2
*Sqrt[-a^2 + b^2]*d^3) + ((84*I)*b^3*x^(5/2)*PolyLog[3, (I*a*E^(I*(c + d*Sq
rt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((168*I)*b
*x^(5/2)*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a
^2*Sqrt[-a^2 + b^2]*d^3) + ((1680*I)*b^2*x^(3/2)*PolyLog[4, -((a*E^(I*(c +
d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^5) + ((1680*I)*b
^2*x^(3/2)*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))]
)/(a^2*(a^2 - b^2)*d^5) + (420*b^3*x^2*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^4) - (840*b*x^2*PolyL
og[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 +
 b^2]*d^4) - (420*b^3*x^2*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[
-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^4) + (840*b*x^2*PolyLog[4, (I*a*E^
(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^4) -
(5040*b^2*x*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))
])/(a^2*(a^2 - b^2)*d^6) - (5040*b^2*x*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])
))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^6) + ((1680*I)*b^3*x^(3/2)
*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2
 + b^2)^(3/2)*d^5) - ((3360*I)*b*x^(3/2)*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x
])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^5) - ((1680*I)*b^3*x^
(3/2)*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*
(-a^2 + b^2)^(3/2)*d^5) + ((3360*I)*b*x^(3/2)*PolyLog[5, (I*a*E^(I*(c + d*S
```

```
qrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^5) - ((10080*I)*
b^2*Sqrt[x]*PolyLog[6, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))
])/(a^2*(a^2 - b^2)*d^7) - ((10080*I)*b^2*Sqrt[x]*PolyLog[6, -((a*E^(I*(c +
 d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^7) - (5040*b^3*
x*PolyLog[6, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^
2 + b^2)^(3/2)*d^6) + (10080*b*x*PolyLog[6, (I*a*E^(I*(c + d*Sqrt[x])))/(b
- Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^6) + (5040*b^3*x*PolyLog[6, (
I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)
*d^6) - (10080*b*x*PolyLog[6, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 +
b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^6) + (10080*b^2*PolyLog[7, -((a*E^(I*(c + d
*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^8) + (10080*b^2*P
olyLog[7, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2
- b^2)*d^8) - ((10080*I)*b^3*Sqrt[x]*PolyLog[7, (I*a*E^(I*(c + d*Sqrt[x])))
/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^7) + ((20160*I)*b*Sqrt[
x]*PolyLog[7, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqr
t[-a^2 + b^2]*d^7) + ((10080*I)*b^3*Sqrt[x]*PolyLog[7, (I*a*E^(I*(c + d*Sqr
t[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^7) - ((20160*I)*
b*Sqrt[x]*PolyLog[7, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(
a^2*Sqrt[-a^2 + b^2]*d^7) + (10080*b^3*PolyLog[8, (I*a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^8) - (20160*b*PolyLog
[8, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b
^2]*d^8) - (10080*b^3*PolyLog[8, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2
 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^8) + (20160*b*PolyLog[8, (I*a*E^(I*(c
+ d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^8) - (2*b^2
*x^(7/2)*Cos[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a*Sin[c + d*Sqrt[x]]))
```

<u>Rule 2221</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

<u>Rule 2296</u>

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

<u>Rule 2320</u>

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4617

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2
```

```
] + b*E^(I*(c + d*x)))), x], x] + Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/
(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^7}{(a+b\csc(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^7}{a^2}+\frac{b^2x^7}{a^2(b+a\sin(c+dx))^2}-\frac{2bx^7}{a^2(b+a\sin(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{x^4}{4a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^7}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^7}{(b+a\sin(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{x^4}{4a^2}-\frac{2b^2x^{7/2}\cos\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad-\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^7}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}+\frac{(14b^2)\,\text{Subst}\left(\int \frac{x^6\cos(c+dx)}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}
\end{aligned}$$

$$= -\frac{2ib^2x^{7/2}}{a^2(a^2-b^2)d} + \frac{x^4}{4a^2} - \frac{2b^2x^{7/2}\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)}$$
$$+ \frac{(8ib)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$- \frac{(8ib)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$+ \frac{(14ib^2)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^6}{ib-\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$
$$+ \frac{(14ib^2)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^6}{ib+\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$

$$= -\frac{2ib^2x^{7/2}}{a^2(a^2-b^2)d} + \frac{x^4}{4a^2} + \frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{4ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{2b^2x^{7/2}\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{(4ib^3)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$+ \frac{(4ib^3)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^7}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$- \frac{(84b^2)\operatorname{Subst}\left(\int x^5\log\left(1+\frac{ae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{(84b^2)\operatorname{Subst}\left(\int x^5\log\left(1+\frac{ae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{(28ib)\operatorname{Subst}\left(\int x^6\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+ \frac{(28ib)\operatorname{Subst}\left(\int x^6\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$
\begin{aligned}
&= -\frac{2ib^2x^{7/2}}{a^2(a^2-b^2)d}+\frac{x^4}{4a^2}+\frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} \\
&+\frac{14b^2x^3\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}-\frac{2ib^3x^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&+\frac{4ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}+\frac{2ib^3x^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&-\frac{4ibx^{7/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}-\frac{84ib^2x^{5/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} \\
&-\frac{84ib^2x^{5/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}+\frac{28bx^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&-\frac{28bx^3\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}-\frac{2b^2x^{7/2}\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)} \\
&+\frac{(420ib^2)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3} \\
&+\frac{(420ib^2)\operatorname{Subst}\left(\int x^4\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3} \\
&-\frac{(168b)\operatorname{Subst}\left(\int x^5\operatorname{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+\frac{(168b)\operatorname{Subst}\left(\int x^5\operatorname{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2} \\
&+\frac{(14ib^3)\operatorname{Subst}\left(\int x^6\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&-\frac{(14ib^3)\operatorname{Subst}\left(\int x^6\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}
\end{aligned}
$$

= Too large to display

Mathematica [A] (warning: unable to verify)

Time = 15.83 (sec) , antiderivative size = 3831, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x^3/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] (x^4*Csc[c + d*Sqrt[x]]^2*(b + a*Sin[c + d*Sqrt[x]])^2)/(4*a^2*(a + b*Csc[c
 + d*Sqrt[x]])^2) - ((2*I)*b*E^(I*c)*Csc[c + d*Sqrt[x]]^2*(2*b*E^(I*c)*x^(7
/2) - ((-1 + E^((2*I)*c))*((-7*I)*b*d^6*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^3*L
og[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)
*c)])] + (2*I)*a^2*d^7*E^(I*c)*x^(7/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/
(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - I*b^2*d^7*E^(I*c)*x^(7/2)*
Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I
)*c)])] - (7*I)*b*d^6*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^3*Log[1 + (a*E^(I*(2*
c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (2*I)*a^2
*d^7*E^(I*c)*x^(7/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqr
t[(a^2 - b^2)*E^((2*I)*c)])] + I*b^2*d^7*E^(I*c)*x^(7/2)*Log[1 + (a*E^(I*(2
*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 7*d^5*(6
*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*
Sqrt[x])*x^(5/2)*PolyLog[2, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sq
rt[(a^2 - b^2)*E^((2*I)*c)])] + 7*d^5*(-6*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] -
 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*x^(5/2)*PolyLog[2, -((a*E
^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (
210*I)*b*d^4*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*PolyLog[3, (I*a*E^(I*(2*c +
d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (84*I)*a^2*d^
5*E^(I*c)*x^(5/2)*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*S
qrt[(a^2 - b^2)*E^((2*I)*c)])] - (42*I)*b^2*d^5*E^(I*c)*x^(5/2)*PolyLog[3,
(I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])
] - (210*I)*b*d^4*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*PolyLog[3, -((a*E^(I*(2
*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (84*I)*
a^2*d^5*E^(I*c)*x^(5/2)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*
c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + (42*I)*b^2*d^5*E^(I*c)*x^(5/2)*Poly
Log[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*
I)*c)]))] + 840*b*d^3*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^(3/2)*PolyLog[4, (I*a
*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] -
420*a^2*d^4*E^(I*c)*x^2*PolyLog[4, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c)
 + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 210*b^2*d^4*E^(I*c)*x^2*PolyLog[4, (
I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])]
 + 840*b*d^3*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^(3/2)*PolyLog[4, -((a*E^(I*(2*
c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + 420*a^2*
d^4*E^(I*c)*x^2*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqr
t[(a^2 - b^2)*E^((2*I)*c)]))] - 210*b^2*d^4*E^(I*c)*x^2*PolyLog[4, -((a*E^(
```

```
I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + (25
20*I)*b*d^2*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x*PolyLog[5, (I*a*E^(I*(2*c + d*S
qrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (1680*I)*a^2*d^3
*E^(I*c)*x^(3/2)*PolyLog[5, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sq
rt[(a^2 - b^2)*E^((2*I)*c)])] + (840*I)*b^2*d^3*E^(I*c)*x^(3/2)*PolyLog[5,
(I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])
] + (2520*I)*b*d^2*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x*PolyLog[5, -((a*E^(I*(2*
c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + (1680*I)
*a^2*d^3*E^(I*c)*x^(3/2)*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I
*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (840*I)*b^2*d^3*E^(I*c)*x^(3/2)*Po
lyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((
2*I)*c)]))] - 5040*b*d*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[6, (I*
a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] +
 5040*a^2*d^2*E^(I*c)*x*PolyLog[6, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c)
 + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 2520*b^2*d^2*E^(I*c)*x*PolyLog[6, (I
*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])]
- 5040*b*d*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[6, -((a*E^(I*(2*c
+ d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - 5040*a^2*d
^2*E^(I*c)*x*PolyLog[6, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(
a^2 - b^2)*E^((2*I)*c)]))] + 2520*b^2*d^2*E^(I*c)*x*PolyLog[6, -((a*E^(I*(2
*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (5040*I
)*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*PolyLog[7, (I*a*E^(I*(2*c + d*Sqrt[x])))/
(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (10080*I)*a^2*d*E^(I*c)*Sq
rt[x]*PolyLog[7, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b
^2)*E^((2*I)*c)])] - (5040*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[7, (I*a*E^(I*(2
*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (5040*I)
*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*PolyLog[7, -((a*E^(I*(2*c + d*Sqrt[x])))/(
I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (10080*I)*a^2*d*E^(I*c)*Sq
rt[x]*PolyLog[7, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b
^2)*E^((2*I)*c)]))] + (5040*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[7, -((a*E^(I*(
2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - 10080*
a^2*E^(I*c)*PolyLog[8, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a
^2 - b^2)*E^((2*I)*c)])] + 5040*b^2*E^(I*c)*PolyLog[8, (I*a*E^(I*(2*c + d*S
qrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 10080*a^2*E^(I*c
)*PolyLog[8, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*
E^((2*I)*c)]))] - 5040*b^2*E^(I*c)*PolyLog[8, -((a*E^(I*(2*c + d*Sqrt[x])))
/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))]))/(d^7*E^(I*c)*Sqrt[(a^2 -
 b^2)*E^((2*I)*c)]))*(b + a*Sin[c + d*Sqrt[x]])^2)/(a^2*(a^2 - b^2)*d*(-1 +
 E^((2*I)*c))*(a + b*Csc[c + d*Sqrt[x]])^2) + (Csc[c/2]*Csc[c + d*Sqrt[x]]^
2*Sec[c/2]*(b + a*Sin[c + d*Sqrt[x]])*(-(b^3*x^(7/2)*Cos[c]) - a*b^2*x^(7/2
)*Sin[d*Sqrt[x]]))/(a^2*(-a + b)*(a + b)*d*(a + b*Csc[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(x^3/(a+b*csc(c+d*x^(1/2)))^2,x)
[Out] int(x^3/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^3}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^3/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x^3/(b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2), x
)
```

Sympy [F]

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**3/(a+b*csc(c+d*x**(1/2)))**2,x)
[Out] Integral(x**3/(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^3}{\left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2}\, dx = \int \frac{x^3}{\left(b\csc\left(d\sqrt{x} + c\right) + a\right)^2}\, dx$$

```
[In] integrate(x^3/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(x^3/(b*csc(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2}\, dx = \int \frac{x^3}{\left(a + \frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\, dx$$

```
[In] int(x^3/(a + b/sin(c + d*x^(1/2)))^2,x)
[Out] int(x^3/(a + b/sin(c + d*x^(1/2)))^2, x)
```

3.47 $\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 2385

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ib^2x^{5/2}}{a^2\left(a^2-b^2\right)d}+\frac{x^3}{3a^2}+\frac{10b^2x^2\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}$$

$$+\frac{10b^2x^2\log\left(1+\frac{ae^{i\left(c+d\sqrt{x}\right)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}-\frac{2ib^3x^{5/2}\log\left(1-\frac{iae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$

$$+\frac{4ibx^{5/2}\log\left(1-\frac{iae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$+\frac{2ib^3x^{5/2}\log\left(1-\frac{iae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$

$$-\frac{4ibx^{5/2}\log\left(1-\frac{iae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$-\frac{40ib^2x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$

$$-\frac{40ib^2x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$

$$-\frac{10b^3x^2\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$

$$+\frac{20bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$+\frac{10b^3x^2\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$

$$-\frac{20bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$

$$+\frac{120b^2x\operatorname{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}$$

$$+\frac{120b^2x\operatorname{PolyLog}\left(3,-\frac{ae^{i\left(c+d\sqrt{x}\right)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}$$

$$-\frac{40ib^3x^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$

$$+\frac{80ibx^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i\left(c+d\sqrt{x}\right)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$

$$+\frac{40ib^3x^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i\left(c+d\sqrt{x}\right)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$

$$80ibx^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i\left(c+d\sqrt{x}\right)}}{\cdots}\right)$$

```
[Out] 480*I*b*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^
2/d^5/(-a^2+b^2)^(1/2)+2*I*b^3*x^(5/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a
^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+40*I*b^3*x^(3/2)*polylog(3,I*a*exp(I
*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+4*I*b*x^(5/2
)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2
)+80*I*b*x^(3/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a
^2/d^3/(-a^2+b^2)^(1/2)+240*I*b^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a
^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^5+240*I*b^2*polylog(4,-a*exp(I*(c+d
*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^5+240*I*b^3*polyl
og(5,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^
(3/2)/d^5-2*b^2*x^(5/2)*cos(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*sin(c+d*x^(1/2)
))-2*I*b^3*x^(5/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/
(-a^2+b^2)^(3/2)/d-40*I*b^2*x^(3/2)*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b-
(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-40*I*b^2*x^(3/2)*polylog(2,-a*exp(I*(c+
d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-40*I*b^3*x^(3/2)*polyl
og(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^
3-4*I*b*x^(5/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(
-a^2+b^2)^(1/2)-80*I*b*x^(3/2)*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+
b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)-240*I*b^3*polylog(5,I*a*exp(I*(c+d*x^
(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^5-480*I*b*poly
log(5,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^5/(-a^2+
b^2)^(1/2)+1/3*x^3/a^2+10*b^2*x^2*ln(1+a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2
)^(1/2)))/a^2/(a^2-b^2)/d^2+10*b^2*x^2*ln(1+a*exp(I*(c+d*x^(1/2)))/(I*b+(a^
2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2-10*b^3*x^2*polylog(2,I*a*exp(I*(c+d*x^(1/2
)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+10*b^3*x^2*polylog(2,I*a
*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+120*b^
2*x*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/
d^4+120*b^2*x*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/
(a^2-b^2)/d^4+120*b^3*x*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1
/2)))/a^2/(-a^2+b^2)^(3/2)/d^4-120*b^3*x*polylog(4,I*a*exp(I*(c+d*x^(1/2)))
/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4+20*b*x^2*polylog(2,I*a*exp(
I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-20*b*x^2*po
lylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(
1/2)-240*b*x*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d
^4/(-a^2+b^2)^(1/2)+240*b*x*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2
)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1/2)-2*I*b^2*x^(5/2)/a^2/(a^2-b^2)/d-240*b^2*
polylog(5,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^6-
240*b^2*polylog(5,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b
^2)/d^6-240*b^3*polylog(6,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^
2/(-a^2+b^2)^(3/2)/d^6+240*b^3*polylog(6,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+
b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^6+480*b*polylog(6,I*a*exp(I*(c+d*x^(1/2
)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)-480*b*polylog(6,I*a*exp(
I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^6/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 4.12 (sec) , antiderivative size = 2385, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules

used = {4290, 4276, 3405, 3404, 2296, 2221, 2611, 6744, 2320, 6724, 4617}

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ix^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} + \frac{2ix^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} - \frac{10x^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} + \frac{10x^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} - \frac{40ix^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{40ix^{3/2}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{120x\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4} - \frac{120x\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4} + \frac{240i\sqrt{x}\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5} - \frac{240i\sqrt{x}\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5} - \frac{240\operatorname{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^6} + \frac{240\operatorname{PolyLog}\left(6,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^6} - \frac{2ix^{5/2}b^2}{a^2\left(a^2-b^2\right)d} + \frac{10x^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{ib-\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} + \frac{10x^2\log\left(\frac{e^{i(c+d\sqrt{x})}a}{ib+\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} - \frac{40ix^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} - \frac{40ix^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} + \frac{120x\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4} + \frac{120x\operatorname{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)b^2}{}$$

```
[In] Int[x^2/(a + b*Csc[c + d*Sqrt[x]])^2,x]

[Out] ((-2*I)*b^2*x^(5/2))/(a^2*(a^2 - b^2)*d) + x^3/(3*a^2) + (10*b^2*x^2*Log[1
+ (a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2)
 + (10*b^2*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2])])/
(a^2*(a^2 - b^2)*d^2) - ((2*I)*b^3*x^(5/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x]
)))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((4*I)*b*x^(5/2)*
Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2
 + b^2]*d) + ((2*I)*b^3*x^(5/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sq
rt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((4*I)*b*x^(5/2)*Log[1 - (I*
a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d)
- ((40*I)*b^2*x^(3/2)*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^
2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - ((40*I)*b^2*x^(3/2)*PolyLog[2, -((a*E^
(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (10
*b^3*x^2*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a
^2*(-a^2 + b^2)^(3/2)*d^2) + (20*b*x^2*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (10*b^3*x^2*PolyLo
g[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)
^(3/2)*d^2) - (20*b*x^2*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a
^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (120*b^2*x*PolyLog[3, -((a*E^(I*(
c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) + (120*b^
2*x*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*
(a^2 - b^2)*d^4) - ((40*I)*b^3*x^(3/2)*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((80*I)*b*x^(3/2
)*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt
[-a^2 + b^2]*d^3) + ((40*I)*b^3*x^(3/2)*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x]
)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((80*I)*b*x^(3/
2)*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqr
t[-a^2 + b^2]*d^3) + ((240*I)*b^2*Sqrt[x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[
x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^5) + ((240*I)*b^2*Sqrt[
x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(
a^2 - b^2)*d^5) + (120*b^3*x*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sq
rt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^4) - (240*b*x*PolyLog[4, (I*a*E
^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^4) -
 (120*b^3*x*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])
/(a^2*(-a^2 + b^2)^(3/2)*d^4) + (240*b*x*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x
])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (240*b^2*PolyLog
[5, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)
*d^6) - (240*b^2*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b
^2]))])/(a^2*(a^2 - b^2)*d^6) + ((240*I)*b^3*Sqrt[x]*PolyLog[5, (I*a*E^(I*(
c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^5) - ((
480*I)*b*Sqrt[x]*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^
2])])/(a^2*Sqrt[-a^2 + b^2]*d^5) - ((240*I)*b^3*Sqrt[x]*PolyLog[5, (I*a*E^(
I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^5) +
 ((480*I)*b*Sqrt[x]*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 +
```

```
 b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^5) - (240*b^3*PolyLog[6, (I*a*E^(I*(c + d*
Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^6) + (480*b*P
olyLog[6, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a
^2 + b^2]*d^6) + (240*b^3*PolyLog[6, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[
-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^6) - (480*b*PolyLog[6, (I*a*E^(I*(
c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^6) - (2*b
^2*x^(5/2)*Cos[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a*Sin[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4617

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2
] + b*E^(I*(c + d*x)))), x], x] + Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/
(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^5}{(a+b\csc(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^5}{a^2}+\frac{b^2x^5}{a^2(b+a\sin(c+dx))^2}-\frac{2bx^5}{a^2(b+a\sin(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{x^3}{3a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^5}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^5}{(b+a\sin(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{x^3}{3a^2}-\frac{2b^2x^{5/2}\cos\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad-\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^5}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}+\frac{(10b^2)\,\text{Subst}\left(\int \frac{x^4\cos(c+dx)}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}\\
&= -\frac{2ib^2x^{5/2}}{a^2\left(a^2-b^2\right)d}+\frac{x^3}{3a^2}-\frac{2b^2x^{5/2}\cos\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}\\
&\quad-\frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}\\
&\quad+\frac{(8ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}\\
&\quad-\frac{(8ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}\\
&\quad+\frac{(10ib^2)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{ib-\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}\\
&\quad+\frac{(10ib^2)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{ib+\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}
\end{aligned}
$$

$$= -\frac{2ib^2x^{5/2}}{a^2(a^2-b^2)d} + \frac{x^3}{3a^2} + \frac{10b^2x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$

$$+ \frac{10b^2x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$- \frac{4ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{2b^2x^{5/2}\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}$$

$$- \frac{(4ib^3)\,\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$

$$+ \frac{(4ib^3)\,\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x^5}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$

$$- \frac{(40b^2)\,\mathrm{Subst}\left(\int x^3\log\left(1+\frac{ae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$

$$- \frac{(40b^2)\,\mathrm{Subst}\left(\int x^3\log\left(1+\frac{ae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$

$$- \frac{(20ib)\mathrm{Subst}\left(\int x^4\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$+ \frac{(20ib)\mathrm{Subst}\left(\int x^4\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$= -\frac{2ib^2x^{5/2}}{a^2(a^2-b^2)d} + \frac{x^3}{3a^2} + \frac{10b^2x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{10b^2x^2\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{4ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{4ibx^{5/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{40ib^2x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{40ib^2x^{3/2}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} + \frac{20bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{20bx^2\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{2b^2x^{5/2}\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(120ib^2)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{(120ib^2)\operatorname{Subst}\left(\int x^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{(80b)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(80b)\operatorname{Subst}\left(\int x^3\operatorname{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(10ib^3)\operatorname{Subst}\left(\int x^4\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{(10ib^3)\operatorname{Subst}\left(\int x^4\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

= Too large to display

Mathematica [A] (warning: unable to verify)

Time = 14.55 (sec) , antiderivative size = 2829, normalized size of antiderivative = 1.19

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Result too large to show}$$

[In] Integrate[x^2/(a + b*Csc[c + d*Sqrt[x]])^2,x]

[Out] (x^3*Csc[c + d*Sqrt[x]]^2*(b + a*Sin[c + d*Sqrt[x]])^2)/(3*a^2*(a + b*Csc[c + d*Sqrt[x]])^2) - ((2*I)*b*E^(I*c)*Csc[c + d*Sqrt[x]]^2*(2*b*E^(I*c)*x^(5/2) - ((-1 + E^((2*I)*c))*((-5*I)*b*d^4*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (2*I)*a^2*d^5*E^(I*c)*x^(5/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - I*b^2*d^5*E^(I*c)*x^(5/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (5*I)*b*d^4*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (2*I)*a^2*d^5*E^(I*c)*x^(5/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + I*b^2*d^5*E^(I*c)*x^(5/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 5*d^3*(4*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*x^(3/2)*PolyLog[2, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 5*d^3*(-4*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*x^(3/2)*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (60*I)*b*d^2*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (40*I)*a^2*d^3*E^(I*c)*x^(3/2)*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (20*I)*b^2*d^3*E^(I*c)*x^(3/2)*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (60*I)*b*d^2*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (40*I)*a^2*d^3*E^(I*c)*x^(3/2)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + (20*I)*b^2*d^3*E^(I*c)*x^(3/2)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + 120*b*d*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[4, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 120*a^2*d^2*E^(I*c)*x*PolyLog[4, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 60*b^2*d^2*E^(I*c)*x*PolyLog[4, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 120*b*d*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + 120*a^2*d^2*E^(I*c)*x*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - 60*b^2*d^2*E^(I*c)*x*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x]))

```
)/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + (120*I)*b*Sqrt[(a^2 - b
^2)*E^((2*I)*c)]*PolyLog[5, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sq
rt[(a^2 - b^2)*E^((2*I)*c)])] - (240*I)*a^2*d*E^(I*c)*Sqrt[x]*PolyLog[5, (I
*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])]
+ (120*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[5, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b
*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (120*I)*b*Sqrt[(a^2 - b^2)*E
^((2*I)*c)]*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a
^2 - b^2)*E^((2*I)*c)]))] + (240*I)*a^2*d*E^(I*c)*Sqrt[x]*PolyLog[5, -((a*E
^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (
120*I)*b^2*d*E^(I*c)*Sqrt[x]*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*
E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + 240*a^2*E^(I*c)*PolyLog[6, (I*
a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] -
 120*b^2*E^(I*c)*PolyLog[6, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sq
rt[(a^2 - b^2)*E^((2*I)*c)])] - 240*a^2*E^(I*c)*PolyLog[6, -((a*E^(I*(2*c +
 d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + 120*b^2*E^(
I*c)*PolyLog[6, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^
2)*E^((2*I)*c)]))]))/(d^5*E^(I*c)*Sqrt[(a^2 - b^2)*E^((2*I)*c)]))*(b + a*Si
n[c + d*Sqrt[x]])^2)/(a^2*(a^2 - b^2)*d*(-1 + E^((2*I)*c))*(a + b*Csc[c + d
*Sqrt[x]])^2) + (Csc[c/2]*Csc[c + d*Sqrt[x]]^2*Sec[c/2]*(b + a*Sin[c + d*Sq
rt[x]])*(-(b^3*x^(5/2)*Cos[c]) - a*b^2*x^(5/2)*Sin[d*Sqrt[x]]))/(a^2*(-a +
b)*(a + b)*d*(a + b*Csc[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
[Out] int(x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x^2/(b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2), x
)
```

Sympy [F]

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**2/(a+b*csc(c+d*x**(1/2)))**2,x)
[Out] Integral(x**2/(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(x^2/(b*csc(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x^2}{{\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)}^2}\,dx$$

```
[In] int(x^2/(a + b/sin(c + d*x^(1/2)))^2,x)
[Out] int(x^2/(a + b/sin(c + d*x^(1/2)))^2, x)
```

3.48 $\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 1565

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ib^2x^{3/2}}{a^2\left(a^2-b^2\right)d}+\frac{x^2}{2a^2}+\frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}$$
$$+\frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}-\frac{2ib^3x^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$
$$+\frac{4ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+\frac{2ib^3x^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}$$
$$-\frac{4ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$-\frac{12ib^2\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{12ib^2\sqrt{x}\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{6b^3x\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$
$$+\frac{12bx\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+\frac{6b^3x\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}$$
$$-\frac{12bx\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+\frac{12b^2\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}$$
$$+\frac{12b^2\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}$$
$$-\frac{12ib^3\sqrt{x}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$
$$+\frac{24ib\sqrt{x}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+\frac{12ib^3\sqrt{x}\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}$$

```
[Out] 24*I*b*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2
/d^3/(-a^2+b^2)^(1/2)+2*I*b^3*x^(3/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^
2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+4*I*b*x^(3/2)*ln(1-I*a*exp(I*(c+d*x^(
1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)+12*I*b^3*polylog(3,I*a*
exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^3
-2*b^2*x^(3/2)*cos(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*sin(c+d*x^(1/2)))-2*I*b^
3*x^(3/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2
)^(3/2)/d-4*I*b*x^(3/2)*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))
/a^2/d/(-a^2+b^2)^(1/2)-12*I*b^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^
2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^3-12*I*b^2*polylog(2,-a*exp(I*(c+d*x
^(1/2)))/(I*b+(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^3-12*I*b^3*polylog(
3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/
2)/d^3-24*I*b*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1
/2)/a^2/d^3/(-a^2+b^2)^(1/2)-6*b^3*x*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b-
(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+6*b^3*x*polylog(2,I*a*exp(I*(c+
d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+12*b*x*polylog(2
,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-12
*b*x*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2
+b^2)^(1/2)+6*b^2*x*ln(1+a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a^2/
(a^2-b^2)/d^2+6*b^2*x*ln(1+a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^
2/(a^2-b^2)/d^2-2*I*b^2*x^(3/2)/a^2/(a^2-b^2)/d+1/2*x^2/a^2+12*b^3*polylog(
4,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^4-1
2*b^3*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^
2)^(3/2)/d^4-24*b*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/
a^2/d^4/(-a^2+b^2)^(1/2)+24*b*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b
^2)^(1/2)))/a^2/d^4/(-a^2+b^2)^(1/2)+12*b^2*polylog(3,-a*exp(I*(c+d*x^(1/2)
))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4+12*b^2*polylog(3,-a*exp(I*(c+d*
x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^4
```

Rubi [A] (verified)

Time = 2.97 (sec) , antiderivative size = 1565, normalized size of antiderivative = 1.00, number of steps used = 37, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules

used = {4290, 4276, 3405, 3404, 2296, 2221, 2611, 6744, 2320, 6724, 4617}

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ix^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} + \frac{2ix^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} - \frac{6x\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} + \frac{6x\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} - \frac{12i\sqrt{x}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{12i\sqrt{x}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{12\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4} - \frac{12\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4} - \frac{2ix^{3/2}b^2}{a^2\left(a^2-b^2\right)d} + \frac{6x\log\left(\frac{e^{i(c+d\sqrt{x})}a}{ib-\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} + \frac{6x\log\left(\frac{e^{i(c+d\sqrt{x})}a}{ib+\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} - \frac{12i\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} - \frac{12i\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} + \frac{12\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4} + \frac{12\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4} - \frac{2x^{3/2}\cos\left(c+d\sqrt{x}\right)b^2}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)} + \frac{4ix^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d} - \frac{4ix^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d} + \frac{12x\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2} - \frac{12x\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2}$$

```
[In] Int[x/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x^(3/2))/(a^2*(a^2 - b^2)*d) + x^2/(2*a^2) + (6*b^2*x*Log[1 + (
a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2) +
(6*b^2*x*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2])])/(a^2*(
a^2 - b^2)*d^2) - ((2*I)*b^3*x^(3/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b
 - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((4*I)*b*x^(3/2)*Log[1
- (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2
]*d) + ((2*I)*b^3*x^(3/2)*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^
2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((4*I)*b*x^(3/2)*Log[1 - (I*a*E^(I
*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((12
*I)*b^2*Sqrt[x]*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^
2]))])/(a^2*(a^2 - b^2)*d^3) - ((12*I)*b^2*Sqrt[x]*PolyLog[2, -((a*E^(I*(c
+ d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (6*b^3*x*
PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2
+ b^2)^(3/2)*d^2) + (12*b*x*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqr
t[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (6*b^3*x*PolyLog[2, (I*a*E^(I
*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) -
(12*b*x*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^
2*Sqrt[-a^2 + b^2]*d^2) + (12*b^2*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I
*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) + (12*b^2*PolyLog[3, -((a*E^
(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) - ((1
2*I)*b^3*Sqrt[x]*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^
2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((24*I)*b*Sqrt[x]*PolyLog[3, (I*a*E^(I
*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3) + ((
12*I)*b^3*Sqrt[x]*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b
^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((24*I)*b*Sqrt[x]*PolyLog[3, (I*a*E^(
I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3) + (
12*b^3*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2
*(-a^2 + b^2)^(3/2)*d^4) - (24*b*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b
- Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (12*b^3*PolyLog[4, (I*a*
E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^4
) + (24*b*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(
a^2*Sqrt[-a^2 + b^2]*d^4) - (2*b^2*x^(3/2)*Cos[c + d*Sqrt[x]])/(a*(a^2 - b^
2)*d*(b + a*Sin[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
```

```
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4617

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2
] + b*E^(I*(c + d*x)))), x], x] + Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/
(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^3}{(a+b\csc(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^3}{a^2}+\frac{b^2x^3}{a^2(b+a\sin(c+dx))^2}-\frac{2bx^3}{a^2(b+a\sin(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{x^2}{2a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^3}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^3}{(b+a\sin(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{x^2}{2a^2}-\frac{2b^2x^{3/2}\cos\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad -\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^3}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}+\frac{(6b^2)\,\text{Subst}\left(\int \frac{x^2\cos(c+dx)}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}
\end{aligned}$$

$$= -\frac{2ib^2x^{3/2}}{a^2(a^2-b^2)d} + \frac{x^2}{2a^2} - \frac{2b^2x^{3/2}\cos(c+d\sqrt{x})}{a(a^2-b^2)d(b+a\sin(c+d\sqrt{x}))}$$
$$- \frac{(4b^3)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)}$$
$$+ \frac{(8ib)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$- \frac{(8ib)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$+ \frac{(6ib^2)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ib-\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$
$$+ \frac{(6ib^2)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ib+\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$

$$= -\frac{2ib^2x^{3/2}}{a^2(a^2-b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{4ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{2b^2x^{3/2}\cos(c+d\sqrt{x})}{a(a^2-b^2)d(b+a\sin(c+d\sqrt{x}))}$$
$$- \frac{(4ib^3)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$+ \frac{(4ib^3)\operatorname{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$- \frac{(12b^2)\operatorname{Subst}\left(\int x\log\left(1+\frac{ae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{(12b^2)\operatorname{Subst}\left(\int x\log\left(1+\frac{ae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{(12ib)\operatorname{Subst}\left(\int x^2\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+ \frac{(12ib)\operatorname{Subst}\left(\int x^2\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$= -\frac{2ib^2x^{3/2}}{a^2(a^2-b^2)d} + \frac{x^2}{2a^2} + \frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{6b^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{4ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{4ibx^{3/2}\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{12ib^2\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{12ib^2\sqrt{x}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} + \frac{12bx\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{12bx\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{2b^2x^{3/2}\cos(c+d\sqrt{x})}{a(a^2-b^2)d(b+a\sin(c+d\sqrt{x}))}$$
$$+ \frac{(12ib^2)\,\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{(12ib^2)\,\mathrm{Subst}\left(\int \mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{(24b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(24b)\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(6ib^3)\,\mathrm{Subst}\left(\int x^2\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{(6ib^3)\,\mathrm{Subst}\left(\int x^2\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

= Too large to display

Mathematica [A] (warning: unable to verify)

Time = 15.34 (sec) , antiderivative size = 1752, normalized size of antiderivative = 1.12

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \frac{\csc^2\left(c+d\sqrt{x}\right)\left(b+a\sin\left(c+d\sqrt{x}\right)\right)\left(x^2\left(b+a\sin\left(c+d\sqrt{x}\right)\right) - \frac{4ibe^{ic}\left(2be^{ic}x^{3/2}+\frac{e^{-ic}\left(-1+e^{2ic}\right)\left(3ibd^2\sqrt{\left(a^2-b^2\right)e^{2ic}} \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[x/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] (Csc[c + d*Sqrt[x]]^2*(b + a*Sin[c + d*Sqrt[x]])*(x^2*(b + a*Sin[c + d*Sqrt
[x]]) - ((4*I)*b*E^(I*c)*(2*b*E^(I*c)*x^(3/2) + ((-1 + E^((2*I)*c))*((3*I)*
b*d^2*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(
I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (2*I)*a^2*d^3*E^(I*c)*x^(3/
2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((
2*I)*c)])] + I*b^2*d^3*E^(I*c)*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/
(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (3*I)*b*d^2*Sqrt[(a^2 - b^
2)*E^((2*I)*c)]*x*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(
a^2 - b^2)*E^((2*I)*c)])] + (2*I)*a^2*d^3*E^(I*c)*x^(3/2)*Log[1 + (a*E^(I*(
2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - I*b^2*d
^3*E^(I*c)*x^(3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[
(a^2 - b^2)*E^((2*I)*c)])] + 3*d*(2*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2
*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*Sqrt[x]*PolyLog[2, (I*a*E^(I*(2
*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 3*d*(2*b
*Sqrt[(a^2 - b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I*c)*Sqrt[x] - b^2*d*E^(I*c)*Sq
rt[x])*Sqrt[x]*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt
[(a^2 - b^2)*E^((2*I)*c)]))] + (6*I)*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*PolyLo
g[3, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)
*c)])] - (12*I)*a^2*d*E^(I*c)*Sqrt[x]*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x]
)))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (6*I)*b^2*d*E^(I*c)*Sq
rt[x]*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b
^2)*E^((2*I)*c)])] + (6*I)*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*PolyLog[3, -((a*
E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] +
(12*I)*a^2*d*E^(I*c)*Sqrt[x]*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*
E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (6*I)*b^2*d*E^(I*c)*Sqrt[x]*Po
lyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((
```

```
2*I)*c)]))] + 12*a^2*E^(I*c)*PolyLog[4, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^
(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 6*b^2*E^(I*c)*PolyLog[4, (I*a*E
^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 12
*a^2*E^(I*c)*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(
a^2 - b^2)*E^((2*I)*c)]))] + 6*b^2*E^(I*c)*PolyLog[4, -((a*E^(I*(2*c + d*Sq
rt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))]))/(d^3*E^(I*c)*Sqr
t[(a^2 - b^2)*E^((2*I)*c)]))*(b + a*Sin[c + d*Sqrt[x]]))/((a^2 - b^2)*d*(-1
 + E^((2*I)*c))) + (4*b^2*x^(3/2)*Csc[c]*(b*Cos[c] + a*Sin[d*Sqrt[x]]))/((a
 - b)*(a + b)*d)))/(2*a^2*(a + b*Csc[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(x/(a+b*csc(c+d*x^(1/2)))^2,x)
[Out] int(x/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(x/(b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2), x)
```

Sympy [F]

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x/(a+b*csc(c+d*x**(1/2)))**2,x)
[Out] Integral(x/(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate(x/(b*csc(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x}{\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x/(a + b/sin(c + d*x^(1/2)))^2,x)
```

```
[Out] int(x/(a + b/sin(c + d*x^(1/2)))^2, x)
```

3.49 $\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x*(a + b*Csc[c + d*Sqrt[x]])^2),x]
[Out] Defer[Int][1/(x*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 61.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*Csc[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x/(a+b*csc(c+d*x^(1/2)))^2,x)
[Out] int(1/x/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*csc(d*sqrt(x) + c)^2 + 2*a*b*x*csc(d*sqrt(x) + c) + a^2*x
), x)
```

Sympy [N/A]

Not integrable

Time = 3.88 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x/(a+b*csc(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x*(a + b*csc(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 13.44 (sec) , antiderivative size = 4411, normalized size of antiderivative = 220.55

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] -((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*b
^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*
a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*c
os(d*sqrt(x))^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(
x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(c)*sin(d*sqrt(x)) + 4*((a^6*b
^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)
^2)*d*sin(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))*s
in(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*si
n(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x)) - (a^6*b^2 -
 a^4*b^4)*d*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 -
a^3*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2*(a^6*b^2*d
*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) + 2*(a^7*b
 - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) + 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*
sin(c) + (a^8 - a^6*b^2)*d)*cos(2*d*sqrt(x) + 2*c) - 2*(2*((a^5*b^3 - a^3*b
^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x))
 + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*si
n(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x)) -
 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(2*
c) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*si
n(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x*integrate(2*(((2*a^5*b - a^3
*b^3)*d*cos(d*sqrt(x) + c)*sin(2*d*sqrt(x) + 2*c) - (2*a^5*b - a^3*b^3)*d*c
os(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + (2*(2*a^4*b^2 - 3*a^2*b^4 + b^6)
```

```
*d*cos(d*sqrt(x))*cos(c) - (2*a^3*b^3 - a*b^5)*d*cos(2*c)*sin(2*d*sqrt(x))
- (2*a^3*b^3 - a*b^5)*d*cos(2*d*sqrt(x))*sin(2*c) - 2*(2*a^4*b^2 - 3*a^2*b^
4 + b^6)*d*sin(d*sqrt(x))*sin(c))*cos(d*sqrt(x) + c) + ((2*a^3*b^3 - a*b^5)
*d*cos(2*d*sqrt(x))*cos(2*c) + 2*(2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*cos(c)*sin
(d*sqrt(x)) - (2*a^3*b^3 - a*b^5)*d*sin(2*d*sqrt(x))*sin(2*c) + 2*(2*a^4*b^
2 - 3*a^2*b^4 + b^6)*d*cos(d*sqrt(x))*sin(c) + (2*a^5*b - 3*a^3*b^3 + a*b^5
)*d)*sin(d*sqrt(x) + c))*x + (a^3*b^3*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x)
+ c) - a^2*b^4*cos(2*c)*sin(2*d*sqrt(x)) - a^2*b^4*cos(2*d*sqrt(x))*sin(2*c
) + 2*(a^3*b^3 - a*b^5)*cos(d*sqrt(x))*cos(c) - 2*(a^3*b^3 - a*b^5)*sin(d*s
qrt(x))*sin(c) - (a*b^5*cos(2*d*sqrt(x))*cos(2*c) - a*b^5*sin(2*d*sqrt(x))*
sin(2*c) + a^3*b^3 - a*b^5 + 2*(a^2*b^4 - b^6)*cos(c)*sin(d*sqrt(x)) + 2*(a
^2*b^4 - b^6)*cos(d*sqrt(x))*sin(c))*cos(d*sqrt(x) + c) + (a^3*b^3*sin(d*sq
rt(x) + c) + a^4*b^2)*sin(2*d*sqrt(x) + 2*c) - (a*b^5*cos(2*c)*sin(2*d*sqrt
(x)) + a*b^5*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^2*b^4 - b^6)*cos(d*sqrt(x))*c
os(c) + 2*(a^2*b^4 - b^6)*sin(d*sqrt(x))*sin(c))*sin(d*sqrt(x) + c))*sqrt(x
))/((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4
*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 -
2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d
*cos(d*sqrt(x))^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*sqr
t(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(c)*sin(d*sqrt(x)) + 4*((a^6
*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(
c)^2)*d*sin(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))
*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*
sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x)) - (a^6*b^2
 - a^4*b^4)*d*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3
- a^3*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2*(a^6*b^2
*d*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) + 2*(a^7
*b - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) + 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x)
)*sin(c) + (a^8 - a^6*b^2)*d)*cos(2*d*sqrt(x) + 2*c) - 2*(2*((a^5*b^3 - a^3
*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x
)) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*
sin(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x))
 - 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(
2*c) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*
sin(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x^2), x) - (a^6*d*cos(2*d*sq
rt(x) + 2*c)^2 + a^6*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^2*b^4*cos(2*c)^2 + a^2
*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(
c)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(c)^2)*d*cos(d*sqrt(x))^2 + (a^2*b^4*
cos(2*c)^2 + a^2*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(x))^2 + 4*(a^5*b - 2*a^3*b^
3 + a*b^5)*d*cos(c)*sin(d*sqrt(x)) + 4*((a^4*b^2 - 2*a^2*b^4 + b^6)*cos(c)^
2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(c)^2)*d*sin(d*sqrt(x))^2 + 4*(a^5*b - 2
*a^3*b^3 + a*b^5)*d*cos(d*sqrt(x))*sin(c) + (a^6 - 2*a^4*b^2 + a^2*b^4)*d -
 2*(2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3 - a*b^5)*cos(2*c)*sin(c
))*d*cos(d*sqrt(x)) - (a^4*b^2 - a^2*b^4)*d*cos(2*c) - 2*((a^3*b^3 - a*b^5)
*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos
```

```
(2*d*sqrt(x)) - 2*(a^4*b^2*d*cos(2*d*sqrt(x))*cos(2*c) - a^4*b^2*d*sin(2*d*
sqrt(x))*sin(2*c) + 2*(a^5*b - a^3*b^3)*d*cos(c)*sin(d*sqrt(x)) + 2*(a^5*b
- a^3*b^3)*d*cos(d*sqrt(x))*sin(c) + (a^6 - a^4*b^2)*d)*cos(2*d*sqrt(x) + 2
*c) - 2*(2*((a^3*b^3 - a*b^5)*cos(2*c)*cos(c) + (a^3*b^3 - a*b^5)*sin(2*c)*
sin(c))*d*cos(d*sqrt(x)) + 2*((a^3*b^3 - a*b^5)*cos(c)*sin(2*c) - (a^3*b^3
- a*b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x)) + (a^4*b^2 - a^2*b^4)*d*sin(2*c)
)*sin(2*d*sqrt(x)) - 2*(a^4*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^4*b^2*d*cos
(2*d*sqrt(x))*sin(2*c) - 2*(a^5*b - a^3*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a
^5*b - a^3*b^3)*d*sin(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x*log(x) +
 4*(a^3*b^3*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) - a^2*b^4*cos(2*c)*si
n(2*d*sqrt(x)) - a^2*b^4*cos(2*d*sqrt(x))*sin(2*c) + 2*(a^3*b^3 - a*b^5)*co
s(d*sqrt(x))*cos(c) - 2*(a^3*b^3 - a*b^5)*sin(d*sqrt(x))*sin(c) - (a*b^5*co
s(2*d*sqrt(x))*cos(2*c) - a*b^5*sin(2*d*sqrt(x))*sin(2*c) + a^3*b^3 - a*b^5
 + 2*(a^2*b^4 - b^6)*cos(c)*sin(d*sqrt(x)) + 2*(a^2*b^4 - b^6)*cos(d*sqrt(x
))*sin(c))*cos(d*sqrt(x) + c) + (a^3*b^3*sin(d*sqrt(x) + c) + a^4*b^2)*sin(
2*d*sqrt(x) + 2*c) - (a*b^5*cos(2*c)*sin(2*d*sqrt(x)) + a*b^5*cos(2*d*sqrt(
x))*sin(2*c) - 2*(a^2*b^4 - b^6)*cos(d*sqrt(x))*cos(c) + 2*(a^2*b^4 - b^6)*
sin(d*sqrt(x))*sin(c))*sin(d*sqrt(x) + c))*sqrt(x))/((a^8*d*cos(2*d*sqrt(x)
 + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*
sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c
)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*cos(d*sqrt(x))^2 + (a^4*b
^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(x))^2 + 4*(a^7*b - 2*a^5
*b^3 + a^3*b^5)*d*cos(c)*sin(d*sqrt(x)) + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6
)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*sin(d*sqrt(x))^2 +
 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))*sin(c) + (a^8 - 2*a^6*b^2
 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*
b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x)) - (a^6*b^2 - a^4*b^4)*d*cos(2*c) - 2
*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c)
)*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*d*sqrt(x))*cos(2*
c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) + 2*(a^7*b - a^5*b^3)*d*cos(c)*sin
(d*sqrt(x)) + 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*sin(c) + (a^8 - a^6*b^2)
*d)*cos(2*d*sqrt(x) + 2*c) - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a
^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x)) + 2*((a^5*b^3 - a^3*b^5
)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x)) +
 (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*c)*s
in(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)
*d*cos(d*sqrt(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*sin(d*sqrt(x))*sin(c))*sin
(2*d*sqrt(x) + 2*c))*x)
```

Giac [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*csc(d*sqrt(x) + c) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 18.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(1/(x*(a + b/sin(c + d*x^(1/2)))^2),x)
[Out] int(1/(x*(a + b/sin(c + d*x^(1/2)))^2), x)
```

3.50 $\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 59.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Integrate[1/(x^2*(a + b*Csc[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^2*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

```
[In] int(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
[Out] int(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{x^2 \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2 x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*csc(d*sqrt(x) + c)^2 + 2*a*b*x^2*csc(d*sqrt(x) + c) + a
^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 8.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*csc(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**2*(a + b*csc(c + d*sqrt(x)))**2), x)
```

Maxima [N/A]

Not integrable

Time = 18.93 (sec) , antiderivative size = 4411, normalized size of antiderivative = 220.55

$$\int \frac{1}{x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] -((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*b
^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*
a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*c
os(d*sqrt(x))^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(
x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(c)*sin(d*sqrt(x)) + 4*((a^6*b
^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)
^2)*d*sin(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))*s
in(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*si
n(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x)) - (a^6*b^2 -
 a^4*b^4)*d*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 -
a^3*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2*(a^6*b^2*d
*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) + 2*(a^7*b
 - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) + 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*
sin(c) + (a^8 - a^6*b^2)*d)*cos(2*d*sqrt(x) + 2*c) - 2*(2*((a^5*b^3 - a^3*b
^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x))
 + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*si
n(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x)) -
 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(2*
c) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*si
n(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x^2*integrate(2*(((2*a^5*b - a
^3*b^3)*d*cos(d*sqrt(x) + c)*sin(2*d*sqrt(x) + 2*c) - (2*a^5*b - a^3*b^3)*d
*cos(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + (2*(2*a^4*b^2 - 3*a^2*b^4 + b^
```

```
6)*d*cos(d*sqrt(x))*cos(c) - (2*a^3*b^3 - a*b^5)*d*cos(2*c)*sin(2*d*sqrt(x)
) - (2*a^3*b^3 - a*b^5)*d*cos(2*d*sqrt(x))*sin(2*c) - 2*(2*a^4*b^2 - 3*a^2*
b^4 + b^6)*d*sin(d*sqrt(x))*sin(c))*cos(d*sqrt(x) + c) + ((2*a^3*b^3 - a*b^
5)*d*cos(2*d*sqrt(x))*cos(2*c) + 2*(2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*cos(c)*s
in(d*sqrt(x)) - (2*a^3*b^3 - a*b^5)*d*sin(2*d*sqrt(x))*sin(2*c) + 2*(2*a^4*
b^2 - 3*a^2*b^4 + b^6)*d*cos(d*sqrt(x))*sin(c) + (2*a^5*b - 3*a^3*b^3 + a*b
^5)*d)*sin(d*sqrt(x) + c))*x + 3*(a^3*b^3*cos(2*d*sqrt(x) + 2*c)*cos(d*sqrt
(x) + c) - a^2*b^4*cos(2*c)*sin(2*d*sqrt(x)) - a^2*b^4*cos(2*d*sqrt(x))*sin
(2*c) + 2*(a^3*b^3 - a*b^5)*cos(d*sqrt(x))*cos(c) - 2*(a^3*b^3 - a*b^5)*sin
(d*sqrt(x))*sin(c) - (a*b^5*cos(2*d*sqrt(x))*cos(2*c) - a*b^5*sin(2*d*sqrt(
x))*sin(2*c) + a^3*b^3 - a*b^5 + 2*(a^2*b^4 - b^6)*cos(c)*sin(d*sqrt(x)) +
2*(a^2*b^4 - b^6)*cos(d*sqrt(x))*sin(c))*cos(d*sqrt(x) + c) + (a^3*b^3*sin(
d*sqrt(x) + c) + a^4*b^2)*sin(2*d*sqrt(x) + 2*c) - (a*b^5*cos(2*c)*sin(2*d*
sqrt(x)) + a*b^5*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^2*b^4 - b^6)*cos(d*sqrt(x
))*cos(c) + 2*(a^2*b^4 - b^6)*sin(d*sqrt(x))*sin(c))*sin(d*sqrt(x) + c))*sq
rt(x))/((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a^8*d*sin(2*d*sqrt(x) + 2*c)^2 +
(a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*cos(2*d*sqrt(x))^2 + 4*((a^6*b^
2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^
2)*d*cos(d*sqrt(x))^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*sin(2*d
*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(c)*sin(d*sqrt(x)) + 4*(
(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*
sin(c)^2)*d*sin(d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt
(x))*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d - 2*(2*((a^5*b^3 - a^3*b^5)*cos
(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*cos(d*sqrt(x)) - (a^6
*b^2 - a^4*b^4)*d*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*
b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*sin(d*sqrt(x)))*cos(2*d*sqrt(x)) - 2*(a^6
*b^2*d*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*d*sin(2*d*sqrt(x))*sin(2*c) + 2*
(a^7*b - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) + 2*(a^7*b - a^5*b^3)*d*cos(d*sqr
t(x))*sin(c) + (a^8 - a^6*b^2)*d)*cos(2*d*sqrt(x) + 2*c) - 2*(2*((a^5*b^3 -
 a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*cos(d*sq
rt(x)) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2
*c)*sin(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a^4*b^4)*d*sin(2*c))*sin(2*d*sqrt
(x)) - 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x)) + a^6*b^2*d*cos(2*d*sqrt(x))*
sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*cos(c) + 2*(a^7*b - a^5*b^3
)*d*sin(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x) + 2*c))*x^3), x) + ((a^6 - a^4*b
^2)*d*cos(2*d*sqrt(x) + 2*c)^2 + (a^4*b^2 - a^2*b^4)*d*cos(2*d*sqrt(x))*cos
(2*c) + (a^6 - a^4*b^2)*d*sin(2*d*sqrt(x) + 2*c)^2 + 2*(a^5*b - 2*a^3*b^3 +
 a*b^5)*d*cos(c)*sin(d*sqrt(x)) - (a^4*b^2 - a^2*b^4)*d*sin(2*d*sqrt(x))*si
n(2*c) + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*sqrt(x))*sin(c) + (a^6 - 2*a
^4*b^2 + a^2*b^4)*d - ((a^4*b^2 - a^2*b^4)*d*cos(2*d*sqrt(x))*cos(2*c) + 2*
(a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(c)*sin(d*sqrt(x)) - (a^4*b^2 - a^2*b^4)*d
*sin(2*d*sqrt(x))*sin(2*c) + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*sqrt(x))
*sin(c) + 2*(a^5*b - a^3*b^3)*d*sin(d*sqrt(x) + c) + (2*a^6 - 3*a^4*b^2 + a
^2*b^4)*d)*cos(2*d*sqrt(x) + 2*c) + 2*(2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(
d*sqrt(x))*cos(c) - (a^3*b^3 - a*b^5)*d*cos(2*c)*sin(2*d*sqrt(x)) - (a^3*b^
```

```
3 - a*b^5)*d*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*si
n(d*sqrt(x))*sin(c))*cos(d*sqrt(x) + c) + (2*(a^5*b - 2*a^3*b^3 + a*b^5)*d*
cos(d*sqrt(x))*cos(c) - (a^4*b^2 - a^2*b^4)*d*cos(2*c)*sin(2*d*sqrt(x)) - (
a^4*b^2 - a^2*b^4)*d*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^5*b - 2*a^3*b^3 + a*b
^5)*d*sin(d*sqrt(x))*sin(c) + 2*(a^5*b - a^3*b^3)*d*cos(d*sqrt(x) + c))*sin
(2*d*sqrt(x) + 2*c) + 2*((a^3*b^3 - a*b^5)*d*cos(2*d*sqrt(x))*cos(2*c) + 2*
(a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(c)*sin(d*sqrt(x)) - (a^3*b^3 - a*b^5)*d*s
in(2*d*sqrt(x))*sin(2*c) + 2*(a^4*b^2 - 2*a^2*b^4 + b^6)*d*cos(d*sqrt(x))*s
in(c) + (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*sqrt(x) + c))*x + 4*(a^3*b^3*c
os(2*d*sqrt(x) + 2*c)*cos(d*sqrt(x) + c) - a^2*b^4*cos(2*c)*sin(2*d*sqrt(x)
) - a^2*b^4*cos(2*d*sqrt(x))*sin(2*c) + 2*(a^3*b^3 - a*b^5)*cos(d*sqrt(x))*
cos(c) - 2*(a^3*b^3 - a*b^5)*sin(d*sqrt(x))*sin(c) - (a*b^5*cos(2*d*sqrt(x)
)*cos(2*c) - a*b^5*sin(2*d*sqrt(x))*sin(2*c) + a^3*b^3 - a*b^5 + 2*(a^2*b^4
 - b^6)*cos(c)*sin(d*sqrt(x)) + 2*(a^2*b^4 - b^6)*cos(d*sqrt(x))*sin(c))*co
s(d*sqrt(x) + c) + (a^3*b^3*sin(d*sqrt(x) + c) + a^4*b^2)*sin(2*d*sqrt(x) +
 2*c) - (a*b^5*cos(2*c)*sin(2*d*sqrt(x)) + a*b^5*cos(2*d*sqrt(x))*sin(2*c)
- 2*(a^2*b^4 - b^6)*cos(d*sqrt(x))*cos(c) + 2*(a^2*b^4 - b^6)*sin(d*sqrt(x)
)*sin(c))*sin(d*sqrt(x) + c))*sqrt(x))/((a^8*d*cos(2*d*sqrt(x) + 2*c)^2 + a
^8*d*sin(2*d*sqrt(x) + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d
*cos(2*d*sqrt(x))^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^
2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*cos(d*sqrt(x))^2 + (a^4*b^4*cos(2*c)^2
 + a^4*b^4*sin(2*c)^2)*d*sin(2*d*sqrt(x))^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^
5)*d*cos(c)*sin(d*sqrt(x)) + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 +
(a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*sin(d*sqrt(x))^2 + 4*(a^7*b - 2
*a^5*b^3 + a^3*b^5)*d*cos(d*sqrt(x))*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d
 - 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)
*sin(c))*d*cos(d*sqrt(x)) - (a^6*b^2 - a^4*b^4)*d*cos(2*c) - 2*((a^5*b^3 -
a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*sin(d*sqr
t(x)))*cos(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*d*sqrt(x))*cos(2*c) - a^6*b^2*
d*sin(2*d*sqrt(x))*sin(2*c) + 2*(a^7*b - a^5*b^3)*d*cos(c)*sin(d*sqrt(x)) +
 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt(x))*sin(c) + (a^8 - a^6*b^2)*d)*cos(2*d*s
qrt(x) + 2*c) - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*
b^5)*sin(2*c)*sin(c))*d*cos(d*sqrt(x)) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(
2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*sin(d*sqrt(x)) + (a^6*b^2 - a
^4*b^4)*d*sin(2*c))*sin(2*d*sqrt(x)) - 2*(a^6*b^2*d*cos(2*c)*sin(2*d*sqrt(x
)) + a^6*b^2*d*cos(2*d*sqrt(x))*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*cos(d*sqrt
(x))*cos(c) + 2*(a^7*b - a^5*b^3)*d*sin(d*sqrt(x))*sin(c))*sin(2*d*sqrt(x)
+ 2*c))*x^2)
```

Giac [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2} \, dx = \int \frac{1}{\left(b\csc\left(d\sqrt{x} + c\right) + a\right)^2 x^2} \, dx$$

```
[In] integrate(1/x^2/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*csc(d*sqrt(x) + c) + a)^2*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 17.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{x^2 \left(a + b\csc\left(c + d\sqrt{x}\right)\right)^2} \, dx = \int \frac{1}{x^2 \left(a + \frac{b}{\sin\left(c + d\sqrt{x}\right)}\right)^2} \, dx$$

```
[In] int(1/(x^2*(a + b/sin(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^2*(a + b/sin(c + d*x^(1/2)))^2), x)
```

3.51 $\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 258

$$\begin{aligned}\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx &= \frac{2}{5}ax^{5/2}-\frac{4bx^2\operatorname{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}\\ &+\frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\ &-\frac{24bx\operatorname{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{24bx\operatorname{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\ &-\frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}\\ &+\frac{48b\operatorname{PolyLog}\left(5,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{48b\operatorname{PolyLog}\left(5,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}\end{aligned}$$

```
[Out] 2/5*a*x^(5/2)-4*b*x^2*arctanh(exp(I*(c+d*x^(1/2))))/d+8*I*b*x^(3/2)*polylog
(2,-exp(I*(c+d*x^(1/2))))/d^2-8*I*b*x^(3/2)*polylog(2,exp(I*(c+d*x^(1/2))))
/d^2-24*b*x*polylog(3,-exp(I*(c+d*x^(1/2))))/d^3+24*b*x*polylog(3,exp(I*(c+
d*x^(1/2))))/d^3+48*b*polylog(5,-exp(I*(c+d*x^(1/2))))/d^5-48*b*polylog(5,e
xp(I*(c+d*x^(1/2))))/d^5-48*I*b*polylog(4,-exp(I*(c+d*x^(1/2))))*x^(1/2)/d^
4+48*I*b*polylog(4,exp(I*(c+d*x^(1/2))))*x^(1/2)/d^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {14, 4290, 4268, 2611, 6744, 2320, 6724}

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{5}ax^{5/2}-\frac{4bx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}+\frac{48b\,\text{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{48b\,\text{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5}-\frac{48ib\sqrt{x}\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4}+\frac{48ib\sqrt{x}\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4}-\frac{24bx\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{24bx\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{8ibx^{3/2}\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{8ibx^{3/2}\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}$$

```
[In] Int[x^(3/2)*(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (2*a*x^(5/2))/5 - (4*b*x^2*ArcTanh[E^(I*(c + d*Sqrt[x]))])/d + ((8*I)*b*x^(
3/2)*PolyLog[2, -E^(I*(c + d*Sqrt[x]))])/d^2 - ((8*I)*b*x^(3/2)*PolyLog[2,
E^(I*(c + d*Sqrt[x]))])/d^2 - (24*b*x*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/d
^3 + (24*b*x*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3 - ((48*I)*b*Sqrt[x]*Pol
yLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((48*I)*b*Sqrt[x]*PolyLog[4, E^(I*(c
 + d*Sqrt[x]))])/d^4 + (48*b*PolyLog[5, -E^(I*(c + d*Sqrt[x]))])/d^5 - (48*
b*PolyLog[5, E^(I*(c + d*Sqrt[x]))])/d^5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(ax^{3/2}+bx^{3/2}\csc\left(c+d\sqrt{x}\right)\right)\,dx\\
&= \frac{2}{5}ax^{5/2}+b\int x^{3/2}\csc\left(c+d\sqrt{x}\right)\,dx\\
&= \frac{2}{5}ax^{5/2}+(2b)\text{Subst}\left(\int x^4\csc(c+dx)\,dx,x,\sqrt{x}\right)\\
&= \frac{2}{5}ax^{5/2}-\frac{4bx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{(8b)\text{Subst}\left(\int x^3\log\left(1-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d}\\
&\quad+\frac{(8b)\text{Subst}\left(\int x^3\log\left(1+e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d}
\end{aligned}$$

$$= \frac{2}{5}ax^{5/2} - \frac{4bx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(24ib)\text{Subst}\left(\int x^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2} + \frac{(24ib)\text{Subst}\left(\int x^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= \frac{2}{5}ax^{5/2} - \frac{4bx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{24bx\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24bx\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{(48b)\text{Subst}\left(\int x\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3} - \frac{(48b)\text{Subst}\left(\int x\operatorname{PolyLog}\left(3,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= \frac{2}{5}ax^{5/2} - \frac{4bx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ibx^{3/2}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{24bx\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24bx\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{48ib\sqrt{x}\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(48ib)\text{Subst}\left(\int\operatorname{PolyLog}\left(4,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(48ib)\text{Subst}\left(\int\operatorname{PolyLog}\left(4,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$= \frac{2}{5}ax^{5/2} - \frac{4bx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{24bx\,\text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24bx\,\text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(48b)\text{Subst}\left(\int \frac{\text{PolyLog}(4,-x)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{(48b)\text{Subst}\left(\int \frac{\text{PolyLog}(4,x)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^5}$$

$$= \frac{2}{5}ax^{5/2} - \frac{4bx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ibx^{3/2}\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{24bx\,\text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{24bx\,\text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4, -e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{48ib\sqrt{x}\,\text{PolyLog}\left(4, e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{48b\,\text{PolyLog}\left(5, -e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{48b\,\text{PolyLog}\left(5, e^{i(c+d\sqrt{x})}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.11

$$\int x^{3/2}(a + b\csc(c+d\sqrt{x}))\,dx = \frac{2\left(ad^5x^{5/2} + 5bd^4x^2\log\left(1 - e^{i(c+d\sqrt{x})}\right) - 5bd^4x^2\log\left(1 + e^{i(c+d\sqrt{x})}\right) + 20ibd^3x^{3/2}\,\text{Po}\right.}{}$$

```
[In] Integrate[x^(3/2)*(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(a*d^5*x^(5/2) + 5*b*d^4*x^2*Log[1 - E^(I*(c + d*Sqrt[x]))] - 5*b*d^4*x^
2*Log[1 + E^(I*(c + d*Sqrt[x]))] + (20*I)*b*d^3*x^(3/2)*PolyLog[2, -E^(I*(c
 + d*Sqrt[x]))] - (20*I)*b*d^3*x^(3/2)*PolyLog[2, E^(I*(c + d*Sqrt[x]))] -
60*b*d^2*x*PolyLog[3, -E^(I*(c + d*Sqrt[x]))] + 60*b*d^2*x*PolyLog[3, E^(I*
(c + d*Sqrt[x]))] - (120*I)*b*d*Sqrt[x]*PolyLog[4, -E^(I*(c + d*Sqrt[x]))]
+ (120*I)*b*d*Sqrt[x]*PolyLog[4, E^(I*(c + d*Sqrt[x]))] + 120*b*PolyLog[5,
-E^(I*(c + d*Sqrt[x]))] - 120*b*PolyLog[5, E^(I*(c + d*Sqrt[x]))]))/(5*d^5)
```

Maple [F]

$$\int x^{\frac{3}{2}} \left(a + b \csc \left(c + d\sqrt{x}\right)\right) dx$$

```
[In] int(x^(3/2)*(a+b*csc(c+d*x^(1/2))),x)
[Out] int(x^(3/2)*(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int x^{3/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right) \, dx = \int \left(b \csc \left(d\sqrt{x} + c\right) + a\right) x^{\frac{3}{2}} \, dx$$

```
[In] integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(b*x^(3/2)*csc(d*sqrt(x) + c) + a*x^(3/2), x)
```

Sympy [F]

$$\int x^{3/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right) \, dx = \int x^{\frac{3}{2}} \left(a + b \csc \left(c + d\sqrt{x}\right)\right) \, dx$$

```
[In] integrate(x**(3/2)*(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(x**(3/2)*(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(200) = 400$.

Time = 0.30 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.83

$$\int x^{3/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right) \, dx = \frac{2 \left(d\sqrt{x} + c\right)^5 a - 10 \left(d\sqrt{x} + c\right)^4 ac + 20 \left(d\sqrt{x} + c\right)^3 ac^2 - 20 \left(d\sqrt{x} + c\right)^2 ac^3 + 10 \left(d\sqrt{\right.}}{}$$

```
[In] integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
[Out] 1/5*(2*(d*sqrt(x) + c)^5*a - 10*(d*sqrt(x) + c)^4*a*c + 20*(d*sqrt(x) + c)^
3*a*c^2 - 20*(d*sqrt(x) + c)^2*a*c^3 + 10*(d*sqrt(x) + c)*a*c^4 - 10*b*c^4*
log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) + 10*(-I*(d*sqrt(x) + c)^4*b +
```

```
 4*I*(d*sqrt(x) + c)^3*b*c - 6*I*(d*sqrt(x) + c)^2*b*c^2 + 4*I*(d*sqrt(x) +
 c)*b*c^3)*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) + 1) + 10*(-I*(d*
sqrt(x) + c)^4*b + 4*I*(d*sqrt(x) + c)^3*b*c - 6*I*(d*sqrt(x) + c)^2*b*c^2
+ 4*I*(d*sqrt(x) + c)*b*c^3)*arctan2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c
) + 1) + 40*(I*(d*sqrt(x) + c)^3*b - 3*I*(d*sqrt(x) + c)^2*b*c + 3*I*(d*sqr
t(x) + c)*b*c^2 - I*b*c^3)*dilog(-e^(I*d*sqrt(x) + I*c)) + 40*(-I*(d*sqrt(x
) + c)^3*b + 3*I*(d*sqrt(x) + c)^2*b*c - 3*I*(d*sqrt(x) + c)*b*c^2 + I*b*c^
3)*dilog(e^(I*d*sqrt(x) + I*c)) - 5*((d*sqrt(x) + c)^4*b - 4*(d*sqrt(x) + c
)^3*b*c + 6*(d*sqrt(x) + c)^2*b*c^2 - 4*(d*sqrt(x) + c)*b*c^3)*log(cos(d*sq
rt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*cos(d*sqrt(x) + c) + 1) + 5*((d*sqr
t(x) + c)^4*b - 4*(d*sqrt(x) + c)^3*b*c + 6*(d*sqrt(x) + c)^2*b*c^2 - 4*(d*
sqrt(x) + c)*b*c^3)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos
(d*sqrt(x) + c) + 1) + 240*b*polylog(5, -e^(I*d*sqrt(x) + I*c)) - 240*b*pol
ylog(5, e^(I*d*sqrt(x) + I*c)) + 240*(-I*(d*sqrt(x) + c)*b + I*b*c)*polylog
(4, -e^(I*d*sqrt(x) + I*c)) + 240*(I*(d*sqrt(x) + c)*b - I*b*c)*polylog(4,
e^(I*d*sqrt(x) + I*c)) - 120*((d*sqrt(x) + c)^2*b - 2*(d*sqrt(x) + c)*b*c +
 b*c^2)*polylog(3, -e^(I*d*sqrt(x) + I*c)) + 120*((d*sqrt(x) + c)^2*b - 2*(
d*sqrt(x) + c)*b*c + b*c^2)*polylog(3, e^(I*d*sqrt(x) + I*c)))/d^5
```

Giac [F]

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int x^{3/2}\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^(3/2)*(a + b/sin(c + d*x^(1/2))),x)
[Out] int(x^(3/2)*(a + b/sin(c + d*x^(1/2))), x)
```

3.52 $\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{3}ax^{3/2}-\frac{4bx\text{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}+\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{4b\,\text{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{4b\,\text{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}$$

```
[Out] 2/3*a*x^(3/2)-4*b*x*arctanh(exp(I*(c+d*x^(1/2))))/d-4*b*polylog(3,-exp(I*(c
+d*x^(1/2))))/d^3+4*b*polylog(3,exp(I*(c+d*x^(1/2))))/d^3+4*I*b*polylog(2,-
exp(I*(c+d*x^(1/2))))*x^(1/2)/d^2-4*I*b*polylog(2,exp(I*(c+d*x^(1/2))))*x^(
1/2)/d^2
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used

= {14, 4290, 4268, 2611, 2320, 6724}

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\frac{2}{3}ax^{3/2}-\frac{4bx\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{4b\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{4b\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{4ib\sqrt{x}\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}$$

```
[In] Int[Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (2*a*x^(3/2))/3 - (4*b*x*ArcTanh[E^(I*(c + d*Sqrt[x]))])/d + ((4*I)*b*Sqrt[
x]*PolyLog[2, -E^(I*(c + d*Sqrt[x]))])/d^2 - ((4*I)*b*Sqrt[x]*PolyLog[2, E^
(I*(c + d*Sqrt[x]))])/d^2 - (4*b*PolyLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3 +
(4*b*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a\sqrt{x} + b\sqrt{x}\csc\left(c + d\sqrt{x}\right)\right) dx \\
&= \frac{2}{3}ax^{3/2} + b\int \sqrt{x}\csc\left(c + d\sqrt{x}\right) dx \\
&= \frac{2}{3}ax^{3/2} + (2b)\text{Subst}\left(\int x^2\csc(c + dx)\,dx, x, \sqrt{x}\right) \\
&= \frac{2}{3}ax^{3/2} - \frac{4bx\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{(4b)\text{Subst}\left(\int x\log\left(1 - e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d} \\
&\quad + \frac{(4b)\text{Subst}\left(\int x\log\left(1 + e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d} \\
&= \frac{2}{3}ax^{3/2} - \frac{4bx\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(4ib)\text{Subst}\left(\int \text{PolyLog}\left(2, -e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^2} \\
&\quad + \frac{(4ib)\text{Subst}\left(\int \text{PolyLog}\left(2, e^{i(c+dx)}\right) dx, x, \sqrt{x}\right)}{d^2} \\
&= \frac{2}{3}ax^{3/2} - \frac{4bx\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} \\
&\quad - \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(4b)\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^3} \\
&\quad + \frac{(4b)\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx, x, e^{i(c+d\sqrt{x})}\right)}{d^3}
\end{aligned}
$$

$$= \frac{2}{3}ax^{3/2} - \frac{4bx\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} + \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, -e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{4ib\sqrt{x}\,\text{PolyLog}\left(2, e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{4b\,\text{PolyLog}\left(3, -e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{4b\,\text{PolyLog}\left(3, e^{i(c+d\sqrt{x})}\right)}{d^3}$$

Mathematica [A] (verified)

Time = 4.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.33

$$\int \sqrt{x}(a + b\csc(c + d\sqrt{x}))\, dx = \frac{2(ad^3x^{3/2} - 6bd^2x\text{arctanh}(\cos(c + d\sqrt{x}) + i\sin(c + d\sqrt{x})) + 6ibd\sqrt{x}\,\text{PolyLog}(2, -\cos(c + d\sqrt{x}) - i\sin$$

```
[In] Integrate[Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(a*d^3*x^(3/2) - 6*b*d^2*x*ArcTanh[Cos[c + d*Sqrt[x]] + I*Sin[c + d*Sqrt
[x]]] + (6*I)*b*d*Sqrt[x]*PolyLog[2, -Cos[c + d*Sqrt[x]] - I*Sin[c + d*Sqrt
[x]]] - (6*I)*b*d*Sqrt[x]*PolyLog[2, Cos[c + d*Sqrt[x]] + I*Sin[c + d*Sqrt[
x]]] - 6*b*PolyLog[3, -Cos[c + d*Sqrt[x]] - I*Sin[c + d*Sqrt[x]]] + 6*b*Pol
yLog[3, Cos[c + d*Sqrt[x]] + I*Sin[c + d*Sqrt[x]]]))/(3*d^3)
```

Maple [F]

$$\int (a + b\csc(c + d\sqrt{x}))\sqrt{x}dx$$

```
[In] int((a+b*csc(c+d*x^(1/2)))*x^(1/2),x)
```

```
[Out] int((a+b*csc(c+d*x^(1/2)))*x^(1/2),x)
```

Fricas [F]

$$\int \sqrt{x}(a + b\csc(c + d\sqrt{x}))\, dx = \int (b\csc(d\sqrt{x} + c) + a)\sqrt{x}\, dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))*x^(1/2),x, algorithm="fricas")
```

```
[Out] integral(b*sqrt(x)*csc(d*sqrt(x) + c) + a*sqrt(x), x)
```

Sympy [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))*x**(1/2),x)
[Out] Integral(sqrt(x)*(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(110) = 220.

Time = 0.27 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.57

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx$$
$$=\frac{2\left(d\sqrt{x}+c\right)^{3}a-6\left(d\sqrt{x}+c\right)^{2}ac+6\left(d\sqrt{x}+c\right)ac^{2}-6\,bc^{2}\log\left(\cot\left(d\sqrt{x}+c\right)+\csc\left(d\sqrt{x}+c\right)\right)+6\left(-i\,(\right.}{}$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))*x^(1/2),x, algorithm="maxima")
[Out] 1/3*(2*(d*sqrt(x) + c)^3*a - 6*(d*sqrt(x) + c)^2*a*c + 6*(d*sqrt(x) + c)*a*
c^2 - 6*b*c^2*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) + 6*(-I*(d*sqrt(
x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b*c)*arctan2(sin(d*sqrt(x) + c), cos(d*sq
rt(x) + c) + 1) + 6*(-I*(d*sqrt(x) + c)^2*b + 2*I*(d*sqrt(x) + c)*b*c)*arct
an2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c) + 1) + 12*(I*(d*sqrt(x) + c)*b
- I*b*c)*dilog(-e^(I*d*sqrt(x) + I*c)) + 12*(-I*(d*sqrt(x) + c)*b + I*b*c)*
dilog(e^(I*d*sqrt(x) + I*c)) - 3*((d*sqrt(x) + c)^2*b - 2*(d*sqrt(x) + c)*b
*c)*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*cos(d*sqrt(x) + c)
+ 1) + 3*((d*sqrt(x) + c)^2*b - 2*(d*sqrt(x) + c)*b*c)*log(cos(d*sqrt(x) +
c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1) - 12*b*polylog(3, -
e^(I*d*sqrt(x) + I*c)) + 12*b*polylog(3, e^(I*d*sqrt(x) + I*c)))/d^3
```

Giac [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)\,dx=\int \left(b\csc\left(d\sqrt{x}+c\right)+a\right)\sqrt{x}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))*x^(1/2),x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(a+b\csc(c+d\sqrt{x}))\,dx=\int \sqrt{x}\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)\,dx$$

```
[In] int(x^(1/2)*(a + b/sin(c + d*x^(1/2))),x)
[Out] int(x^(1/2)*(a + b/sin(c + d*x^(1/2))), x)
```

3.53 $\int \frac{a+b\csc(c+d\sqrt{x})}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 26

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2a\sqrt{x} - \frac{2b\text{arctanh}\left(\cos\left(c+d\sqrt{x}\right)\right)}{d}$$

```
[Out] -2*b*arctanh(cos(c+d*x^(1/2)))/d+2*a*x^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {14, 4290, 3855}

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2a\sqrt{x} - \frac{2b\text{arctanh}\left(\cos\left(c+d\sqrt{x}\right)\right)}{d}$$

```
[In] Int[(a + b*Csc[c + d*Sqrt[x]])/Sqrt[x],x]
```

```
[Out] 2*a*Sqrt[x] - (2*b*ArcTanh[Cos[c + d*Sqrt[x]]])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{a}{\sqrt{x}} + \frac{b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\right) dx \\
&= 2a\sqrt{x} + b\int \frac{\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx \\
&= 2a\sqrt{x} + (2b)\text{Subst}\left(\int \csc(c+dx)\,dx, x, \sqrt{x}\right) \\
&= 2a\sqrt{x} - \frac{2b\text{arctanh}\left(\cos\left(c+d\sqrt{x}\right)\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(a\left(c+d\sqrt{x}\right)-b\log\left(\cos\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)+b\log\left(\sin\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)\right)}{d}$$

```
[In] Integrate[(a + b*Csc[c + d*Sqrt[x]])/Sqrt[x],x]
[Out] (2*(a*(c + d*Sqrt[x]) - b*Log[Cos[(c + d*Sqrt[x])/2]] + b*Log[Sin[(c + d*Sq
rt[x])/2]]))/d
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$2a\sqrt{x}-\frac{2b\ln\left(\csc\left(c+d\sqrt{x}\right)+\cot\left(c+d\sqrt{x}\right)\right)}{d}$	32
default	$2a\sqrt{x}-\frac{2b\ln\left(\csc\left(c+d\sqrt{x}\right)+\cot\left(c+d\sqrt{x}\right)\right)}{d}$	32
parts	$2a\sqrt{x}-\frac{2b\ln\left(\csc\left(c+d\sqrt{x}\right)+\cot\left(c+d\sqrt{x}\right)\right)}{d}$	32

```
[In] int((a+b*csc(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a*x^(1/2)-2*b/d*ln(csc(c+d*x^(1/2))+cot(c+d*x^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\,ad\sqrt{x}-b\log\left(\frac{1}{2}\,\cos\left(d\sqrt{x}+c\right)+\frac{1}{2}\right)+b\log\left(-\frac{1}{2}\,\cos\left(d\sqrt{x}+c\right)+\frac{1}{2}\right)}{d}$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^(1/2),x, algorithm="fricas")
[Out] (2*a*d*sqrt(x) - b*log(1/2*cos(d*sqrt(x) + c) + 1/2) + b*log(-1/2*cos(d*sqr
t(x) + c) + 1/2))/d
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2a\sqrt{x}+2b\left(\begin{cases}\frac{\sqrt{x}\left(\cot\left(c\right)\csc\left(c\right)+\csc^{2}\left(c\right)\right)}{\cot\left(c\right)+\csc\left(c\right)} & \text{for}\ d=0\\ -\frac{\log\left(\cot\left(c+d\sqrt{x}\right)+\csc\left(c+d\sqrt{x}\right)\right)}{d} & \text{otherwise}\end{cases}\right)$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))/x**(1/2),x)
[Out] 2*a*sqrt(x) + 2*b*Piecewise((sqrt(x)*(cot(c)*csc(c) + csc(c)**2)/(cot(c) +
csc(c)), Eq(d, 0)), (-log(cot(c + d*sqrt(x)) + csc(c + d*sqrt(x)))/d, True)
)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,a\sqrt{x}-\frac{2\,b\log\left(\cot\left(d\sqrt{x}+c\right)+\csc\left(d\sqrt{x}+c\right)\right)}{d}$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")
[Out] 2*a*sqrt(x) - 2*b*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c))/d
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{a+b\csc \left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2\left(\left(d\sqrt{x}+c\right)a+b\log\left(\left|\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)\right|\right)\right)}{d}$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")
[Out] 2*((d*sqrt(x) + c)*a + b*log(abs(tan(1/2*d*sqrt(x) + 1/2*c))))/d
```

Mupad [B] (verification not implemented)

Time = 19.86 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.81

$$\int \frac{a+b\csc \left(c+d\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,a\,\sqrt{x}+\frac{2\,b\,\ln\left(\frac{b\,2\mathrm{i}-b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}}{\sqrt{x}}\right)}{d}-\frac{2\,b\,\ln\left(\frac{b\,2\mathrm{i}+b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i}}{\sqrt{x}}\right)}{d}$$

```
[In] int((a + b/sin(c + d*x^(1/2)))/x^(1/2),x)
[Out] 2*a*x^(1/2) + (2*b*log((b*2i - b*exp(d*x^(1/2)*1i)*exp(c*1i)*2i)/x^(1/2)))/
d - (2*b*log((b*2i + b*exp(d*x^(1/2)*1i)*exp(c*1i)*2i)/x^(1/2)))/d
```

3.54 $\int \frac{a+b\csc(c+d\sqrt{x})}{x^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=-\frac{2a}{\sqrt{x}}+b\text{Int}\left(\frac{\csc\left(c+d\sqrt{x}\right)}{x^{3/2}},x\right)$$

```
[Out] -2*a/x^(1/2)+b*Unintegrable(csc(c+d*x^(1/2))/x^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx$$

```
[In] Int[(a + b*Csc[c + d*Sqrt[x]])/x^(3/2),x]
```

```
[Out] (-2*a)/Sqrt[x] + b*Defer[Int][Csc[c + d*Sqrt[x]]/x^(3/2), x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^{3/2}}+\frac{b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\right)dx\\&=-\frac{2a}{\sqrt{x}}+b\int \frac{\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 25.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^(3/2),x]
[Out] Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{\frac{3}{2}}}dx$$

```
[In] int((a+b*csc(c+d*x^(1/2)))/x^(3/2),x)
[Out] int((a+b*csc(c+d*x^(1/2)))/x^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^(3/2),x, algorithm="fricas")
[Out] integral((b*sqrt(x)*csc(d*sqrt(x) + c) + a*sqrt(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{3/2}}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))/x**(3/2),x)
[Out] Integral((a + b*csc(c + d*sqrt(x)))/x**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 113, normalized size of antiderivative = 5.65

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^{3/2}}\,dx = \int \frac{b\csc\left(d\sqrt{x} + c\right) + a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^(3/2),x, algorithm="maxima")
```

```
[Out] ((b*integrate(sin(d*sqrt(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c
)^2 + 2*cos(d*sqrt(x) + c) + 1)*x^(3/2)), x) + b*integrate(sin(d*sqrt(x) +
c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1
)*x^(3/2)), x))*sqrt(x) - 2*a)/sqrt(x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^{3/2}}\,dx = \int \frac{b\csc\left(d\sqrt{x} + c\right) + a}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*sqrt(x) + c) + a)/x^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 18.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b\csc\left(c + d\sqrt{x}\right)}{x^{3/2}}\,dx = \int \frac{a + \frac{b}{\sin\left(c+d\sqrt{x}\right)}}{x^{3/2}}\,dx$$

```
[In] int((a + b/sin(c + d*x^(1/2)))/x^(3/2),x)
```

```
[Out] int((a + b/sin(c + d*x^(1/2)))/x^(3/2), x)
```

3.55 $\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=-\frac{2a}{3x^{3/2}}+b\text{Int}\left(\frac{\csc\left(c+d\sqrt{x}\right)}{x^{5/2}},x\right)$$

```
[Out] -2/3*a/x^(3/2)+b*Unintegrable(csc(c+d*x^(1/2))/x^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$$

```
[In] Int[(a + b*Csc[c + d*Sqrt[x]])/x^(5/2),x]
[Out] (-2*a)/(3*x^(3/2)) + b*Defer[Int][Csc[c + d*Sqrt[x]]/x^(5/2), x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\int \left(\frac{a}{x^{5/2}}+\frac{b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\right)\,dx\\&=-\frac{2a}{3x^{3/2}}+b\int \frac{\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx\end{aligned}$$

Mathematica [N/A]

Not integrable

Time = 28.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^(5/2),x]
[Out] Integrate[(a + b*Csc[c + d*Sqrt[x]])/x^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{\frac{5}{2}}}dx$$

```
[In] int((a+b*csc(c+d*x^(1/2)))/x^(5/2),x)
[Out] int((a+b*csc(c+d*x^(1/2)))/x^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^(5/2),x, algorithm="fricas")
[Out] integral((b*sqrt(x)*csc(d*sqrt(x) + c) + a*sqrt(x))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 6.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))/x**(5/2),x)
[Out] Integral((a + b*csc(c + d*sqrt(x)))/x**(5/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 115, normalized size of antiderivative = 5.75

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(b*integrate(sin(d*sqrt(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(
x) + c)^2 + 2*cos(d*sqrt(x) + c) + 1)*x^(5/2)), x) + b*integrate(sin(d*sqrt
(x) + c)/((cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) +
c) + 1)*x^(5/2)), x))*x^(3/2) - 2*a)/x^(3/2)
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{b\csc\left(d\sqrt{x}+c\right)+a}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))/x^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*sqrt(x) + c) + a)/x^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 18.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\csc\left(c+d\sqrt{x}\right)}{x^{5/2}}\,dx=\int \frac{a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}}{x^{5/2}}\,dx$$

```
[In] int((a + b/sin(c + d*x^(1/2)))/x^(5/2),x)
```

```
[Out] int((a + b/sin(c + d*x^(1/2)))/x^(5/2), x)
```

3.56 $\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 421

$$\begin{aligned}\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx &= -\frac{2ib^2x^2}{d}+\frac{2}{5}a^2x^{5/2}\\
&-\frac{8abx^2\operatorname{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}-\frac{2b^2x^2\cot\left(c+d\sqrt{x}\right)}{d}\\
&+\frac{8b^2x^{3/2}\log\left(1-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2}+\frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\
&-\frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{12ib^2x\operatorname{PolyLog}\left(2,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\
&-\frac{48abx\operatorname{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{48abx\operatorname{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\
&+\frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}\\
&+\frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}+\frac{6ib^2\operatorname{PolyLog}\left(4,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^5}\\
&+\frac{96ab\operatorname{PolyLog}\left(5,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{96ab\operatorname{PolyLog}\left(5,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}\end{aligned}$$

```
[Out] 6*I*b^2*polylog(4,exp(2*I*(c+d*x^(1/2))))/d^5+2/5*a^2*x^(5/2)-8*a*b*x^2*arc
tanh(exp(I*(c+d*x^(1/2))))/d-2*b^2*x^2*cot(c+d*x^(1/2))/d+8*b^2*x^(3/2)*ln(
1-exp(2*I*(c+d*x^(1/2))))/d^2-16*I*a*b*x^(3/2)*polylog(2,exp(I*(c+d*x^(1/2)
)))/d^2-12*I*b^2*x*polylog(2,exp(2*I*(c+d*x^(1/2))))/d^3+96*I*a*b*polylog(4
,exp(I*(c+d*x^(1/2))))*x^(1/2)/d^4-48*a*b*x*polylog(3,-exp(I*(c+d*x^(1/2)))
)/d^3+48*a*b*x*polylog(3,exp(I*(c+d*x^(1/2))))/d^3+16*I*a*b*x^(3/2)*polylog
```

```
(2,-exp(I*(c+d*x^(1/2))))/d^2+96*a*b*polylog(5,-exp(I*(c+d*x^(1/2))))/d^5-9
6*a*b*polylog(5,exp(I*(c+d*x^(1/2))))/d^5+12*b^2*polylog(3,exp(2*I*(c+d*x^(
1/2))))*x^(1/2)/d^4-96*I*a*b*polylog(4,-exp(I*(c+d*x^(1/2))))*x^(1/2)/d^4-2
*I*b^2*x^2/d
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {4290, 4275, 4268, 2611, 6744, 2320, 6724, 4269, 3798, 2221}

$$\begin{aligned}\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx&=\frac{2}{5}a^2x^{5/2}\\&-\frac{8abx^2\operatorname{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d}+\frac{96ab\operatorname{PolyLog}\left(5,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}\\&-\frac{96ab\operatorname{PolyLog}\left(5,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^5}-\frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}\\&+\frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{48abx\operatorname{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\&+\frac{48abx\operatorname{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}+\frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}\\&-\frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2}+\frac{6ib^2\operatorname{PolyLog}\left(4,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^5}\\&+\frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^4}-\frac{12ib^2x\operatorname{PolyLog}\left(2,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3}\\&+\frac{8b^2x^{3/2}\log\left(1-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2}-\frac{2b^2x^2\cot\left(c+d\sqrt{x}\right)}{d}-\frac{2ib^2x^2}{d}\end{aligned}$$

```
[In] Int[x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x^2)/d + (2*a^2*x^(5/2))/5 - (8*a*b*x^2*ArcTanh[E^(I*(c + d*Sqr
t[x]))])/d - (2*b^2*x^2*Cot[c + d*Sqrt[x]])/d + (8*b^2*x^(3/2)*Log[1 - E^((
2*I)*(c + d*Sqrt[x]))])/d^2 + ((16*I)*a*b*x^(3/2)*PolyLog[2, -E^(I*(c + d*S
qrt[x]))])/d^2 - ((16*I)*a*b*x^(3/2)*PolyLog[2, E^(I*(c + d*Sqrt[x]))])/d^2
 - ((12*I)*b^2*x*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (48*a*b*x*Pol
yLog[3, -E^(I*(c + d*Sqrt[x]))])/d^3 + (48*a*b*x*PolyLog[3, E^(I*(c + d*Sqr
t[x]))])/d^3 + (12*b^2*Sqrt[x]*PolyLog[3, E^((2*I)*(c + d*Sqrt[x]))])/d^4 -
 ((96*I)*a*b*Sqrt[x]*PolyLog[4, -E^(I*(c + d*Sqrt[x]))])/d^4 + ((96*I)*a*b*
Sqrt[x]*PolyLog[4, E^(I*(c + d*Sqrt[x]))])/d^4 + ((6*I)*b^2*PolyLog[4, E^((
2*I)*(c + d*Sqrt[x]))])/d^5 + (96*a*b*PolyLog[5, -E^(I*(c + d*Sqrt[x]))])/d
^5 - (96*a*b*PolyLog[5, E^(I*(c + d*Sqrt[x]))])/d^5
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^4(a+b\csc(c+dx))^2\,dx,x,\sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^4+2abx^4\csc(c+dx)+b^2x^4\csc^2(c+dx)\right)\,dx,x,\sqrt{x}\right)\\
&= \frac{2}{5}a^2x^{5/2}+(4ab)\text{Subst}\left(\int x^4\csc(c+dx)\,dx,x,\sqrt{x}\right)\\
&\quad +\left(2b^2\right)\text{Subst}\left(\int x^4\csc^2(c+dx)\,dx,x,\sqrt{x}\right)\\
&= \frac{2}{5}a^2x^{5/2}-\frac{8abx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{2b^2x^2\cot\left(c+d\sqrt{x}\right)}{d}\\
&\quad -\frac{(16ab)\text{Subst}\left(\int x^3\log\left(1-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d}\\
&\quad +\frac{(16ab)\text{Subst}\left(\int x^3\log\left(1+e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d}\\
&\quad +\frac{\left(8b^2\right)\text{Subst}\left(\int x^3\cot(c+dx)\,dx,x,\sqrt{x}\right)}{d}
\end{aligned}$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^2\cot\left(c+d\sqrt{x}\right)}{d} + \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(48iab)\text{Subst}\left(\int x^2\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2} + \frac{(48iab)\text{Subst}\left(\int x^2\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2} - \frac{(16ib^2)\,\text{Subst}\left(\int\frac{e^{2i(c+dx)}x^3}{1-e^{2i(c+dx)}}\,dx,x,\sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^2\cot\left(c+d\sqrt{x}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{48abx\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{48abx\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{(96ab)\text{Subst}\left(\int x\operatorname{PolyLog}\left(3,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3} - \frac{(96ab)\text{Subst}\left(\int x\operatorname{PolyLog}\left(3,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3} - \frac{(24b^2)\,\text{Subst}\left(\int x^2\log\left(1-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^2\cot\left(c+d\sqrt{x}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12ib^2x\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{48abx\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{48abx\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(96iab)\text{Subst}\left(\int \text{PolyLog}\left(4,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} - \frac{(96iab)\text{Subst}\left(\int \text{PolyLog}\left(4,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4} + \frac{(24ib^2)\,\text{Subst}\left(\int x\,\text{PolyLog}\left(2,e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^3}$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^2\cot\left(c+d\sqrt{x}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12ib^2x\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{48abx\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{48abx\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b^2\sqrt{x}\,\text{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{96iab\sqrt{x}\,\text{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{(96ab)\text{Subst}\left(\int \frac{\text{PolyLog}(4,-x)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{(96ab)\text{Subst}\left(\int \frac{\text{PolyLog}(4,x)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{(12b^2)\,\text{Subst}\left(\int \text{PolyLog}\left(3,e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^4}$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^2\cot\left(c+d\sqrt{x}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12ib^2x\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{48abx\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{48abx\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{96ab\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{96ab\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5} + \frac{(6ib^2)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(3,x)}{x}\,dx,x,e^{2i(c+d\sqrt{x})}\right)}{d^5}$$

$$= -\frac{2ib^2x^2}{d} + \frac{2}{5}a^2x^{5/2} - \frac{8abx^2\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x^2\cot\left(c+d\sqrt{x}\right)}{d} + \frac{8b^2x^{3/2}\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{16iabx^{3/2}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{12ib^2x\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{48abx\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{48abx\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{12b^2\sqrt{x}\operatorname{PolyLog}\left(3,e^{2i(c+d\sqrt{x})}\right)}{d^4} - \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,-e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{96iab\sqrt{x}\operatorname{PolyLog}\left(4,e^{i(c+d\sqrt{x})}\right)}{d^4} + \frac{6ib^2\operatorname{PolyLog}\left(4,e^{2i(c+d\sqrt{x})}\right)}{d^5} + \frac{96ab\operatorname{PolyLog}\left(5,-e^{i(c+d\sqrt{x})}\right)}{d^5} - \frac{96ab\operatorname{PolyLog}\left(5,e^{i(c+d\sqrt{x})}\right)}{d^5}$$

Mathematica [A] (verified)

Time = 8.64 (sec) , antiderivative size = 749, normalized size of antiderivative = 1.78

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2a^2x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\sin^2\left(c+d\sqrt{x}\right)}{5\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}$$
$$+\frac{4b\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\left(-\frac{ibd^4x^2}{-1+e^{2ic}}+2bd^3x^{3/2}\log\left(1-e^{-i(c+d\sqrt{x})}\right)+ad^4x^2\log\left(1-e^{-i(c+d\sqrt{x})}\right)+2bd^3x\cdots\right.}{\cdots}$$
$$+\frac{b^2x^2\csc\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\sin^2\left(c+d\sqrt{x}\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}$$
$$+\frac{b^2x^2\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\sin^2\left(c+d\sqrt{x}\right)\sin\left(\frac{d\sqrt{x}}{2}\right)}{d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)^2}$$

```
[In] Integrate[x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*a^2*x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])^2*Sin[c + d*Sqrt[x]]^2)/(5*(b + a
*Sin[c + d*Sqrt[x]])^2) + (4*b*(a + b*Csc[c + d*Sqrt[x]])^2*(((-I)*b*d^4*x^
2)/(-1 + E^((2*I)*c)) + 2*b*d^3*x^(3/2)*Log[1 - E^((-I)*(c + d*Sqrt[x]))] +
 a*d^4*x^2*Log[1 - E^((-I)*(c + d*Sqrt[x]))] + 2*b*d^3*x^(3/2)*Log[1 + E^((
-I)*(c + d*Sqrt[x]))] - a*d^4*x^2*Log[1 + E^((-I)*(c + d*Sqrt[x]))] - (2*I)
*d^2*(-3*b + 2*a*d*Sqrt[x])*x*PolyLog[2, -E^((-I)*(c + d*Sqrt[x]))] + (2*I)
*d^2*(3*b + 2*a*d*Sqrt[x])*x*PolyLog[2, E^((-I)*(c + d*Sqrt[x]))] + 12*b*d*
Sqrt[x]*PolyLog[3, -E^((-I)*(c + d*Sqrt[x]))] - 12*a*d^2*x*PolyLog[3, -E^((
-I)*(c + d*Sqrt[x]))] + 12*b*d*Sqrt[x]*PolyLog[3, E^((-I)*(c + d*Sqrt[x]))]
 + 12*a*d^2*x*PolyLog[3, E^((-I)*(c + d*Sqrt[x]))] - (12*I)*b*PolyLog[4, -E
^((-I)*(c + d*Sqrt[x]))] + (24*I)*a*d*Sqrt[x]*PolyLog[4, -E^((-I)*(c + d*Sq
rt[x]))] - (12*I)*b*PolyLog[4, E^((-I)*(c + d*Sqrt[x]))] - (24*I)*a*d*Sqrt[
x]*PolyLog[4, E^((-I)*(c + d*Sqrt[x]))] + 24*a*PolyLog[5, -E^((-I)*(c + d*S
qrt[x]))] - 24*a*PolyLog[5, E^((-I)*(c + d*Sqrt[x]))])*Sin[c + d*Sqrt[x]]^2
)/(d^5*(b + a*Sin[c + d*Sqrt[x]])^2) + (b^2*x^2*Csc[c/2]*Csc[c/2 + (d*Sqrt[
x])/2]*(a + b*Csc[c + d*Sqrt[x]])^2*Sin[c + d*Sqrt[x]]^2*Sin[(d*Sqrt[x])/2]
)/(d*(b + a*Sin[c + d*Sqrt[x]])^2) + (b^2*x^2*(a + b*Csc[c + d*Sqrt[x]])^2*
Sec[c/2]*Sec[c/2 + (d*Sqrt[x])/2]*Sin[c + d*Sqrt[x]]^2*Sin[(d*Sqrt[x])/2])/
(d*(b + a*Sin[c + d*Sqrt[x]])^2)
```

Maple [F]

$$\int x^{\frac{3}{2}}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}dx$$

[In] int(x^(3/2)*(a+b*csc(c+d*x^(1/2)))^2,x)

[Out] int(x^(3/2)*(a+b*csc(c+d*x^(1/2)))^2,x)

Fricas [F]

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^{2}x^{\frac{3}{2}}\,dx$$

[In] integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(b^2*x^(3/2)*csc(d*sqrt(x) + c)^2 + 2*a*b*x^(3/2)*csc(d*sqrt(x) + c
) + a^2*x^(3/2), x)

Sympy [F]

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}\,dx=\int x^{\frac{3}{2}}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}\,dx$$

[In] integrate(x**(3/2)*(a+b*csc(c+d*x**(1/2)))**2,x)

[Out] Integral(x**(3/2)*(a + b*csc(c + d*sqrt(x)))**2, x)

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2836 vs. $2(334)=668$.

Time = 0.37 (sec) , antiderivative size = 2836, normalized size of antiderivative = 6.74

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}\,dx=\text{Too large to display}$$

[In] integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")

[Out] 2/5*((d*sqrt(x) + c)^5*a^2 - 5*(d*sqrt(x) + c)^4*a^2*c + 10*(d*sqrt(x) + c)
^3*a^2*c^2 - 10*(d*sqrt(x) + c)^2*a^2*c^3 + 5*(d*sqrt(x) + c)*a^2*c^4 - 10*
a*b*c^4*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) - 5*(2*b^2*c^4 - 2*((d
*sqrt(x) + c)^4*a*b + 2*b^2*c^3 - 2*(2*a*b*c + b^2)*(d*sqrt(x) + c)^3 + 6*(
a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^2 - 2*(2*a*b*c^3 + 3*b^2*c^2)*(d*sqrt(x) +

```
 c) - ((d*sqrt(x) + c)^4*a*b + 2*b^2*c^3 - 2*(2*a*b*c + b^2)*(d*sqrt(x) + c
)^3 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c)^2 - 2*(2*a*b*c^3 + 3*b^2*c^2)*(d*
sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-I*(d*sqrt(x) + c)^4*a*b - 2*I*b^2*
c^3 + 2*(2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^3 + 6*(-I*a*b*c^2 - I*b^2*c)*(d
*sqrt(x) + c)^2 + 2*(2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) + c))*sin(2*d*sq
rt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) + 1) + 4*(b^2*
c^3*cos(2*d*sqrt(x) + 2*c) + I*b^2*c^3*sin(2*d*sqrt(x) + 2*c) - b^2*c^3)*ar
ctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) - 1) - 2*((d*sqrt(x) + c)^4*a*
b - 2*(2*a*b*c - b^2)*(d*sqrt(x) + c)^3 + 6*(a*b*c^2 - b^2*c)*(d*sqrt(x) +
c)^2 - 2*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c) - ((d*sqrt(x) + c)^4*a*b -
 2*(2*a*b*c - b^2)*(d*sqrt(x) + c)^3 + 6*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c)^
2 - 2*(2*a*b*c^3 - 3*b^2*c^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-I
*(d*sqrt(x) + c)^4*a*b + 2*(2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^3 + 6*(-I*a*
b*c^2 + I*b^2*c)*(d*sqrt(x) + c)^2 + 2*(2*I*a*b*c^3 - 3*I*b^2*c^2)*(d*sqrt(
x) + c))*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c), -cos(d*sqrt(x)
 + c) + 1) + 2*((d*sqrt(x) + c)^4*b^2 - 4*(d*sqrt(x) + c)^3*b^2*c + 6*(d*sq
rt(x) + c)^2*b^2*c^2 - 4*(d*sqrt(x) + c)*b^2*c^3)*cos(2*d*sqrt(x) + 2*c) +
4*(2*(d*sqrt(x) + c)^3*a*b - 2*a*b*c^3 - 3*b^2*c^2 - 3*(2*a*b*c + b^2)*(d*s
qrt(x) + c)^2 + 6*(a*b*c^2 + b^2*c)*(d*sqrt(x) + c) - (2*(d*sqrt(x) + c)^3*
a*b - 2*a*b*c^3 - 3*b^2*c^2 - 3*(2*a*b*c + b^2)*(d*sqrt(x) + c)^2 + 6*(a*b*
c^2 + b^2*c)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) - (2*I*(d*sqrt(x) + c)
^3*a*b - 2*I*a*b*c^3 - 3*I*b^2*c^2 + 3*(-2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)
^2 + 6*(I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*dilog
(-e^(I*d*sqrt(x) + I*c)) - 4*(2*(d*sqrt(x) + c)^3*a*b - 2*a*b*c^3 + 3*b^2*c
^2 - 3*(2*a*b*c - b^2)*(d*sqrt(x) + c)^2 + 6*(a*b*c^2 - b^2*c)*(d*sqrt(x) +
 c) - (2*(d*sqrt(x) + c)^3*a*b - 2*a*b*c^3 + 3*b^2*c^2 - 3*(2*a*b*c - b^2)*
(d*sqrt(x) + c)^2 + 6*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) +
2*c) + (-2*I*(d*sqrt(x) + c)^3*a*b + 2*I*a*b*c^3 - 3*I*b^2*c^2 + 3*(2*I*a*b
*c - I*b^2)*(d*sqrt(x) + c)^2 + 6*(-I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c))*s
in(2*d*sqrt(x) + 2*c))*dilog(e^(I*d*sqrt(x) + I*c)) + (I*(d*sqrt(x) + c)^4*
a*b + 2*I*b^2*c^3 - 2*(2*I*a*b*c + I*b^2)*(d*sqrt(x) + c)^3 - 6*(-I*a*b*c^2
 - I*b^2*c)*(d*sqrt(x) + c)^2 - 2*(2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) +
c) + (-I*(d*sqrt(x) + c)^4*a*b - 2*I*b^2*c^3 - 2*(-2*I*a*b*c - I*b^2)*(d*sq
rt(x) + c)^3 - 6*(I*a*b*c^2 + I*b^2*c)*(d*sqrt(x) + c)^2 - 2*(-2*I*a*b*c^3
- 3*I*b^2*c^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)^4
*a*b + 2*b^2*c^3 - 2*(2*a*b*c + b^2)*(d*sqrt(x) + c)^3 + 6*(a*b*c^2 + b^2*c
)*(d*sqrt(x) + c)^2 - 2*(2*a*b*c^3 + 3*b^2*c^2)*(d*sqrt(x) + c))*sin(2*d*sq
rt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*cos(d*sqr
t(x) + c) + 1) + (-I*(d*sqrt(x) + c)^4*a*b + 2*I*b^2*c^3 - 2*(-2*I*a*b*c +
I*b^2)*(d*sqrt(x) + c)^3 - 6*(I*a*b*c^2 - I*b^2*c)*(d*sqrt(x) + c)^2 - 2*(-
2*I*a*b*c^3 + 3*I*b^2*c^2)*(d*sqrt(x) + c) + (I*(d*sqrt(x) + c)^4*a*b - 2*I
*b^2*c^3 - 2*(2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)^3 - 6*(-I*a*b*c^2 + I*b^2*
c)*(d*sqrt(x) + c)^2 - 2*(2*I*a*b*c^3 - 3*I*b^2*c^2)*(d*sqrt(x) + c))*cos(2
*d*sqrt(x) + 2*c) - ((d*sqrt(x) + c)^4*a*b - 2*b^2*c^3 - 2*(2*a*b*c - b^2)*
(d*sqrt(x) + c)^3 + 6*(a*b*c^2 - b^2*c)*(d*sqrt(x) + c)^2 - 2*(2*a*b*c^3 -
```

```
3*b^2*c^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^
2 + sin(d*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1) - 48*(-I*a*b*cos(2*d*s
qrt(x) + 2*c) + a*b*sin(2*d*sqrt(x) + 2*c) + I*a*b)*polylog(5, -e^(I*d*sqrt
(x) + I*c)) - 48*(I*a*b*cos(2*d*sqrt(x) + 2*c) - a*b*sin(2*d*sqrt(x) + 2*c)
 - I*a*b)*polylog(5, e^(I*d*sqrt(x) + I*c)) - 24*(2*(d*sqrt(x) + c)*a*b - 2
*a*b*c - b^2 - (2*(d*sqrt(x) + c)*a*b - 2*a*b*c - b^2)*cos(2*d*sqrt(x) + 2*
c) + (-2*I*(d*sqrt(x) + c)*a*b + 2*I*a*b*c + I*b^2)*sin(2*d*sqrt(x) + 2*c))
*polylog(4, -e^(I*d*sqrt(x) + I*c)) + 24*(2*(d*sqrt(x) + c)*a*b - 2*a*b*c +
 b^2 - (2*(d*sqrt(x) + c)*a*b - 2*a*b*c + b^2)*cos(2*d*sqrt(x) + 2*c) - (2*
I*(d*sqrt(x) + c)*a*b - 2*I*a*b*c + I*b^2)*sin(2*d*sqrt(x) + 2*c))*polylog(
4, e^(I*d*sqrt(x) + I*c)) - 24*(-I*(d*sqrt(x) + c)^2*a*b - I*a*b*c^2 - I*b^
2*c + (2*I*a*b*c + I*b^2)*(d*sqrt(x) + c) + (I*(d*sqrt(x) + c)^2*a*b + I*a*
b*c^2 + I*b^2*c + (-2*I*a*b*c - I*b^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2
*c) - ((d*sqrt(x) + c)^2*a*b + a*b*c^2 + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x)
 + c))*sin(2*d*sqrt(x) + 2*c))*polylog(3, -e^(I*d*sqrt(x) + I*c)) - 24*(I*(
d*sqrt(x) + c)^2*a*b + I*a*b*c^2 - I*b^2*c + (-2*I*a*b*c + I*b^2)*(d*sqrt(x
) + c) + (-I*(d*sqrt(x) + c)^2*a*b - I*a*b*c^2 + I*b^2*c + (2*I*a*b*c - I*b
^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)^2*a*b + a*b*
c^2 - b^2*c - (2*a*b*c - b^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*poly
log(3, e^(I*d*sqrt(x) + I*c)) - 2*(-I*(d*sqrt(x) + c)^4*b^2 + 4*I*(d*sqrt(x
) + c)^3*b^2*c - 6*I*(d*sqrt(x) + c)^2*b^2*c^2 + 4*I*(d*sqrt(x) + c)*b^2*c^
3)*sin(2*d*sqrt(x) + 2*c))/(-I*cos(2*d*sqrt(x) + 2*c) + sin(2*d*sqrt(x) + 2
*c) + I))/d^5
```

Giac [F]

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{3}{2}}\,dx$$

```
[In] integrate(x^(3/2)*(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)^2*x^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int x^{3/2}\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2\,dx$$

```
[In] int(x^(3/2)*(a + b/sin(c + d*x^(1/2)))^2,x)
[Out] int(x^(3/2)*(a + b/sin(c + d*x^(1/2)))^2, x)
```

3.57 $\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 241

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx = -\frac{2ib^2x}{d}+\frac{2}{3}a^2x^{3/2}-\frac{8abx\operatorname{arctanh}\left(e^{i\left(c+d\sqrt{x}\right)}\right)}{d} -\frac{2b^2x\cot\left(c+d\sqrt{x}\right)}{d}+\frac{4b^2\sqrt{x}\log\left(1-e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^2} +\frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} -\frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^2} -\frac{2ib^2\operatorname{PolyLog}\left(2,e^{2i\left(c+d\sqrt{x}\right)}\right)}{d^3} -\frac{8ab\operatorname{PolyLog}\left(3,-e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3} +\frac{8ab\operatorname{PolyLog}\left(3,e^{i\left(c+d\sqrt{x}\right)}\right)}{d^3}$$

```
[Out] -2*I*b^2*x/d+2/3*a^2*x^(3/2)-8*a*b*x*arctanh(exp(I*(c+d*x^(1/2))))/d-2*b^2*
x*cot(c+d*x^(1/2))/d-2*I*b^2*polylog(2,exp(2*I*(c+d*x^(1/2))))/d^3-8*a*b*po
lylog(3,-exp(I*(c+d*x^(1/2))))/d^3+8*a*b*polylog(3,exp(I*(c+d*x^(1/2))))/d^
3+4*b^2*ln(1-exp(2*I*(c+d*x^(1/2))))*x^(1/2)/d^2+8*I*a*b*polylog(2,-exp(I*(
c+d*x^(1/2))))*x^(1/2)/d^2-8*I*a*b*polylog(2,exp(I*(c+d*x^(1/2))))*x^(1/2)/
d^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4290, 4275, 4268, 2611, 2320, 6724, 4269, 3798, 2221, 2317, 2438}

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\frac{2}{3}a^2x^{3/2}-\frac{8abx\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{8ab\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{8ab\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}+\frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2}-\frac{2ib^2\operatorname{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3}+\frac{4b^2\sqrt{x}\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2}-\frac{2b^2x\cot\left(c+d\sqrt{x}\right)}{d}-\frac{2ib^2x}{d}$$

```
[In] Int[Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x)/d + (2*a^2*x^(3/2))/3 - (8*a*b*x*ArcTanh[E^(I*(c + d*Sqrt[x]
))])/d - (2*b^2*x*Cot[c + d*Sqrt[x]])/d + (4*b^2*Sqrt[x]*Log[1 - E^((2*I)*(
c + d*Sqrt[x]))])/d^2 + ((8*I)*a*b*Sqrt[x]*PolyLog[2, -E^(I*(c + d*Sqrt[x])
)])/d^2 - ((8*I)*a*b*Sqrt[x]*PolyLog[2, E^(I*(c + d*Sqrt[x]))])/d^2 - ((2*I
)*b^2*PolyLog[2, E^((2*I)*(c + d*Sqrt[x]))])/d^3 - (8*a*b*PolyLog[3, -E^(I*
(c + d*Sqrt[x]))])/d^3 + (8*a*b*PolyLog[3, E^(I*(c + d*Sqrt[x]))])/d^3
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int x^2(a+b\csc(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(a^2x^2+2abx^2\csc(c+dx)+b^2x^2\csc^2(c+dx)\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}a^2x^{3/2}+(4ab)\text{Subst}\left(\int x^2\csc(c+dx)\,dx, x, \sqrt{x}\right)\\
&\qquad +\left(2b^2\right)\text{Subst}\left(\int x^2\csc^2(c+dx)\,dx, x, \sqrt{x}\right)\\
&= \frac{2}{3}a^2x^{3/2}-\frac{8abx\text{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d}-\frac{2b^2x\cot\left(c+d\sqrt{x}\right)}{d}\\
&\quad -\frac{(8ab)\text{Subst}\left(\int x\log\left(1-e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad +\frac{(8ab)\text{Subst}\left(\int x\log\left(1+e^{i(c+dx)}\right)\,dx, x, \sqrt{x}\right)}{d}\\
&\quad +\frac{\left(4b^2\right)\text{Subst}\left(\int x\cot(c+dx)\,dx, x, \sqrt{x}\right)}{d}
\end{aligned}
$$

$$= -\frac{2ib^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8abx\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x\cot\left(c+d\sqrt{x}\right)}{d} + \frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(8iab)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,-e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2} + \frac{(8iab)\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,e^{i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2} - \frac{(8ib^2)\operatorname{Subst}\left(\int\frac{e^{2i(c+dx)}x}{1-e^{2i(c+dx)}}\,dx,x,\sqrt{x}\right)}{d}$$

$$= -\frac{2ib^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8abx\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x\cot\left(c+d\sqrt{x}\right)}{d} + \frac{4b^2\sqrt{x}\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{(8ab)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,-x)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{(8ab)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}(2,x)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{d^3} - \frac{(4b^2)\operatorname{Subst}\left(\int\log\left(1-e^{2i(c+dx)}\right)\,dx,x,\sqrt{x}\right)}{d^2}$$

$$= -\frac{2ib^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8abx\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x\cot\left(c+d\sqrt{x}\right)}{d} + \frac{4b^2\sqrt{x}\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8iab\sqrt{x}\operatorname{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8ab\operatorname{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{8ab\operatorname{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{(2ib^2)\operatorname{Subst}\left(\int\frac{\log(1-x)}{x}\,dx,x,e^{2i(c+d\sqrt{x})}\right)}{d^3}$$

$$= -\frac{2ib^2x}{d} + \frac{2}{3}a^2x^{3/2} - \frac{8abx\operatorname{arctanh}\left(e^{i(c+d\sqrt{x})}\right)}{d} - \frac{2b^2x\cot\left(c+d\sqrt{x}\right)}{d} + \frac{4b^2\sqrt{x}\log\left(1-e^{2i(c+d\sqrt{x})}\right)}{d^2} + \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,-e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{8iab\sqrt{x}\,\text{PolyLog}\left(2,e^{i(c+d\sqrt{x})}\right)}{d^2} - \frac{2ib^2\,\text{PolyLog}\left(2,e^{2i(c+d\sqrt{x})}\right)}{d^3} - \frac{8ab\,\text{PolyLog}\left(3,-e^{i(c+d\sqrt{x})}\right)}{d^3} + \frac{8ab\,\text{PolyLog}\left(3,e^{i(c+d\sqrt{x})}\right)}{d^3}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 681 vs. $2(241) = 482$.

Time = 5.22 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.83

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx$$

$$= \frac{-12ib^2d^2x - 2a^2d^3x^{3/2} + 2a^2d^3e^{2ic}x^{3/2} - 12b^2d\sqrt{x}\log\left(1-e^{-i(c+d\sqrt{x})}\right) + 12b^2de^{2ic}\sqrt{x}\log\left(1-e^{-i(c+d\sqrt{x})}\right)}{}$$

```
[In] Integrate[Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-12*I)*b^2*d^2*x - 2*a^2*d^3*x^(3/2) + 2*a^2*d^3*E^((2*I)*c)*x^(3/2) - 12
*b^2*d*Sqrt[x]*Log[1 - E^((-I)*(c + d*Sqrt[x]))] + 12*b^2*d*E^((2*I)*c)*Sqr
t[x]*Log[1 - E^((-I)*(c + d*Sqrt[x]))] - 12*a*b*d^2*x*Log[1 - E^((-I)*(c +
d*Sqrt[x]))] + 12*a*b*d^2*E^((2*I)*c)*x*Log[1 - E^((-I)*(c + d*Sqrt[x]))] -
 12*b^2*d*Sqrt[x]*Log[1 + E^((-I)*(c + d*Sqrt[x]))] + 12*b^2*d*E^((2*I)*c)*
Sqrt[x]*Log[1 + E^((-I)*(c + d*Sqrt[x]))] + 12*a*b*d^2*x*Log[1 + E^((-I)*(c
 + d*Sqrt[x]))] - 12*a*b*d^2*E^((2*I)*c)*x*Log[1 + E^((-I)*(c + d*Sqrt[x]))
] + (12*I)*b*(-1 + E^((2*I)*c))*(b - 2*a*d*Sqrt[x])*PolyLog[2, -E^((-I)*(c
+ d*Sqrt[x]))] + (12*I)*b*(-1 + E^((2*I)*c))*(b + 2*a*d*Sqrt[x])*PolyLog[2,
 E^((-I)*(c + d*Sqrt[x]))] + 24*a*b*PolyLog[3, -E^((-I)*(c + d*Sqrt[x]))] -
 24*a*b*E^((2*I)*c)*PolyLog[3, -E^((-I)*(c + d*Sqrt[x]))] - 24*a*b*PolyLog[
3, E^((-I)*(c + d*Sqrt[x]))] + 24*a*b*E^((2*I)*c)*PolyLog[3, E^((-I)*(c + d
*Sqrt[x]))] - 3*b^2*d^2*x*Csc[c/2]*Csc[(c + d*Sqrt[x])/2]*Sin[(d*Sqrt[x])/2
] + 3*b^2*d^2*E^((2*I)*c)*x*Csc[c/2]*Csc[(c + d*Sqrt[x])/2]*Sin[(d*Sqrt[x])
/2] - 3*b^2*d^2*x*Sec[c/2]*Sec[(c + d*Sqrt[x])/2]*Sin[(d*Sqrt[x])/2] + 3*b^
2*d^2*E^((2*I)*c)*x*Sec[c/2]*Sec[(c + d*Sqrt[x])/2]*Sin[(d*Sqrt[x])/2])/(3*
d^3*(-1 + E^((2*I)*c)))
```

Maple [F]

$$\int \left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}\sqrt{x}dx$$

```
[In] int((a+b*csc(c+d*x^(1/2)))^2*x^(1/2),x)
[Out] int((a+b*csc(c+d*x^(1/2)))^2*x^(1/2),x)
```

Fricas [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}\,dx=\int \left(b\csc\left(d\sqrt{x}+c\right)+a\right)^{2}\sqrt{x}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="fricas")
[Out] integral(b^2*sqrt(x)*csc(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*csc(d*sqrt(x) + c
) + a^2*sqrt(x), x)
```

Sympy [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}\,dx=\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}\,dx$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))**2*x**(1/2),x)
[Out] Integral(sqrt(x)*(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(190) = 380$.

Time = 0.30 (sec) , antiderivative size = 1217, normalized size of antiderivative = 5.05

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="maxima")
[Out] 2/3*((d*sqrt(x) + c)^3*a^2 - 3*(d*sqrt(x) + c)^2*a^2*c + 3*(d*sqrt(x) + c)*
a^2*c^2 - 6*a*b*c^2*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c)) - 3*(2*b^2
*c^2 - 2*((d*sqrt(x) + c)^2*a*b + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x) + c) -
 ((d*sqrt(x) + c)^2*a*b + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x) + c))*cos(2*d*
sqrt(x) + 2*c) + (-I*(d*sqrt(x) + c)^2*a*b - I*b^2*c + (2*I*a*b*c + I*b^2)*
```

```
(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*arctan2(sin(d*sqrt(x) + c), cos(d*
sqrt(x) + c) + 1) + 2*(b^2*c*cos(2*d*sqrt(x) + 2*c) + I*b^2*c*sin(2*d*sqrt(
x) + 2*c) - b^2*c)*arctan2(sin(d*sqrt(x) + c), cos(d*sqrt(x) + c) - 1) - 2*
((d*sqrt(x) + c)^2*a*b - (2*a*b*c - b^2)*(d*sqrt(x) + c) - ((d*sqrt(x) + c)
^2*a*b - (2*a*b*c - b^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + (-I*(d*s
qrt(x) + c)^2*a*b + (2*I*a*b*c - I*b^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) +
2*c))*arctan2(sin(d*sqrt(x) + c), -cos(d*sqrt(x) + c) + 1) + 2*((d*sqrt(x)
+ c)^2*b^2 - 2*(d*sqrt(x) + c)*b^2*c)*cos(2*d*sqrt(x) + 2*c) + 2*(2*(d*sqrt
(x) + c)*a*b - 2*a*b*c - b^2 - (2*(d*sqrt(x) + c)*a*b - 2*a*b*c - b^2)*cos(
2*d*sqrt(x) + 2*c) - (2*I*(d*sqrt(x) + c)*a*b - 2*I*a*b*c - I*b^2)*sin(2*d*
sqrt(x) + 2*c))*dilog(-e^(I*d*sqrt(x) + I*c)) - 2*(2*(d*sqrt(x) + c)*a*b -
2*a*b*c + b^2 - (2*(d*sqrt(x) + c)*a*b - 2*a*b*c + b^2)*cos(2*d*sqrt(x) + 2
*c) + (-2*I*(d*sqrt(x) + c)*a*b + 2*I*a*b*c - I*b^2)*sin(2*d*sqrt(x) + 2*c)
)*dilog(e^(I*d*sqrt(x) + I*c)) + (I*(d*sqrt(x) + c)^2*a*b + I*b^2*c + (-2*I
*a*b*c - I*b^2)*(d*sqrt(x) + c) + (-I*(d*sqrt(x) + c)^2*a*b - I*b^2*c + (2*
I*a*b*c + I*b^2)*(d*sqrt(x) + c))*cos(2*d*sqrt(x) + 2*c) + ((d*sqrt(x) + c)
^2*a*b + b^2*c - (2*a*b*c + b^2)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*l
og(cos(d*sqrt(x) + c)^2 + sin(d*sqrt(x) + c)^2 + 2*cos(d*sqrt(x) + c) + 1)
+ (-I*(d*sqrt(x) + c)^2*a*b + I*b^2*c + (2*I*a*b*c - I*b^2)*(d*sqrt(x) + c)
 + (I*(d*sqrt(x) + c)^2*a*b - I*b^2*c + (-2*I*a*b*c + I*b^2)*(d*sqrt(x) + c
))*cos(2*d*sqrt(x) + 2*c) - ((d*sqrt(x) + c)^2*a*b - b^2*c - (2*a*b*c - b^2
)*(d*sqrt(x) + c))*sin(2*d*sqrt(x) + 2*c))*log(cos(d*sqrt(x) + c)^2 + sin(d
*sqrt(x) + c)^2 - 2*cos(d*sqrt(x) + c) + 1) - 4*(I*a*b*cos(2*d*sqrt(x) + 2*
c) - a*b*sin(2*d*sqrt(x) + 2*c) - I*a*b)*polylog(3, -e^(I*d*sqrt(x) + I*c))
 - 4*(-I*a*b*cos(2*d*sqrt(x) + 2*c) + a*b*sin(2*d*sqrt(x) + 2*c) + I*a*b)*p
olylog(3, e^(I*d*sqrt(x) + I*c)) - 2*(-I*(d*sqrt(x) + c)^2*b^2 + 2*I*(d*sqr
t(x) + c)*b^2*c)*sin(2*d*sqrt(x) + 2*c))/(-I*cos(2*d*sqrt(x) + 2*c) + sin(2
*d*sqrt(x) + 2*c) + I))/d^3
```

Giac [F]

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2\sqrt{x}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2*x^(1/2),x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)^2*sqrt(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2\,dx=\int \sqrt{x}\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)^2\,dx$$

```
[In] int(x^(1/2)*(a + b/sin(c + d*x^(1/2)))^2,x)
[Out] int(x^(1/2)*(a + b/sin(c + d*x^(1/2)))^2, x)
```

3.58 $\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 47

$$\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{\sqrt{x}}\,dx = 2a^2\sqrt{x} - \frac{4ab\operatorname{arctanh}(\cos(c+d\sqrt{x}))}{d} - \frac{2b^2\cot(c+d\sqrt{x})}{d}$$

```
[Out] -4*a*b*arctanh(cos(c+d*x^(1/2)))/d-2*b^2*cot(c+d*x^(1/2))/d+2*a^2*x^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4290, 3858, 3855, 3852, 8}

$$\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{\sqrt{x}}\,dx = 2a^2\sqrt{x} - \frac{4ab\operatorname{arctanh}(\cos(c+d\sqrt{x}))}{d} - \frac{2b^2\cot(c+d\sqrt{x})}{d}$$

```
[In] Int[(a + b*Csc[c + d*Sqrt[x]])^2/Sqrt[x],x]
```

```
[Out] 2*a^2*Sqrt[x] - (4*a*b*ArcTanh[Cos[c + d*Sqrt[x]]])/d - (2*b^2*Cot[c + d*Sq
rt[x]])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int (a+b\csc(c+dx))^2\,dx, x, \sqrt{x}\right)\\
&= 2a^2\sqrt{x}+(4ab)\text{Subst}\left(\int \csc(c+dx)\,dx, x, \sqrt{x}\right)+\left(2b^2\right)\text{Subst}\left(\int \csc^2(c+dx)\,dx, x, \sqrt{x}\right)\\
&= 2a^2\sqrt{x}-\frac{4ab\text{arctanh}\left(\cos\left(c+d\sqrt{x}\right)\right)}{d}-\frac{\left(2b^2\right)\text{Subst}\left(\int 1\,dx, x, \cot\left(c+d\sqrt{x}\right)\right)}{d}\\
&= 2a^2\sqrt{x}-\frac{4ab\text{arctanh}\left(\cos\left(c+d\sqrt{x}\right)\right)}{d}-\frac{2b^2\cot\left(c+d\sqrt{x}\right)}{d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.98

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx$$
$$= \frac{-b^2\cot\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)+2a\left(ac+ad\sqrt{x}-2b\log\left(\cos\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)+2b\log\left(\sin\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)\right)+b^2\,\text{ta}}{d}$$

```
[In] Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/Sqrt[x],x]
```

```
[Out] (-(b^2*Cot[(c + d*Sqrt[x])/2]) + 2*a*(a*c + a*d*Sqrt[x] - 2*b*Log[Cos[(c +
d*Sqrt[x])/2]] + 2*b*Log[Sin[(c + d*Sqrt[x])/2]]) + b^2*Tan[(c + d*Sqrt[x])
/2])/d
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

method	result	size
parts	$2a^2\sqrt{x}-\frac{2b^2\cot\left(c+d\sqrt{x}\right)}{d}-\frac{4ab\ln\left(\csc\left(c+d\sqrt{x}\right)+\cot\left(c+d\sqrt{x}\right)\right)}{d}$	51
derivativedivides	$\frac{2a^2\left(c+d\sqrt{x}\right)+4ab\ln\left(\csc\left(c+d\sqrt{x}\right)-\cot\left(c+d\sqrt{x}\right)\right)-2b^2\cot\left(c+d\sqrt{x}\right)}{d}$	55
default	$\frac{2a^2\left(c+d\sqrt{x}\right)+4ab\ln\left(\csc\left(c+d\sqrt{x}\right)-\cot\left(c+d\sqrt{x}\right)\right)-2b^2\cot\left(c+d\sqrt{x}\right)}{d}$	55

```
[In] int((a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*a^2*x^(1/2)-2*b^2*cot(c+d*x^(1/2))/d-4*a*b/d*ln(csc(c+d*x^(1/2))+cot(c+d*
x^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx$$
$$=\frac{2\left(a^2d\sqrt{x}\sin\left(d\sqrt{x}+c\right)-ab\log\left(\frac{1}{2}\cos\left(d\sqrt{x}+c\right)+\frac{1}{2}\right)\sin\left(d\sqrt{x}+c\right)+ab\log\left(-\frac{1}{2}\cos\left(d\sqrt{x}+c\right)+\frac{1}{2}\right)\sin}{d\sin\left(d\sqrt{x}+c\right)}$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")
[Out] 2*(a^2*d*sqrt(x)*sin(d*sqrt(x) + c) - a*b*log(1/2*cos(d*sqrt(x) + c) + 1/2)
*sin(d*sqrt(x) + c) + a*b*log(-1/2*cos(d*sqrt(x) + c) + 1/2)*sin(d*sqrt(x)
+ c) - b^2*cos(d*sqrt(x) + c))/(d*sin(d*sqrt(x) + c))
```

Sympy [A] (verification not implemented)

Time = 7.86 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.87

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx$$
$$=\begin{cases}\frac{2a^2\left(c+d\sqrt{x}\right)-4ab\log\left(\cot\left(c+d\sqrt{x}\right)+\csc\left(c+d\sqrt{x}\right)\right)-2b^2\cot\left(c+d\sqrt{x}\right)}{d} & \text{for } d\neq 0\\ -\sqrt{x}\left(-2a^2-4ab\csc\left(c\right)-2b^2\csc^2\left(c\right)\right) & \text{otherwise}\end{cases}$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))**2/x**(1/2),x)
[Out] Piecewise(((2*a**2*(c + d*sqrt(x)) - 4*a*b*log(cot(c + d*sqrt(x)) + csc(c +
 d*sqrt(x))) - 2*b**2*cot(c + d*sqrt(x)))/d, Ne(d, 0)), (-sqrt(x)*(-2*a**2
- 4*a*b*csc(c) - 2*b**2*csc(c)**2), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = 2\,a^2\sqrt{x} - \frac{4\,ab\log\left(\cot\left(d\sqrt{x}+c\right)+\csc\left(d\sqrt{x}+c\right)\right)}{d} - \frac{2\,b^2}{d\tan\left(d\sqrt{x}+c\right)}$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")
[Out] 2*a^2*sqrt(x) - 4*a*b*log(cot(d*sqrt(x) + c) + csc(d*sqrt(x) + c))/d - 2*b^
2/(d*tan(d*sqrt(x) + c))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(41) = 82.

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.77

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = \frac{2\left(d\sqrt{x}+c\right)a^2 + 4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)\right|\right) + b^2\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right) - \frac{4\,ab\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)+b^2}{\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)}}{d}$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")
[Out] (2*(d*sqrt(x) + c)*a^2 + 4*a*b*log(abs(tan(1/2*d*sqrt(x) + 1/2*c))) + b^2*t
an(1/2*d*sqrt(x) + 1/2*c) - (4*a*b*tan(1/2*d*sqrt(x) + 1/2*c) + b^2)/tan(1/
2*d*sqrt(x) + 1/2*c))/d
```

Mupad [B] (verification not implemented)

Time = 19.64 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.36

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{\sqrt{x}}\,dx = 2\,a^2\,\sqrt{x} - \frac{b^2\,4\mathrm{i}}{d\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,\sqrt{x}\,2\mathrm{i}}-1\right)} - \frac{4\,a\,b\,\ln\left(-\frac{a\,b\,4\mathrm{i}}{\sqrt{x}}-\frac{a\,b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,4\mathrm{i}}{\sqrt{x}}\right)}{d} + \frac{4\,a\,b\,\ln\left(\frac{a\,b\,4\mathrm{i}}{\sqrt{x}}-\frac{a\,b\,\mathrm{e}^{d\,\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,4\mathrm{i}}{\sqrt{x}}\right)}{d}$$

```
[In] int((a + b/sin(c + d*x^(1/2)))^2/x^(1/2),x)
```

```
[Out] 2*a^2*x^(1/2) - (b^2*4i)/(d*(exp(c*2i + d*x^(1/2)*2i) - 1)) - (4*a*b*log(-
(a*b*4i)/x^(1/2) - (a*b*exp(d*x^(1/2)*1i)*exp(c*1i)*4i)/x^(1/2)))/d + (4*a*
b*log((a*b*4i)/x^(1/2) - (a*b*exp(d*x^(1/2)*1i)*exp(c*1i)*4i)/x^(1/2)))/d
```

3.59 $\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx = \text{Int}\left(\frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx = \int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$$

```
[In] Int[(a + b*Csc[c + d*Sqrt[x]])^2/x^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Csc[c + d*Sqrt[x]])^2/x^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 76.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^(3/2),x]
[Out] Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{3}{2}}}dx$$

```
[In] int((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x)
[Out] int((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="fricas")
[Out] integral((b^2*sqrt(x)*csc(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*csc(d*sqrt(x) +
c) + a^2*sqrt(x))/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx = \int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))**2/x**(3/2),x)
[Out] Integral((a + b*csc(c + d*sqrt(x)))**2/x**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 794, normalized size of antiderivative = 36.09

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx = \int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="maxima")
[Out] -(4*b^2*sin(2*d*sqrt(x) + 2*c) - ((d*integrate(2*(a*b*d*sqrt(x)*sin(d*sqrt(
x) + c) + b^2*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x
) + c)^2 + 2*d*cos(d*sqrt(x) + c) + d)*x^2), x) + d*integrate(2*(a*b*d*sqrt
(x)*sin(d*sqrt(x) + c) - b^2*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 +
 d*sin(d*sqrt(x) + c)^2 - 2*d*cos(d*sqrt(x) + c) + d)*x^2), x))*cos(2*d*sqr
t(x) + 2*c)^2 + (d*integrate(2*(a*b*d*sqrt(x)*sin(d*sqrt(x) + c) + b^2*sin(
d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 + 2*d*cos
(d*sqrt(x) + c) + d)*x^2), x) + d*integrate(2*(a*b*d*sqrt(x)*sin(d*sqrt(x)
+ c) - b^2*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) +
 c)^2 - 2*d*cos(d*sqrt(x) + c) + d)*x^2), x))*sin(2*d*sqrt(x) + 2*c)^2 - 2*
(d*integrate(2*(a*b*d*sqrt(x)*sin(d*sqrt(x) + c) + b^2*sin(d*sqrt(x) + c))/
((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 + 2*d*cos(d*sqrt(x) + c)
+ d)*x^2), x) + d*integrate(2*(a*b*d*sqrt(x)*sin(d*sqrt(x) + c) - b^2*sin(d
*sqrt(x) + c))/((d*cos(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 - 2*d*cos(
d*sqrt(x) + c) + d)*x^2), x))*cos(2*d*sqrt(x) + 2*c) + d*integrate(2*(a*b*d
*sqrt(x)*sin(d*sqrt(x) + c) + b^2*sin(d*sqrt(x) + c))/((d*cos(d*sqrt(x) + c
)^2 + d*sin(d*sqrt(x) + c)^2 + 2*d*cos(d*sqrt(x) + c) + d)*x^2), x) + d*int
egrate(2*(a*b*d*sqrt(x)*sin(d*sqrt(x) + c) - b^2*sin(d*sqrt(x) + c))/((d*co
s(d*sqrt(x) + c)^2 + d*sin(d*sqrt(x) + c)^2 - 2*d*cos(d*sqrt(x) + c) + d)*x
^2), x))*x + 2*(a^2*d*cos(2*d*sqrt(x) + 2*c)^2 + a^2*d*sin(2*d*sqrt(x) + 2*
c)^2 - 2*a^2*d*cos(2*d*sqrt(x) + 2*c) + a^2*d)*sqrt(x))/((d*cos(2*d*sqrt(x)
 + 2*c)^2 + d*sin(2*d*sqrt(x) + 2*c)^2 - 2*d*cos(2*d*sqrt(x) + 2*c) + d)*x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^(3/2),x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)^2/x^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 18.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{3/2}}\,dx=\int \frac{\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)^2}{x^{3/2}}\,dx$$

```
[In] int((a + b/sin(c + d*x^(1/2)))^2/x^(3/2),x)
[Out] int((a + b/sin(c + d*x^(1/2)))^2/x^(3/2), x)
```

3.60 $\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^{5/2}}\,dx = \operatorname{Int}\left(\frac{(a+b\csc(c+d\sqrt{x}))^2}{x^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^{5/2}}\,dx = \int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^{5/2}}\,dx$$

```
[In] Int[(a + b*Csc[c + d*Sqrt[x]])^2/x^(5/2),x]
```

```
[Out] Defer[Int][(a + b*Csc[c + d*Sqrt[x]])^2/x^(5/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\csc(c+d\sqrt{x}))^2}{x^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 84.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx$$

```
[In] Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^(5/2),x]
[Out] Integrate[(a + b*Csc[c + d*Sqrt[x]])^2/x^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{5}{2}}}dx$$

```
[In] int((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x)
[Out] int((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="fricas")
[Out] integral((b^2*sqrt(x)*csc(d*sqrt(x) + c)^2 + 2*a*b*sqrt(x)*csc(d*sqrt(x) +
c) + a^2*sqrt(x))/x^3, x)
```

Sympy [N/A]

Not integrable

Time = 7.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x**(1/2)))**2/x**(5/2),x)
[Out] Integral((a + b*csc(c + d*sqrt(x)))**2/x**(5/2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\text{Timed out}$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="maxima")
[Out] Timed out
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}{x^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*csc(c+d*x^(1/2)))^2/x^(5/2),x, algorithm="giac")
[Out] integrate((b*csc(d*sqrt(x) + c) + a)^2/x^(5/2), x)
```

Mupad [N/A]

Not integrable

Time = 18.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}{x^{5/2}}\,dx=\int \frac{\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)^2}{x^{5/2}}\,dx$$

```
[In] int((a + b/sin(c + d*x^(1/2)))^2/x^(5/2),x)
[Out] int((a + b/sin(c + d*x^(1/2)))^2/x^(5/2), x)
```

3.61 $\int \frac{\csc^3(\sqrt{x})}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 24

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}} \, dx = -\text{arctanh}\left(\cos\left(\sqrt{x}\right)\right) - \cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)$$

```
[Out] -arctanh(cos(x^(1/2)))-cot(x^(1/2))*csc(x^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4290, 3853, 3855}

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}} \, dx = -\text{arctanh}\left(\cos\left(\sqrt{x}\right)\right) - \cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)$$

```
[In] Int[Csc[Sqrt[x]]^3/Sqrt[x],x]
```

```
[Out] -ArcTanh[Cos[Sqrt[x]]] - Cot[Sqrt[x]]*Csc[Sqrt[x]]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \csc^3(x)\,dx, x, \sqrt{x}\right)\\ &= -\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right) + \text{Subst}\left(\int \csc(x)\,dx, x, \sqrt{x}\right)\\ &= -\text{arctanh}\left(\cos\left(\sqrt{x}\right)\right) - \cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(24) = 48.

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{1}{4}\csc^2\left(\frac{\sqrt{x}}{2}\right) - \log\left(\cos\left(\frac{\sqrt{x}}{2}\right)\right) + \log\left(\sin\left(\frac{\sqrt{x}}{2}\right)\right) + \frac{1}{4}\sec^2\left(\frac{\sqrt{x}}{2}\right)$$

```
[In] Integrate[Csc[Sqrt[x]]^3/Sqrt[x],x]
[Out] -1/4*Csc[Sqrt[x]/2]^2 - Log[Cos[Sqrt[x]/2]] + Log[Sin[Sqrt[x]/2]] + Sec[Sqr
t[x]/2]^2/4
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)+\ln\left(\csc\left(\sqrt{x}\right)-\cot\left(\sqrt{x}\right)\right)$	24
default	$-\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)+\ln\left(\csc\left(\sqrt{x}\right)-\cot\left(\sqrt{x}\right)\right)$	24

```
[In] int(csc(x^(1/2))^3/x^(1/2),x,method=_RETURNVERBOSE)
[Out] -cot(x^(1/2))*csc(x^(1/2))+ln(csc(x^(1/2))-cot(x^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(18) = 36$.

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{\left(\cos\left(\sqrt{x}\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(\sqrt{x}\right)+\frac{1}{2}\right)-\left(\cos\left(\sqrt{x}\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(\sqrt{x}\right)+\frac{1}{2}\right)-2\cos\left(\sqrt{x}\right)}{2\left(\cos\left(\sqrt{x}\right)^2-1\right)}$$

```
[In] integrate(csc(x^(1/2))^3/x^(1/2),x, algorithm="fricas")
[Out] -1/2*((cos(sqrt(x))^2 - 1)*log(1/2*cos(sqrt(x)) + 1/2) - (cos(sqrt(x))^2 -
1)*log(-1/2*cos(sqrt(x)) + 1/2) - 2*cos(sqrt(x)))/(cos(sqrt(x))^2 - 1)
```

Sympy [F]

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx$$

```
[In] integrate(csc(x**(1/2))**3/x**(1/2),x)
[Out] Integral(csc(sqrt(x))**3/sqrt(x), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{\cos\left(\sqrt{x}\right)}{\cos\left(\sqrt{x}\right)^2-1}-\frac{1}{2}\log\left(\cos\left(\sqrt{x}\right)+1\right)+\frac{1}{2}\log\left(\cos\left(\sqrt{x}\right)-1\right)$$

```
[In] integrate(csc(x^(1/2))^3/x^(1/2),x, algorithm="maxima")
[Out] cos(sqrt(x))/(cos(sqrt(x))^2 - 1) - 1/2*log(cos(sqrt(x)) + 1) + 1/2*log(cos
(sqrt(x)) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(18) = 36$.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{\left(\frac{2\left(\cos\left(\sqrt{x}\right)-1\right)}{\cos\left(\sqrt{x}\right)+1}-1\right)\left(\cos\left(\sqrt{x}\right)+1\right)}{4\left(\cos\left(\sqrt{x}\right)-1\right)} - \frac{\cos\left(\sqrt{x}\right)-1}{4\left(\cos\left(\sqrt{x}\right)+1\right)} + \frac{1}{2}\log\left(-\frac{\cos\left(\sqrt{x}\right)-1}{\cos\left(\sqrt{x}\right)+1}\right)$$

```
[In] integrate(csc(x^(1/2))^3/x^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*(cos(sqrt(x)) - 1)/(cos(sqrt(x)) + 1) - 1)*(cos(sqrt(x)) + 1)/(cos(
sqrt(x)) - 1) - 1/4*(cos(sqrt(x)) - 1)/(cos(sqrt(x)) + 1) + 1/2*log(-(cos(s
qrt(x)) - 1)/(cos(sqrt(x)) + 1))
```

Mupad [B] (verification not implemented)

Time = 19.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.92

$$\int \frac{\csc^3\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\ln\left(-\frac{e^{\sqrt{x}\,1i}\,1i}{\sqrt{x}}-\frac{1i}{\sqrt{x}}\right)+\ln\left(-\frac{e^{\sqrt{x}\,1i}\,1i}{\sqrt{x}}+\frac{1i}{\sqrt{x}}\right) + \frac{4\,e^{\sqrt{x}\,1i}}{1+e^{\sqrt{x}\,4i}-2\,e^{\sqrt{x}\,2i}} + \frac{2\,e^{\sqrt{x}\,1i}}{e^{\sqrt{x}\,2i}-1}$$

```
[In] int(1/(x^(1/2)*sin(x^(1/2))^3),x)
```

```
[Out] log(1i/x^(1/2) - (exp(x^(1/2)*1i)*1i)/x^(1/2)) - log(- (exp(x^(1/2)*1i)*1i)
/x^(1/2) - 1i/x^(1/2)) + (4*exp(x^(1/2)*1i))/(exp(x^(1/2)*4i) - 2*exp(x^(1/
2)*2i) + 1) + (2*exp(x^(1/2)*1i))/(exp(x^(1/2)*2i) - 1)
```

3.62 $\int \frac{x^{3/2}}{a+b\csc(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 675

$$\begin{aligned}\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx &= \frac{2x^{5/2}}{5a}+\frac{2ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}\\
&-\frac{2ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{8bx^{3/2}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}\\
&-\frac{8bx^{3/2}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2}+\frac{24ibx\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}\\
&-\frac{24ibx\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}\\
&+\frac{48b\sqrt{x}\operatorname{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4}\\
&-\frac{48ib\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}+\frac{48ib\operatorname{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}\end{aligned}$$

```
[Out] 2/5*x^(5/2)/a+2*I*b*x^2*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))
/a/d/(-a^2+b^2)^(1/2)-2*I*b*x^2*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)
^(1/2)))/a/d/(-a^2+b^2)^(1/2)+8*b*x^(3/2)*polylog(2,I*a*exp(I*(c+d*x^(1/2))
)/(b-(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)-8*b*x^(3/2)*polylog(2,I*a*ex
p(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^2/(-a^2+b^2)^(1/2)+24*I*b*x*po
lylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/
2)-24*I*b*x*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a/d^3/
(-a^2+b^2)^(1/2)-48*I*b*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1
/2)))/a/d^5/(-a^2+b^2)^(1/2)+48*I*b*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b+(
```

```
-a^2+b^2)^(1/2)))/a/d^5/(-a^2+b^2)^(1/2)-48*b*polylog(4,I*a*exp(I*(c+d*x^(1
/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^4/(-a^2+b^2)^(1/2)+48*b*polylog(4,I
*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^4/(-a^2+b^2)^(1/2
)
```

Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 675, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4290, 4276, 3404, 2296, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx &= -\frac{48ib\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}} \\ &+\frac{48ib\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^5\sqrt{b^2-a^2}}-\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}} \\ &+\frac{48b\sqrt{x}\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^4\sqrt{b^2-a^2}}+\frac{24ibx\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} \\ &-\frac{24ibx\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}}+\frac{8bx^{3/2}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} \\ &-\frac{8bx^{3/2}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}}+\frac{2ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}} \\ &-\frac{2ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad\sqrt{b^2-a^2}}+\frac{2x^{5/2}}{5a}\end{aligned}$$

```
[In] Int[x^(3/2)/(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (2*x^(5/2))/(5*a) + ((2*I)*b*x^2*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - S
qrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((2*I)*b*x^2*Log[1 - (I*a*E^(I*
(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (8*b*x^
(3/2)*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sq
rt[-a^2 + b^2]*d^2) - (8*b*x^(3/2)*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(
b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((24*I)*b*x*PolyLog[3, (
I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3
) - ((24*I)*b*x*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2
])])/(a*Sqrt[-a^2 + b^2]*d^3) - (48*b*Sqrt[x]*PolyLog[4, (I*a*E^(I*(c + d*S
qrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^4) + (48*b*Sqrt[x]
*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a
^2 + b^2]*d^4) - ((48*I)*b*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt
[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^5) + ((48*I)*b*PolyLog[5, (I*a*E^(I*(
c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^5)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^4}{a+b\csc(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^4}{a}-\frac{bx^4}{a(b+a\sin(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2x^{5/2}}{5a}-\frac{(2b)\text{Subst}\left(\int \frac{x^4}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2x^{5/2}}{5a}-\frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2x^{5/2}}{5a}+\frac{(4ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}\\
&\quad -\frac{(4ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}
\end{aligned}$$

$$= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{(8ib)\text{Subst}\left(\int x^3 \log\left(1 - \frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d} + \frac{(8ib)\text{Subst}\left(\int x^3 \log\left(1 - \frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}$$

$$= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{8bx^{3/2} \text{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{8bx^{3/2} \text{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{(24b)\text{Subst}\left(\int x^2 \text{PolyLog}\left(2, \frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(24b)\text{Subst}\left(\int x^2 \text{PolyLog}\left(2, \frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{8bx^{3/2} \text{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{8bx^{3/2} \text{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{24ibx \text{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{24ibx \text{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{(48ib)\text{Subst}\left(\int x \text{PolyLog}\left(3, \frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3} + \frac{(48ib)\text{Subst}\left(\int x \text{PolyLog}\left(3, \frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{24ibx \operatorname{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{24ibx \operatorname{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{(48b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(4, \frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4} - \frac{(48b)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(4, \frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right) dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^4}$$

$$= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{24ibx \operatorname{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{24ibx \operatorname{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} - \frac{(48ib)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(4, \frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x} dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{(48ib)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}\left(4, \frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x} dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^5}$$

$$= \frac{2x^{5/2}}{5a} + \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx^2 \log\left(1 - \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{8bx^{3/2} \operatorname{PolyLog}\left(2, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{24ibx \operatorname{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{24ibx \operatorname{PolyLog}\left(3, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} + \frac{48b\sqrt{x} \operatorname{PolyLog}\left(4, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^4} - \frac{48ib \operatorname{PolyLog}\left(5, \frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5} + \frac{48ib \operatorname{PolyLog}\left(5, \frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^5}$$

Mathematica [A] (verified)

Time = 2.09 (sec) , antiderivative size = 539, normalized size of antiderivative = 0.80

$$\int \frac{x^{3/2}}{a + b\csc\left(c + d\sqrt{x}\right)}\, dx = \frac{2\left(\sqrt{a^2-b^2}d^5x^{5/2} - 5bd^4x^2 \log\left(1 - \frac{ae^{i(c+d\sqrt{x})}}{-ib+\sqrt{a^2-b^2}}\right) + 5bd^4x^2 \log\left(1 + \frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right) + \cdots\right.}{}$$

```
[In] Integrate[x^(3/2)/(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(Sqrt[a^2 - b^2]*d^5*x^(5/2) - 5*b*d^4*x^2*Log[1 - (a*E^(I*(c + d*Sqrt[x
]))))/((-I)*b + Sqrt[a^2 - b^2])] + 5*b*d^4*x^2*Log[1 + (a*E^(I*(c + d*Sqrt[
x])))/(I*b + Sqrt[a^2 - b^2])] + (20*I)*b*d^3*x^(3/2)*PolyLog[2, (a*E^(I*(c
 + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (20*I)*b*d^3*x^(3/2)*PolyLog[
2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - 60*b*d^2*x*PolyL
og[3, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + 60*b*d^2*x*Po
lyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - (120*I)*b*
d*Sqrt[x]*PolyLog[4, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])]
+ (120*I)*b*d*Sqrt[x]*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^
2 - b^2]))] + 120*b*PolyLog[5, (a*E^(I*(c + d*Sqrt[x])))/((-I)*b + Sqrt[a^2
 - b^2])] - 120*b*PolyLog[5, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 -
b^2]))]))/(5*a*Sqrt[a^2 - b^2]*d^5)
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
[Out] int(x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(x^(3/2)/(b*csc(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x**(3/2)/(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(x**(3/2)/(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{\frac{3}{2}}}{b\csc\left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(x^(3/2)/(b*csc(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{x^{3/2}}{a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}}\,dx$$

```
[In] int(x^(3/2)/(a + b/sin(c + d*x^(1/2))),x)
[Out] int(x^(3/2)/(a + b/sin(c + d*x^(1/2))), x)
```

3.63 $\int \frac{\sqrt{x}}{a+b\csc(c+d\sqrt{x})}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 407

$$\begin{aligned}\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx &= \frac{2x^{3/2}}{3a}+\frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} \\ &\quad -\frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d}+\frac{4b\sqrt{x}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\ &\quad -\frac{4b\sqrt{x}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} \\ &\quad +\frac{4ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}-\frac{4ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}\end{aligned}$$

```
[Out] 2/3*x^(3/2)/a+2*I*b*x*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a
/d/(-a^2+b^2)^(1/2)-2*I*b*x*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/
2)))/a/d/(-a^2+b^2)^(1/2)+4*I*b*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2
+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)-4*I*b*polylog(3,I*a*exp(I*(c+d*x^(1/2)
))/(b+(-a^2+b^2)^(1/2)))/a/d^3/(-a^2+b^2)^(1/2)+4*b*polylog(2,I*a*exp(I*(c+
d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^2/(-a^2+b^2)^(1/2)-4*b*polylo
g(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a/d^2/(-a^2+b^2)
^(1/2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {4290, 4276, 3404, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \frac{4ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} - \frac{4ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^3\sqrt{b^2-a^2}} + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)}{ad^2\sqrt{b^2-a^2}} + \frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)}{ad\sqrt{b^2-a^2}} - \frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{\sqrt{b^2-a^2}+b}\right)}{ad\sqrt{b^2-a^2}} + \frac{2x^{3/2}}{3a}$$

```
[In] Int[Sqrt[x]/(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (2*x^(3/2))/(3*a) + ((2*I)*b*x*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqr
t[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) - ((2*I)*b*x*Log[1 - (I*a*E^(I*(c +
 d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d) + (4*b*Sqrt[x
]*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-
a^2 + b^2]*d^2) - (4*b*Sqrt[x]*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b +
Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2) + ((4*I)*b*PolyLog[3, (I*a*E^(
I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3) - ((4
*I)*b*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a*Sq
rt[-a^2 + b^2]*d^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= 2\text{Subst}\left(\int \frac{x^2}{a+b\csc(c+dx)}\,dx, x, \sqrt{x}\right)\\ &= 2\text{Subst}\left(\int \left(\frac{x^2}{a}-\frac{bx^2}{a(b+a\sin(c+dx))}\right)\,dx, x, \sqrt{x}\right)\end{aligned}$$

$$= \frac{2x^{3/2}}{3a} - \frac{(2b)\text{Subst}\left(\int \frac{x^2}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{2x^{3/2}}{3a} - \frac{(4b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a}$$

$$= \frac{2x^{3/2}}{3a} + \frac{(4ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}} - \frac{(4ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{\sqrt{-a^2+b^2}}$$

$$= \frac{2x^{3/2}}{3a} + \frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{(4ib)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d} + \frac{(4ib)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d}$$

$$= \frac{2x^{3/2}}{3a} + \frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{(4b)\text{Subst}\left(\int \text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(4b)\text{Subst}\left(\int \text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}d^2}$$

$$= \frac{2x^{3/2}}{3a} + \frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{(4ib)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,\frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{(4ib)\text{Subst}\left(\int \frac{\text{PolyLog}\left(2,\frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+d\sqrt{x})}\right)}{a\sqrt{-a^2+b^2}d^3}$$

$$= \frac{2x^{3/2}}{3a} + \frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} - \frac{2ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d} + \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} - \frac{4b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2} + \frac{4ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3} - \frac{4ib\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3}$$

Mathematica [A] (verified)

Time = 3.48 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx = \frac{2\left(\sqrt{a^2-b^2}d^3x^{3/2} - 3bd^2x\log\left(1-\frac{ae^{i(c+d\sqrt{x})}}{-ib+\sqrt{a^2-b^2}}\right) + 3bd^2x\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right) + 6ibd\sqrt{x}\,\text{PolyLog}\left(2,\frac{ae^{i(c\cdots}}{-ib+\sqrt{\cdots}}\right.\right.}{3a\sqrt{a^2\cdots}}$$

```
[In] Integrate[Sqrt[x]/(a + b*Csc[c + d*Sqrt[x]]),x]
```

```
[Out] (2*(Sqrt[a^2 - b^2]*d^3*x^(3/2) - 3*b*d^2*x*Log[1 - (a*E^(I*(c + d*Sqrt[x])
))/((-I)*b + Sqrt[a^2 - b^2])] + 3*b*d^2*x*Log[1 + (a*E^(I*(c + d*Sqrt[x]))
)/(I*b + Sqrt[a^2 - b^2])] + (6*I)*b*d*Sqrt[x]*PolyLog[2, (a*E^(I*(c + d*Sq
rt[x])))/((-I)*b + Sqrt[a^2 - b^2])] - (6*I)*b*d*Sqrt[x]*PolyLog[2, -((a*E^
(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))] - 6*b*PolyLog[3, (a*E^(I*(c
+ d*Sqrt[x])))/((-I)*b + Sqrt[a^2 - b^2])] + 6*b*PolyLog[3, -((a*E^(I*(c +
d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))]))/(3*a*Sqrt[a^2 - b^2]*d^3)
```

Maple [F]

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}dx$$

```
[In] int(x^(1/2)/(a+b*csc(c+d*x^(1/2))),x)
```

```
[Out] int(x^(1/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [F]

$$\int \frac{\sqrt{x}}{a+b\csc \left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{b\csc \left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*csc(d*sqrt(x) + c) + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{x}}{a+b\csc \left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{a+b\csc \left(c+d\sqrt{x}\right)}\,dx$$

```
[In] integrate(x**(1/2)/(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(sqrt(x)/(a + b*csc(c + d*sqrt(x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}}{a+b\csc \left(c+d\sqrt{x}\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{\sqrt{x}}{a+b\csc \left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{b\csc \left(d\sqrt{x}+c\right)+a}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
[Out] integrate(sqrt(x)/(b*csc(d*sqrt(x) + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{a+b\csc\left(c+d\sqrt{x}\right)}\,dx=\int \frac{\sqrt{x}}{a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}}\,dx$$

```
[In] int(x^(1/2)/(a + b/sin(c + d*x^(1/2))),x)
[Out] int(x^(1/2)/(a + b/sin(c + d*x^(1/2))), x)
```

3.64 $\int \frac{1}{\sqrt{x}(a+b\csc(c+d\sqrt{x}))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 66

$$\int \frac{1}{\sqrt{x}\,(a+b\csc(c+d\sqrt{x}))}\,dx=\frac{2\sqrt{x}}{a}+\frac{4b\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d}$$

```
[Out] 4*b*arctanh((a+b*tan(1/2*c+1/2*d*x^(1/2)))/(a^2-b^2)^(1/2))/a/d/(a^2-b^2)^(
1/2)+2*x^(1/2)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4290, 3868, 2739, 632, 212}

$$\int \frac{1}{\sqrt{x}\,(a+b\csc(c+d\sqrt{x}))}\,dx=\frac{4b\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+d\sqrt{x})\right)}{\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}+\frac{2\sqrt{x}}{a}$$

```
[In] Int[1/(Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])),x]
```

```
[Out] (2*Sqrt[x])/a + (4*b*ArcTanh[(a + b*Tan[(c + d*Sqrt[x])/2])/Sqrt[a^2 - b^2]
])/(a*Sqrt[a^2 - b^2]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{1}{a+b\csc(c+dx)}\,dx, x, \sqrt{x}\right)\\
&= \frac{2\sqrt{x}}{a} - \frac{2\text{Subst}\left(\int \frac{1}{1+\frac{a\sin(c+dx)}{b}}\,dx, x, \sqrt{x}\right)}{a}\\
&= \frac{2\sqrt{x}}{a} - \frac{4\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx, x, \tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{ad}\\
&= \frac{2\sqrt{x}}{a} + \frac{8\text{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}+2\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{ad}\\
&= \frac{2\sqrt{x}}{a} + \frac{4b\,\text{arctanh}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}d}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\frac{2\left(\frac{c}{d}+\sqrt{x}-\frac{2b\arctan\left(\frac{a+b\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}d}\right)}{a}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])),x]
```

```
[Out] (2*(c/d + Sqrt[x] - (2*b*ArcTan[(a + b*Tan[(c + d*Sqrt[x])/2])/Sqrt[-a^2 +
b^2]])/(Sqrt[-a^2 + b^2]*d)))/a
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a}-\frac{4b\arctan\left(\frac{2b\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}}{d}$	73
default	$\frac{\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a}-\frac{4b\arctan\left(\frac{2b\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}}{d}$	73

```
[In] int(1/(a+b*csc(c+d*x^(1/2)))/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(2/a*arctan(tan(1/2*c+1/2*d*x^(1/2)))-2*b/a/(-a^2+b^2)^(1/2)*arctan(1/2
*(2*b*tan(1/2*c+1/2*d*x^(1/2))+2*a)/(-a^2+b^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 275, normalized size of antiderivative = 4.17

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$
$$=\left[\frac{2\left(a^2-b^2\right)d\sqrt{x}+\sqrt{a^2-b^2}b\log\left(\frac{\left(a^2-2b^2\right)\cos\left(d\sqrt{x}+c\right)^2+2\sqrt{a^2-b^2}a\cos\left(d\sqrt{x}+c\right)+a^2+b^2+2\left(\sqrt{a^2-b^2}b\cos\left(d\sqrt{x}+c\right)+ab\right)\sin\left(d\sqrt{\right.}\right.}{a^2\cos\left(d\sqrt{x}+c\right)^2-2ab\sin\left(d\sqrt{x}+c\right)-a^2-b^2}}{\left(a^3-ab^2\right)d}\right.$$

```
[In] integrate(1/(a+b*csc(c+d*x^(1/2)))/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*(a^2 - b^2)*d*sqrt(x) + sqrt(a^2 - b^2)*b*log(((a^2 - 2*b^2)*cos(d*sqrt
(x) + c)^2 + 2*sqrt(a^2 - b^2)*a*cos(d*sqrt(x) + c) + a^2 + b^2 + 2*(sqrt(a
^2 - b^2)*b*cos(d*sqrt(x) + c) + a*b)*sin(d*sqrt(x) + c))/(a^2*cos(d*sqrt(x
) + c)^2 - 2*a*b*sin(d*sqrt(x) + c) - a^2 - b^2)))/((a^3 - a*b^2)*d), 2*((a
^2 - b^2)*d*sqrt(x) + sqrt(-a^2 + b^2)*b*arctan(-(sqrt(-a^2 + b^2)*b*sin(d*
sqrt(x) + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*cos(d*sqrt(x) + c))))/((a^3
 - a*b^2)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/(a+b*csc(c+d*x**(1/2)))/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(a + b*csc(c + d*sqrt(x)))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*csc(c+d*x^(1/2)))/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.27

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=-\frac{4\left(\pi\left\lfloor\frac{d\sqrt{x}+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}d\sqrt{x}+\frac{1}{2}c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}ad}+\frac{2\left(d\sqrt{x}+c\right)}{ad}$$

```
[In] integrate(1/(a+b*csc(c+d*x^(1/2)))/x^(1/2),x, algorithm="giac")
```

```
[Out] -4*(pi*floor(1/2*(d*sqrt(x) + c)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*d*sqr
t(x) + 1/2*c) + a)/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2)*a*d) + 2*(d*sqrt(
x) + c)/(a*d)
```

Mupad [B] (verification not implemented)

Time = 19.52 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.41

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx = \frac{2\sqrt{x}}{a} - \frac{2\,b\,\ln\left(b\,\mathrm{e}^{d\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i} - \frac{2\,b\left(a\,1\mathrm{i}+b\,\mathrm{e}^{d\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{a-b}} + \frac{2\,b\,\ln\left(b\,\mathrm{e}^{d\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\,2\mathrm{i} + \frac{2\,b\left(a\,1\mathrm{i}+b\,\mathrm{e}^{d\sqrt{x}\,1\mathrm{i}}\,\mathrm{e}^{c\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)}{a\,d\,\sqrt{a+b}\,\sqrt{a-b}}$$

```
[In] int(1/(x^(1/2)*(a + b/sin(c + d*x^(1/2)))),x)
```

```
[Out] (2*x^(1/2))/a - (2*b*log(b*exp(d*x^(1/2)*1i)*exp(c*1i)*2i - (2*b*(a*1i + b*
exp(d*x^(1/2)*1i)*exp(c*1i)))/((a + b)^(1/2)*(a - b)^(1/2))))/(a*d*(a + b)^
(1/2)*(a - b)^(1/2)) + (2*b*log(b*exp(d*x^(1/2)*1i)*exp(c*1i)*2i + (2*b*(a*
1i + b*exp(d*x^(1/2)*1i)*exp(c*1i)))/((a + b)^(1/2)*(a - b)^(1/2))))/(a*d*(
a + b)^(1/2)*(a - b)^(1/2))
```

3.65 $\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Int}\left(\frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])),x]
[Out] Defer[Int][1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Integrate[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}dx$$

```
[In] int(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
[Out] int(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*x^2*csc(d*sqrt(x) + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{\frac{3}{2}}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] integrate(1/x**(3/2)/(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(1/(x**(3/2)*(a + b*csc(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 244, normalized size of antiderivative = 11.09

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] -2*(a*b*sqrt(x)*integrate((2*b*cos(d*sqrt(x) + c)^2 + a*cos(d*sqrt(x) + c)*
sin(2*d*sqrt(x) + 2*c) - a*cos(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) + 2*b*
sin(d*sqrt(x) + c)^2 + a*sin(d*sqrt(x) + c))/((a^3*cos(2*d*sqrt(x) + 2*c)^2
 + 4*a*b^2*cos(d*sqrt(x) + c)^2 + 4*a^2*b*cos(d*sqrt(x) + c)*sin(2*d*sqrt(x
) + 2*c) + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a*b^2*sin(d*sqrt(x) + c)^2 + 4*
a^2*b*sin(d*sqrt(x) + c) + a^3 - 2*(2*a^2*b*sin(d*sqrt(x) + c) + a^3)*cos(2
*d*sqrt(x) + 2*c))*x^(3/2)), x) + 1)/(a*sqrt(x))
```

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*csc(d*sqrt(x) + c) + a)*x^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 17.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{3/2}\,\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)}\,dx$$

```
[In] int(1/(x^(3/2)*(a + b/sin(c + d*x^(1/2)))),x)
```

```
[Out] int(1/(x^(3/2)*(a + b/sin(c + d*x^(1/2)))), x)
```

3.66 $\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\text{Int}\left(\frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)},x\right)$$

```
[Out] Unintegrable(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

```
[In] Int[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])),x]
[Out] Defer[Int][1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right)} \, dx = \int \frac{1}{x^{5/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right)} \, dx$$

```
[In] Integrate[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])),x]
[Out] Integrate[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{5}{2}} \left(a + b \csc \left(c + d\sqrt{x}\right)\right)} dx$$

```
[In] int(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x)
[Out] int(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^{5/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right)} \, dx = \int \frac{1}{\left(b \csc \left(d\sqrt{x} + c\right) + a\right) x^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="fricas")
[Out] integral(sqrt(x)/(b*x^3*csc(d*sqrt(x) + c) + a*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 8.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right)} \, dx = \int \frac{1}{x^{\frac{5}{2}} \left(a + b \csc \left(c + d\sqrt{x}\right)\right)} \, dx$$

```
[In] integrate(1/x**(5/2)/(a+b*csc(c+d*x**(1/2))),x)
[Out] Integral(1/(x**(5/2)*(a + b*csc(c + d*sqrt(x)))), x)
```

Maxima [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 245, normalized size of antiderivative = 11.14

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="maxima")
```

```
[Out] -2/3*(3*a*b*x^(3/2)*integrate((2*b*cos(d*sqrt(x) + c)^2 + a*cos(d*sqrt(x) +
 c)*sin(2*d*sqrt(x) + 2*c) - a*cos(2*d*sqrt(x) + 2*c)*sin(d*sqrt(x) + c) +
2*b*sin(d*sqrt(x) + c)^2 + a*sin(d*sqrt(x) + c))/((a^3*cos(2*d*sqrt(x) + 2*
c)^2 + 4*a*b^2*cos(d*sqrt(x) + c)^2 + 4*a^2*b*cos(d*sqrt(x) + c)*sin(2*d*sq
rt(x) + 2*c) + a^3*sin(2*d*sqrt(x) + 2*c)^2 + 4*a*b^2*sin(d*sqrt(x) + c)^2
+ 4*a^2*b*sin(d*sqrt(x) + c) + a^3 - 2*(2*a^2*b*sin(d*sqrt(x) + c) + a^3)*c
os(2*d*sqrt(x) + 2*c))*x^(5/2)), x) + 1)/(a*x^(3/2))
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*csc(d*sqrt(x) + c) + a)*x^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 17.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}\,dx=\int \frac{1}{x^{5/2}\,\left(a+\frac{b}{\sin\left(c+d\,\sqrt{x}\right)}\right)}\,dx$$

```
[In] int(1/(x^(5/2)*(a + b/sin(c + d*x^(1/2)))),x)
```

```
[Out] int(1/(x^(5/2)*(a + b/sin(c + d*x^(1/2)))), x)
```

3.67 $\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1977

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ib^2x^2}{a^2\left(a^2-b^2\right)d}+\frac{2x^{5/2}}{5a^2}$$
$$+\frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}+\frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2}$$
$$-\frac{2ib^3x^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}+\frac{4ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+\frac{2ib^3x^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d}-\frac{4ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$-\frac{24ib^2x\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}-\frac{24ib^2x\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^3}$$
$$-\frac{8b^3x^{3/2}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}+\frac{16bx^{3/2}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+\frac{8b^3x^{3/2}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^2}-\frac{16bx^{3/2}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+\frac{48b^2\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}+\frac{48b^2\sqrt{x}\,\text{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^4}$$
$$-\frac{24ib^3x\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}+\frac{48ibx\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+\frac{24ib^3x\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^3}-\frac{48ibx\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3}$$
$$+\frac{48ib^2\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^5}+\frac{48ib^2\,\text{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^5}$$
$$+\frac{48b^3\sqrt{x}\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^4}-\frac{96b\sqrt{x}\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^4}$$
$$-\frac{48b^3\sqrt{x}\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^4}+\frac{96b\sqrt{x}\,\text{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^4}$$
$$+\frac{48ib^3\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^5}-\frac{96ib\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^5}$$
$$-\frac{48ib^3\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}d^5}+\frac{96ib\,\text{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^5}$$
$$-\frac{2b^2x^2\cos\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}$$

```
[Out] -2*b^2*x^2*cos(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*sin(c+d*x^(1/2)))-2*I*b^3*x^
2*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/
d-24*I*b^2*x*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a^2/(
a^2-b^2)/d^3-24*I*b^2*x*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1
/2)))/a^2/(a^2-b^2)/d^3-24*I*b^3*x*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-
a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3-4*I*b*x^2*ln(1-I*a*exp(I*(c+d*x^(
1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-48*I*b*x*polylog(3,I*a*
exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)+2/5*x^(
5/2)/a^2+48*I*b^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/
a^2/(a^2-b^2)/d^5+48*I*b^2*polylog(4,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)
^(1/2)))/a^2/(a^2-b^2)/d^5+48*I*b^3*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b-(
-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5+96*I*b*polylog(5,I*a*exp(I*(c+d*
x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)+2*I*b^3*x^2*ln(1-I
*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+24*I*b
^3*x*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2
)^(3/2)/d^3+4*I*b*x^2*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a
^2/d/(-a^2+b^2)^(1/2)+48*I*b*x*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+
b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)+8*b^2*x^(3/2)*ln(1+a*exp(I*(c+d*x^(1/
2)))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2+8*b^2*x^(3/2)*ln(1+a*exp(I*(c
+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2-8*b^3*x^(3/2)*polylog
(2,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2+
8*b^3*x^(3/2)*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/
(-a^2+b^2)^(3/2)/d^2+16*b*x^(3/2)*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a
^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)-16*b*x^(3/2)*polylog(2,I*a*exp(I*(
c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/(-a^2+b^2)^(1/2)+48*b^2*polylog
(3,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^4
+48*b^2*polylog(3,-a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))*x^(1/2)/a^
2/(a^2-b^2)/d^4+48*b^3*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/
2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^4-48*b^3*polylog(4,I*a*exp(I*(c+d*x^(1/
2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^4-96*b*polylog(4,
I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^4/(-a^2+b^2)^(
1/2)+96*b*polylog(4,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/
a^2/d^4/(-a^2+b^2)^(1/2)-48*I*b^3*polylog(5,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a
^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^5-96*I*b*polylog(5,I*a*exp(I*(c+d*x^
(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/d^5/(-a^2+b^2)^(1/2)-2*I*b^2*x^2/a^2/(a^2
-b^2)/d
```

Rubi [A] (verified)

Time = 3.55 (sec) , antiderivative size = 1977, normalized size of antiderivative = 1.00, number of steps used = 43, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules

used = {4290, 4276, 3405, 3404, 2296, 2221, 2611, 6744, 2320, 6724, 4617}

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ix^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} + \frac{2ix^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} - \frac{8x^{3/2}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} + \frac{8x^{3/2}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} - \frac{24ix\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{24ix\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{48\sqrt{x}\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4} - \frac{48\sqrt{x}\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^4} + \frac{48i\,\mathrm{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5} - \frac{48i\,\mathrm{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^5} - \frac{2ix^2b^2}{a^2\left(a^2-b^2\right)d} + \frac{8x^{3/2}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{ib-\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} + \frac{8x^{3/2}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{ib+\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} - \frac{24ix\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} - \frac{24ix\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} + \frac{48\sqrt{x}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4} + \frac{48\sqrt{x}\,\mathrm{PolyLog}\left(3,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^4} + \frac{48i\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^5} + \frac{48i\,\mathrm{PolyLog}\left(4,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^5} - \frac{2x^2\cos\left(c+d\sqrt{x}\right)b^2}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)} + \frac{4ix^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d} - \frac{4ix^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d} + \frac{16x^{3/2}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2} - \frac{16x^{3/2}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2} + \frac{48ix\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3} - \frac{48ix\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3} - \frac{96\sqrt{x}\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^4} + \frac{96\sqrt{x}\,\mathrm{PolyLog}\left(4,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^4} - \frac{96i\,\mathrm{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^5} + \frac{96i\,\mathrm{PolyLog}\left(5,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^5} + \frac{2x^{5/2}}{5a^2}$$

```
[In] Int[x^(3/2)/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x^2)/(a^2*(a^2 - b^2)*d) + (2*x^(5/2))/(5*a^2) + (8*b^2*x^(3/2)
*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^
2)*d^2) + (8*b^2*x^(3/2)*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2
- b^2])])/(a^2*(a^2 - b^2)*d^2) - ((2*I)*b^3*x^2*Log[1 - (I*a*E^(I*(c + d*S
qrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((4*I)*b*x^
2*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a
^2 + b^2]*d) + ((2*I)*b^3*x^2*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt
[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((4*I)*b*x^2*Log[1 - (I*a*E^(I
*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((24
*I)*b^2*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])
/(a^2*(a^2 - b^2)*d^3) - ((24*I)*b^2*x*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])
))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (8*b^3*x^(3/2)*PolyLo
g[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)
^(3/2)*d^2) + (16*b*x^(3/2)*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqr
t[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (8*b^3*x^(3/2)*PolyLog[2, (I*
a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d
^2) - (16*b*x^(3/2)*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 +
 b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2) + (48*b^2*Sqrt[x]*PolyLog[3, -((a*E^(I*
(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^4) + (48*b^
2*Sqrt[x]*PolyLog[3, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 - b^2]))])
/(a^2*(a^2 - b^2)*d^4) - ((24*I)*b^3*x*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])
))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((48*I)*b*x*Poly
Log[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2
+ b^2]*d^3) + ((24*I)*b^3*x*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqr
t[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((48*I)*b*x*PolyLog[3, (I*a
*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3)
 + ((48*I)*b^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2
]))])/(a^2*(a^2 - b^2)*d^5) + ((48*I)*b^2*PolyLog[4, -((a*E^(I*(c + d*Sqrt[
x])))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^5) + (48*b^3*Sqrt[x]*Po
lyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 +
b^2)^(3/2)*d^4) - (96*b*Sqrt[x]*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b -
 Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^4) - (48*b^3*Sqrt[x]*PolyLog[4
, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3
/2)*d^4) + (96*b*Sqrt[x]*PolyLog[4, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-
a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^4) + ((48*I)*b^3*PolyLog[5, (I*a*E^(I
*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^5) -
((96*I)*b*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(
a^2*Sqrt[-a^2 + b^2]*d^5) - ((48*I)*b^3*PolyLog[5, (I*a*E^(I*(c + d*Sqrt[x]
)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^5) + ((96*I)*b*PolyL
og[5, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 +
 b^2]*d^5) - (2*b^2*x^2*Cos[c + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a*Sin[c +
 d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4617

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2
] + b*E^(I*(c + d*x)))), x], x] + Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/
(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^4}{(a+b\csc(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^4}{a^2}+\frac{b^2x^4}{a^2(b+a\sin(c+dx))^2}-\frac{2bx^4}{a^2(b+a\sin(c+dx))}\right)\,dx, x, \sqrt{x}\right)\\
&= \frac{2x^{5/2}}{5a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^4}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^4}{(b+a\sin(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}
\end{aligned}
$$

$$= \frac{2x^{5/2}}{5a^2} - \frac{2b^2x^2\cos\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)} - \frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}$$
$$- \frac{\left(2b^3\right)\text{Subst}\left(\int \frac{x^4}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)} + \frac{\left(8b^2\right)\text{Subst}\left(\int \frac{x^3\cos(c+dx)}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}$$

$$= -\frac{2ib^2x^2}{a^2\left(a^2-b^2\right)d} + \frac{2x^{5/2}}{5a^2} - \frac{2b^2x^2\cos\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{\left(4b^3\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}$$
$$+ \frac{(8ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$- \frac{(8ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}}$$
$$+ \frac{\left(8ib^2\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{ib-\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}$$
$$+ \frac{\left(8ib^2\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^3}{ib+\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}$$

$$= -\frac{2ib^2x^2}{a^2(a^2-b^2)d} + \frac{2x^{5/2}}{5a^2} + \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$- \frac{4ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{2b^2x^2\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}$$
$$- \frac{(4ib^3)\,\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$+ \frac{(4ib^3)\,\mathrm{Subst}\left(\int \frac{e^{i(c+dx)}x^4}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}}$$
$$- \frac{(24b^2)\,\mathrm{Subst}\left(\int x^2\log\left(1+\frac{ae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{(24b^2)\,\mathrm{Subst}\left(\int x^2\log\left(1+\frac{ae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2}$$
$$- \frac{(16ib)\mathrm{Subst}\left(\int x^3\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$
$$+ \frac{(16ib)\mathrm{Subst}\left(\int x^3\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$= -\frac{2ib^2x^2}{a^2(a^2-b^2)d} + \frac{2x^{5/2}}{5a^2} + \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{8b^2x^{3/2}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{4ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{4ibx^2\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{24ib^2x\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{24ib^2x\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} + \frac{16bx^{3/2}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{16bx^{3/2}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{2b^2x^2\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(48ib^2)\,\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{(48ib^2)\,\mathrm{Subst}\left(\int x\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{(48b)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(48b)\mathrm{Subst}\left(\int x^2\,\mathrm{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(8ib^3)\,\mathrm{Subst}\left(\int x^3\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{(8ib^3)\,\mathrm{Subst}\left(\int x^3\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

= Too large to display

Mathematica [A] (warning: unable to verify)

Time = 14.05 (sec) , antiderivative size = 2316, normalized size of antiderivative = 1.17

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[x^(3/2)/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] (2*x^(5/2)*Csc[c + d*Sqrt[x]]^2*(b + a*Sin[c + d*Sqrt[x]])^2)/(5*a^2*(a + b
*Csc[c + d*Sqrt[x]])^2) - ((2*I)*b*E^(I*c)*Csc[c + d*Sqrt[x]]^2*(2*b*E^(I*c
)*x^2 - ((-1 + E^((2*I)*c))*((-4*I)*b*d^3*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^(
3/2)*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^
((2*I)*c)])] + (2*I)*a^2*d^4*E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x]))
)/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - I*b^2*d^4*E^(I*c)*x^2*Lo
g[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*
c)])] - (4*I)*b*d^3*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*x^(3/2)*Log[1 + (a*E^(I*(
2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (2*I)*a
^2*d^4*E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[
(a^2 - b^2)*E^((2*I)*c)])] + I*b^2*d^4*E^(I*c)*x^2*Log[1 + (a*E^(I*(2*c + d
*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - 4*d^2*(3*b*Sqr
t[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x
])*x*PolyLog[2, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^
2)*E^((2*I)*c)])] + 4*d^2*(-3*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2*d*E^(
I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*x*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[
x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (24*I)*b*d*Sqrt[(a^
2 - b^2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E
^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (24*I)*a^2*d^2*E^(I*c)*x*PolyL
og[3, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I
)*c)])] - (12*I)*b^2*d^2*E^(I*c)*x*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x])))
/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] - (24*I)*b*d*Sqrt[(a^2 - b^
2)*E^((2*I)*c)]*Sqrt[x]*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*
c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (24*I)*a^2*d^2*E^(I*c)*x*PolyLog[3,
 -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]
))] + (12*I)*b^2*d^2*E^(I*c)*x*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*
b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + 24*b*Sqrt[(a^2 - b^2)*E^((2*
I)*c)]*PolyLog[4, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 -
b^2)*E^((2*I)*c)])] - 48*a^2*d*E^(I*c)*Sqrt[x]*PolyLog[4, (I*a*E^(I*(2*c +
d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 24*b^2*d*E^(I
*c)*Sqrt[x]*PolyLog[4, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a
^2 - b^2)*E^((2*I)*c)])] + 24*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)]*PolyLog[4, -(
(a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))]
 + 48*a^2*d*E^(I*c)*Sqrt[x]*PolyLog[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E
^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - 24*b^2*d*E^(I*c)*Sqrt[x]*PolyLo
g[4, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)
```

```
*c)]))] - (48*I)*a^2*E^(I*c)*PolyLog[5, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^
(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (24*I)*b^2*E^(I*c)*PolyLog[5, (
I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])]
 + (48*I)*a^2*E^(I*c)*PolyLog[5, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c)
 + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] - (24*I)*b^2*E^(I*c)*PolyLog[5, -((a*E^
(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))]))/(d
^4*E^(I*c)*Sqrt[(a^2 - b^2)*E^((2*I)*c)]))*(b + a*Sin[c + d*Sqrt[x]])^2)/(a
^2*(a^2 - b^2)*d*(-1 + E^((2*I)*c))*(a + b*Csc[c + d*Sqrt[x]])^2) + (Csc[c/
2]*Csc[c + d*Sqrt[x]]^2*Sec[c/2]*(b + a*Sin[c + d*Sqrt[x]])*(-(b^3*x^2*Cos[
c]) - a*b^2*x^2*Sin[d*Sqrt[x]]))/(a^2*(-a + b)*(a + b)*d*(a + b*Csc[c + d*S
qrt[x]])^2)
```

Maple [F]

$$\int \frac{x^{\frac{3}{2}}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}}dx$$

```
[In] int(x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

```
[Out] int(x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [F]

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^{2}}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
```

```
[Out] integral(x^(3/2)/(b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2
), x)
```

Sympy [F]

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}}\,dx=\int \frac{x^{\frac{3}{2}}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^{2}}\,dx$$

```
[In] integrate(x**(3/2)/(a+b*csc(c+d*x**(1/2)))**2,x)
```

```
[Out] Integral(x**(3/2)/(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^{\frac{3}{2}}}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(x^(3/2)/(b*csc(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{x^{3/2}}{\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^(3/2)/(a + b/sin(c + d*x^(1/2)))^2,x)
[Out] int(x^(3/2)/(a + b/sin(c + d*x^(1/2)))^2, x)
```

3.68 $\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 1157

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ib^2x}{a^2(a^2-b^2)\,d}+\frac{2x^{3/2}}{3a^2}+\frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2}$$
$$+\frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2}-\frac{2ib^3x\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d}$$
$$+\frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d}+\frac{2ib^3x\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d}$$
$$-\frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d}-\frac{4ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3}$$
$$-\frac{4ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3}$$
$$-\frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2}$$
$$+\frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2}$$
$$+\frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2}$$
$$-\frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2}$$
$$-\frac{4ib^3\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3}$$
$$+\frac{8ib\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^3}$$
$$+\frac{4ib^3\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3}$$
$$-\frac{8ib\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^3}$$
$$-\frac{2b^2x\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)\,d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}$$

```
[Out] -4*I*b^3*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2
+b^2)^(3/2)/d^3+2/3*x^(3/2)/a^2+4*I*b*x*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-
a^2+b^2)^(1/2)))/a^2/d/(-a^2+b^2)^(1/2)-4*I*b^2*polylog(2,-a*exp(I*(c+d*x^(
1/2)))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3-2*I*b^2*x/a^2/(a^2-b^2)/d-2
*I*b^3*x*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)
^(3/2)/d-4*I*b^2*polylog(2,-a*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))/a
^2/(a^2-b^2)/d^3-8*I*b*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/
2)))/a^2/d^3/(-a^2+b^2)^(1/2)-2*b^2*x*cos(c+d*x^(1/2))/a/(a^2-b^2)/d/(b+a*s
in(c+d*x^(1/2)))-4*I*b*x*ln(1-I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2))
)/a^2/d/(-a^2+b^2)^(1/2)+8*I*b*polylog(3,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+
b^2)^(1/2)))/a^2/d^3/(-a^2+b^2)^(1/2)+4*I*b^3*polylog(3,I*a*exp(I*(c+d*x^(1
/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3+2*I*b^3*x*ln(1-I*a*exp
(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d+4*b^2*ln(1+a
*exp(I*(c+d*x^(1/2)))/(I*b-(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/d^2+4*b^
2*ln(1+a*exp(I*(c+d*x^(1/2)))/(I*b+(a^2-b^2)^(1/2)))*x^(1/2)/a^2/(a^2-b^2)/
d^2-4*b^3*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/
a^2/(-a^2+b^2)^(3/2)/d^2+4*b^3*polylog(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+
b^2)^(1/2)))*x^(1/2)/a^2/(-a^2+b^2)^(3/2)/d^2+8*b*polylog(2,I*a*exp(I*(c+d*
x^(1/2)))/(b-(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^2/(-a^2+b^2)^(1/2)-8*b*polylo
g(2,I*a*exp(I*(c+d*x^(1/2)))/(b+(-a^2+b^2)^(1/2)))*x^(1/2)/a^2/d^2/(-a^2+b^
2)^(1/2)
```

Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 1157, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules

used = {4290, 4276, 3405, 3404, 2296, 2221, 2611, 2320, 6724, 4617, 2317, 2438}

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{2ix\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d}+\frac{2ix\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d} - \frac{4\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} + \frac{4\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^2} - \frac{4i\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} + \frac{4i\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b^3}{a^2\left(b^2-a^2\right)^{3/2}d^3} - \frac{2ixb^2}{a^2\left(a^2-b^2\right)d} + \frac{4\sqrt{x}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{ib-\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} + \frac{4\sqrt{x}\log\left(\frac{e^{i(c+d\sqrt{x})}a}{ib+\sqrt{a^2-b^2}}+1\right)b^2}{a^2\left(a^2-b^2\right)d^2} - \frac{4i\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} - \frac{4i\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)b^2}{a^2\left(a^2-b^2\right)d^3} - \frac{2x\cos\left(c+d\sqrt{x}\right)b^2}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)} + \frac{4ix\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d} - \frac{4ix\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d} + \frac{8\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2} - \frac{8\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^2} + \frac{8i\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3} - \frac{8i\,\text{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{b^2-a^2}}\right)b}{a^2\sqrt{b^2-a^2}d^3} + \frac{2x^{3/2}}{3a^2}$$

```
[In] Int[Sqrt[x]/(a + b*Csc[c + d*Sqrt[x]])^2,x]
```

```
[Out] ((-2*I)*b^2*x)/(a^2*(a^2 - b^2)*d) + (2*x^(3/2))/(3*a^2) + (4*b^2*Sqrt[x]*L
og[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)
*d^2) + (4*b^2*Sqrt[x]*Log[1 + (a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a^2 -
b^2])])/(a^2*(a^2 - b^2)*d^2) - ((2*I)*b^3*x*Log[1 - (I*a*E^(I*(c + d*Sqrt[
x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) + ((4*I)*b*x*Log[
1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b
^2]*d) + ((2*I)*b^3*x*Log[1 - (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 +
b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d) - ((4*I)*b*x*Log[1 - (I*a*E^(I*(c + d*Sq
rt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d) - ((4*I)*b^2*Pol
yLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b - Sqrt[a^2 - b^2]))])/(a^2*(a^2 -
b^2)*d^3) - ((4*I)*b^2*PolyLog[2, -((a*E^(I*(c + d*Sqrt[x])))/(I*b + Sqrt[a
^2 - b^2]))])/(a^2*(a^2 - b^2)*d^3) - (4*b^3*Sqrt[x]*PolyLog[2, (I*a*E^(I*(
c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) + (8
*b*Sqrt[x]*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/
(a^2*Sqrt[-a^2 + b^2]*d^2) + (4*b^3*Sqrt[x]*PolyLog[2, (I*a*E^(I*(c + d*Sqr
t[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2) - (8*b*Sqrt[x
]*PolyLog[2, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt
[-a^2 + b^2]*d^2) - ((4*I)*b^3*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b -
Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^3) + ((8*I)*b*PolyLog[3, (I*a
*E^(I*(c + d*Sqrt[x])))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3)
 + ((4*I)*b^3*PolyLog[3, (I*a*E^(I*(c + d*Sqrt[x])))/(b + Sqrt[-a^2 + b^2])
])/(a^2*(-a^2 + b^2)^(3/2)*d^3) - ((8*I)*b*PolyLog[3, (I*a*E^(I*(c + d*Sqrt
[x])))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3) - (2*b^2*x*Cos[c
 + d*Sqrt[x]])/(a*(a^2 - b^2)*d*(b + a*Sin[c + d*Sqrt[x]]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4617

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2
] + b*E^(I*(c + d*x)))), x], x] + Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/
(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= 2\text{Subst}\left(\int \frac{x^2}{(a+b\csc(c+dx))^2}\,dx, x, \sqrt{x}\right)\\
&= 2\text{Subst}\left(\int \left(\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\sin(c+dx))^2}-\frac{2bx^2}{a^2(b+a\sin(c+dx))}\right)dx, x, \sqrt{x}\right)\\
&= \frac{2x^{3/2}}{3a^2}-\frac{(4b)\text{Subst}\left(\int \frac{x^2}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2}+\frac{(2b^2)\,\text{Subst}\left(\int \frac{x^2}{(b+a\sin(c+dx))^2}\,dx, x, \sqrt{x}\right)}{a^2}\\
&= \frac{2x^{3/2}}{3a^2}-\frac{2b^2x\cos\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}-\frac{(8b)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2}\\
&\quad-\frac{(2b^3)\,\text{Subst}\left(\int \frac{x^2}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}+\frac{(4b^2)\,\text{Subst}\left(\int \frac{x\cos(c+dx)}{b+a\sin(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)d}
\end{aligned}
$$

$$= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} - \frac{2b^2x\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)} - \frac{(4b^3)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)} + \frac{(8ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}} - \frac{(8ib)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a\sqrt{-a^2+b^2}} + \frac{(4ib^2)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib-\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d} + \frac{(4ib^2)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(a^2-b^2)d}$$

$$= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} + \frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{2b^2x\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)} - \frac{(4ib^3)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}} + \frac{(4ib^3)\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, \sqrt{x}\right)}{a(-a^2+b^2)^{3/2}} - \frac{(4b^2)\,\text{Subst}\left(\int \log\left(1+\frac{ae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2} - \frac{(4b^2)\,\text{Subst}\left(\int \log\left(1+\frac{ae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2(a^2-b^2)d^2} - \frac{(8ib)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{(8ib)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, \sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d}$$

$$= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{8b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$- \frac{8b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{2b^2x\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)}$$
$$+ \frac{(4ib^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ax}{ib-\sqrt{a^2-b^2}}\right)}{x}dx,x,e^{i(c+d\sqrt{x})}\right)}{a^2(a^2-b^2)d^3}$$
$$+ \frac{(4ib^2)\,\text{Subst}\left(\int\frac{\log\left(1+\frac{ax}{ib+\sqrt{a^2-b^2}}\right)}{x}dx,x,e^{i(c+d\sqrt{x})}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{(8b)\text{Subst}\left(\int\text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(8b)\text{Subst}\left(\int\text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2\sqrt{-a^2+b^2}d^2}$$
$$+ \frac{(4ib^3)\,\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{(4ib^3)\,\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d}$$

$$
\begin{aligned}
&= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} \\
&+ \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&+ \frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&- \frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4ib^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} \\
&- \frac{4ib^2\,\text{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{4b^3\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
&+ \frac{8b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{4b^3\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
&- \frac{8b\sqrt{x}\,\text{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{2b^2x\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)} \\
&+ \frac{(8ib)\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,\frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&- \frac{(8ib)\text{Subst}\left(\int\frac{\text{PolyLog}\left(2,\frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+d\sqrt{x})}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&+ \frac{(4b^3)\,\text{Subst}\left(\int\text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
&- \frac{(4b^3)\,\text{Subst}\left(\int\text{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,\sqrt{x}\right)}{a^2(-a^2+b^2)^{3/2}d^2}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} \\
&+ \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&+ \frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d} \\
&- \frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} \\
&- \frac{4ib^2\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
&+ \frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{4b^3\sqrt{x}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2} \\
&- \frac{8b\sqrt{x}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{8ib\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} \\
&- \frac{8ib\operatorname{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} - \frac{2b^2x\cos\left(c+d\sqrt{x}\right)}{a(a^2-b^2)d\left(b+a\sin\left(c+d\sqrt{x}\right)\right)} \\
&- \frac{(4ib^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+d\sqrt{x})}\right)}{a^2(-a^2+b^2)^{3/2}d^3} \\
&+ \frac{(4ib^3)\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+d\sqrt{x})}\right)}{a^2(-a^2+b^2)^{3/2}d^3}
\end{aligned}
$$

$$= -\frac{2ib^2x}{a^2(a^2-b^2)d} + \frac{2x^{3/2}}{3a^2} + \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2}$$
$$+ \frac{4b^2\sqrt{x}\log\left(1+\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2} - \frac{2ib^3x\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$+ \frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} + \frac{2ib^3x\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d}$$
$$- \frac{4ibx\log\left(1-\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d} - \frac{4ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3}$$
$$- \frac{4ib^2\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+d\sqrt{x})}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^3} - \frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$+ \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} + \frac{4b^3\sqrt{x}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2}$$
$$- \frac{8b\sqrt{x}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2} - \frac{4ib^3\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$+ \frac{8ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} + \frac{4ib^3\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^3}$$
$$- \frac{8ib\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+d\sqrt{x})}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3} - \frac{2b^2x\cos(c+d\sqrt{x})}{a(a^2-b^2)d(b+a\sin(c+d\sqrt{x}))}$$

Mathematica [A] (verified)

Time = 9.71 (sec) , antiderivative size = 846, normalized size of antiderivative = 0.73

$$\int \frac{\sqrt{x}}{(a+b\csc(c+d\sqrt{x}))^2}\,dx$$

$$= \frac{\csc^2(c+d\sqrt{x})(b+a\sin(c+d\sqrt{x}))\left(2x^{3/2}(b+a\sin(c+d\sqrt{x})) - \frac{6ib\left(\frac{2bd^2e^{2ic}x}{-1+e^{2ic}} + \frac{2\left(b\sqrt{(a^2-b^2)e^{2ic}}-2a^2de^{ic}\sqrt{x}+b^2d\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

[In] Integrate[Sqrt[x]/(a + b*Csc[c + d*Sqrt[x]])^2,x]

[Out] (Csc[c + d*Sqrt[x]]^2*(b + a*Sin[c + d*Sqrt[x]])*(2*x^(3/2)*(b + a*Sin[c + d*Sqrt[x]]) - ((6*I)*b*((2*b*d^2*E^((2*I)*c)*x)/(-1 + E^((2*I)*c)) + (2*(b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*PolyLog[2, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 2*(b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I*c)*Sqrt[x] - b^2*d*E^(I*c)*Sqrt[x])*PolyLog[2, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))] + I*(d*Sqrt[x]*((2*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] - 2*a^2*d*E^(I*c)*Sqrt[x] + b^2*d*E^(I*c)*Sqrt[x])*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) - Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + (2*b*Sqrt[(a^2 - b^2)*E^((2*I)*c)] + 2*a^2*d*E^(I*c)*Sqrt[x] - b^2*d*E^(I*c)*Sqrt[x])*Log[1 + (a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)])]) - 2*(2*a^2 - b^2)*E^(I*c)*PolyLog[3, (I*a*E^(I*(2*c + d*Sqrt[x])))/(b*E^(I*c) + I*Sqrt[(a^2 - b^2)*E^((2*I)*c)])] + 2*(2*a^2 - b^2)*E^(I*c)*PolyLog[3, -((a*E^(I*(2*c + d*Sqrt[x])))/(I*b*E^(I*c) + Sqrt[(a^2 - b^2)*E^((2*I)*c)]))])))/Sqrt[(a^2 - b^2)*E^((2*I)*c)])*(b + a*Sin[c + d*Sqrt[x]]))/((a^2 - b^2)*d^3) + (6*b^2*x*Csc[c]*(b*Cos[c] + a*Sin[d*Sqrt[x]]))/((a - b)*(a + b)*d)))/(3*a^2*(a + b*Csc[c + d*Sqrt[x]])^2)

Maple [F]

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}dx$$

[In] int(x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x)

[Out] int(x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x)

Fricas [F]

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{\sqrt{x}}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

[In] integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")

[Out] integral(sqrt(x)/(b^2*csc(d*sqrt(x) + c)^2 + 2*a*b*csc(d*sqrt(x) + c) + a^2), x)

Sympy [F]

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(x**(1/2)/(a+b*csc(c+d*x**(1/2)))**2,x)
```

```
[Out] Integral(sqrt(x)/(a + b*csc(c + d*sqrt(x)))**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [F]

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2}\,dx$$

```
[In] integrate(x^(1/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x)/(b*csc(d*sqrt(x) + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}}{\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \int \frac{\sqrt{x}}{\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(x^(1/2)/(a + b/sin(c + d*x^(1/2)))^2,x)
```

```
[Out] int(x^(1/2)/(a + b/sin(c + d*x^(1/2)))^2, x)
```

3.69 $\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\frac{2\sqrt{x}}{a^2}+\frac{4b(2a^2-b^2)\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}d}-\frac{2b^2\cot\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}$$

```
[Out] 4*b*(2*a^2-b^2)*arctanh((a+b*tan(1/2*c+1/2*d*x^(1/2)))/(a^2-b^2)^(1/2))/a^2
/(a^2-b^2)^(3/2)/d-2*b^2*cot(c+d*x^(1/2))/a/(a^2-b^2)/d/(a+b*csc(c+d*x^(1/2
)))+2*x^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4290, 3870, 4004, 3916, 2739, 632, 212}

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\frac{4b(2a^2-b^2)\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d\left(a^2-b^2\right)^{3/2}}-\frac{2b^2\cot\left(c+d\sqrt{x}\right)}{ad\left(a^2-b^2\right)\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}+\frac{2\sqrt{x}}{a^2}$$

```
[In] Int[1/(Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

```
[Out] (2*Sqrt[x])/a^2 + (4*b*(2*a^2 - b^2)*ArcTanh[(a + b*Tan[(c + d*Sqrt[x])/2])
/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(3/2)*d) - (2*b^2*Cot[c + d*Sqrt[x]])/(
a*(a^2 - b^2)*d*(a + b*Csc[c + d*Sqrt[x]]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
```

```
1)/n], 0] && IntegerQ[p]
```

Rubi steps

$$\text{integral} = 2\text{Subst}\left(\int \frac{1}{(a+b\csc(c+dx))^2}\,dx, x, \sqrt{x}\right)$$

$$= -\frac{2b^2\cot\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\csc\left(c+d\sqrt{x}\right)\right)} - \frac{2\text{Subst}\left(\int \frac{-a^2+b^2+ab\csc(c+dx)}{a+b\csc(c+dx)}\,dx, x, \sqrt{x}\right)}{a\left(a^2-b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} - \frac{2b^2\cot\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\csc\left(c+d\sqrt{x}\right)\right)} - \frac{\left(2b\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{\csc(c+dx)}{a+b\csc(c+dx)}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} - \frac{2b^2\cot\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\csc\left(c+d\sqrt{x}\right)\right)} - \frac{\left(2\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{a\sin(c+dx)}{b}}\,dx, x, \sqrt{x}\right)}{a^2\left(a^2-b^2\right)}$$

$$= \frac{2\sqrt{x}}{a^2} - \frac{2b^2\cot\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\csc\left(c+d\sqrt{x}\right)\right)} - \frac{\left(4\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx, x, \tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{a^2\left(a^2-b^2\right)d}$$

$$= \frac{2\sqrt{x}}{a^2} - \frac{2b^2\cot\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\csc\left(c+d\sqrt{x}\right)\right)} + \frac{\left(8\left(2a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}+2\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{a^2\left(a^2-b^2\right)d}$$

$$= \frac{2\sqrt{x}}{a^2} + \frac{4b\left(2a^2-b^2\right)\text{arctanh}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{3/2}d} - \frac{2b^2\cot\left(c+d\sqrt{x}\right)}{a\left(a^2-b^2\right)d\left(a+b\csc\left(c+d\sqrt{x}\right)\right)}$$

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \frac{2\csc\left(c+d\sqrt{x}\right)\left(\frac{ab^2\cot\left(c+d\sqrt{x}\right)}{(-a+b)(a+b)} + \left(c+d\sqrt{x}\right)\left(a+b\csc\left(c+d\sqrt{x}\right)\right) - \frac{2b\left(-2a^2+b^2\right)\arctan\left(\frac{a+b\tan\left(\frac{1}{2}\left(c+d\sqrt{x}\right)\right)}{\sqrt{-a^2+b^2}}\right)(a+}{\left(-a^2+b^2\right)^{3/2}}\right.}{a^2d\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}$$

```
[In] Integrate[1/(Sqrt[x]*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

```
[Out] (2*Csc[c + d*Sqrt[x]]*((a*b^2*Cot[c + d*Sqrt[x]])/((-a + b)*(a + b)) + (c +
 d*Sqrt[x])*(a + b*Csc[c + d*Sqrt[x]]) - (2*b*(-2*a^2 + b^2)*ArcTan[(a + b*
Tan[(c + d*Sqrt[x])/2])/Sqrt[-a^2 + b^2]]*(a + b*Csc[c + d*Sqrt[x]]))/(-a^2
 + b^2)^(3/2))*(b + a*Sin[c + d*Sqrt[x]]))/(a^2*d*(a + b*Csc[c + d*Sqrt[x]]
)^2)
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.43

method	result	size
derivativedivides	$\frac{-\frac{4b\left(\frac{\frac{a^2\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{2a^2-2b^2}+\frac{ab}{2a^2-2b^2}}{\frac{\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2 b}{2}+a\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+\frac{b}{2}}+\frac{2\left(2a^2-b^2\right)\arctan\left(\frac{2b\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{\left(2a^2-2b^2\right)\sqrt{-a^2+b^2}}\right)}{a^2}+\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a^2}}{d}$	179
default	$\frac{-\frac{4b\left(\frac{\frac{a^2\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)}{2a^2-2b^2}+\frac{ab}{2a^2-2b^2}}{\frac{\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)^2 b}{2}+a\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+\frac{b}{2}}+\frac{2\left(2a^2-b^2\right)\arctan\left(\frac{2b\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{\left(2a^2-2b^2\right)\sqrt{-a^2+b^2}}\right)}{a^2}+\frac{4\arctan\left(\tan\left(\frac{c}{2}+\frac{d\sqrt{x}}{2}\right)\right)}{a^2}}{d}$	179

```
[In] int(1/(a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-2/a^2*b*((1/2*a^2/(a^2-b^2)*tan(1/2*c+1/2*d*x^(1/2))+1/2*a*b/(a^2-b^2
))/(1/2*tan(1/2*c+1/2*d*x^(1/2))^2*b+a*tan(1/2*c+1/2*d*x^(1/2))+1/2*b)+2*(2
*a^2-b^2)/(2*a^2-2*b^2)/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*c+1/2*d*x^
(1/2))+2*a)/(-a^2+b^2)^(1/2)))+2/a^2*arctan(tan(1/2*c+1/2*d*x^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(112) = 224.

Time = 0.30 (sec) , antiderivative size = 576, normalized size of antiderivative = 4.61

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

$$= \left[\frac{2\left(a^5-2a^3b^2+ab^4\right)d\sqrt{x}\sin\left(d\sqrt{x}+c\right)+2\left(a^4b-2a^2b^3+b^5\right)d\sqrt{x}-2\left(a^3b^2-ab^4\right)\cos\left(d\sqrt{x}+c\right)+((2}{(a^7-2a^5b^{}}\right.$$

```
[In] integrate(1/(a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] [(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*sin(d*sqrt(x) + c) + 2*(a^4*b - 2*a
^2*b^3 + b^5)*d*sqrt(x) - 2*(a^3*b^2 - a*b^4)*cos(d*sqrt(x) + c) + ((2*a^3*
b - a*b^3)*sqrt(a^2 - b^2)*sin(d*sqrt(x) + c) + (2*a^2*b^2 - b^4)*sqrt(a^2
- b^2))*log(((a^2 - 2*b^2)*cos(d*sqrt(x) + c)^2 + 2*sqrt(a^2 - b^2)*a*cos(d
*sqrt(x) + c) + a^2 + b^2 + 2*(sqrt(a^2 - b^2)*b*cos(d*sqrt(x) + c) + a*b)*
sin(d*sqrt(x) + c))/(a^2*cos(d*sqrt(x) + c)^2 - 2*a*b*sin(d*sqrt(x) + c) -
a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*sin(d*sqrt(x) + c) + (a^6*b - 2
*a^4*b^3 + a^2*b^5)*d), 2*((a^5 - 2*a^3*b^2 + a*b^4)*d*sqrt(x)*sin(d*sqrt(x
) + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*sqrt(x) + ((2*a^3*b - a*b^3)*sqrt(-a^2
 + b^2)*sin(d*sqrt(x) + c) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2))*arctan(-(s
qrt(-a^2 + b^2)*b*sin(d*sqrt(x) + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*cos
(d*sqrt(x) + c))) - (a^3*b^2 - a*b^4)*cos(d*sqrt(x) + c))/((a^7 - 2*a^5*b^2
 + a^3*b^4)*d*sin(d*sqrt(x) + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/(a+b*csc(c+d*x**(1/2)))**2/x**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x)*(a + b*csc(c + d*sqrt(x)))**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(1/(a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.39

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = -\frac{4\,(2\,a^2b-b^3)\left(\pi\left\lfloor\frac{d\sqrt{x}+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)+a}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4d-a^2b^2d)\sqrt{-a^2+b^2}} - \frac{4\left(ab\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)+b^2\right)}{(a^3d-ab^2d)\left(b\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)^2+2\,a\tan\left(\frac{1}{2}\,d\sqrt{x}+\frac{1}{2}\,c\right)+b\right)} + \frac{2\left(d\sqrt{x}+c\right)}{a^2d}$$

```
[In] integrate(1/(a+b*csc(c+d*x^(1/2)))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] -4*(2*a^2*b - b^3)*(pi*floor(1/2*(d*sqrt(x) + c)/pi + 1/2)*sgn(b) + arctan(
(b*tan(1/2*d*sqrt(x) + 1/2*c) + a)/sqrt(-a^2 + b^2)))/((a^4*d - a^2*b^2*d)*
sqrt(-a^2 + b^2)) - 4*(a*b*tan(1/2*d*sqrt(x) + 1/2*c) + b^2)/((a^3*d - a*b^
2*d)*(b*tan(1/2*d*sqrt(x) + 1/2*c)^2 + 2*a*tan(1/2*d*sqrt(x) + 1/2*c) + b))
 + 2*(d*sqrt(x) + c)/(a^2*d)
```

Mupad [B] (verification not implemented)

Time = 22.78 (sec) , antiderivative size = 2737, normalized size of antiderivative = 21.90

$$\int \frac{1}{\sqrt{x}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx = \text{Too large to display}$$

```
[In] int(1/(x^(1/2)*(a + b/sin(c + d*x^(1/2)))^2),x)
```

```
[Out] - (4*atan((512*a^3*b^3*tan(c/2 + (d*x^(1/2))/2))/((512*a^3*b^9)/(a^6 + a^2*
b^4 - 2*a^4*b^2) - (1536*a^5*b^7)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (1024*a^7*b
^5)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (512*a^9*b^3)/(a^6 + a^2*b^4 - 2*a^4*b^2)
 - (512*a^11*b)/(a^6 + a^2*b^4 - 2*a^4*b^2)) - (512*a*b^5*tan(c/2 + (d*x^(1
/2))/2))/((512*a^3*b^9)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (1536*a^5*b^7)/(a^6 +
 a^2*b^4 - 2*a^4*b^2) + (1024*a^7*b^5)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (512*a
^9*b^3)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (512*a^11*b)/(a^6 + a^2*b^4 - 2*a^4*b
^2)) + (512*a^5*b*tan(c/2 + (d*x^(1/2))/2))/((512*a^3*b^9)/(a^6 + a^2*b^4 -
 2*a^4*b^2) - (1536*a^5*b^7)/(a^6 + a^2*b^4 - 2*a^4*b^2) + (1024*a^7*b^5)/(
a^6 + a^2*b^4 - 2*a^4*b^2) + (512*a^9*b^3)/(a^6 + a^2*b^4 - 2*a^4*b^2) - (5
12*a^11*b)/(a^6 + a^2*b^4 - 2*a^4*b^2))))/(a^2*d) - ((4*b^2)/(a*(a^2 - b^2)
) + (4*b*tan(c/2 + (d*x^(1/2))/2))/(a^2 - b^2))/(d*(b + b*tan(c/2 + (d*x^(1
/2))/2)^2 + 2*a*tan(c/2 + (d*x^(1/2))/2))) - (b*atan(((b*(2*a^2 - b^2)*((a
+ b)^3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x^(1/2))/2)*(8*a*b^7 - 8*a^7*b -
```

```
32*a^3*b^5 + 36*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (32*(4*a*b^6 - 8*a^
3*b^4 + 4*a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (2*b*(2*a^2 - b^2)*((a +
b)^3*(a - b)^3)^(1/2)*((32*(2*a^8*b - 2*a^6*b^3))/(a^6 + a^2*b^4 - 2*a^4*b^
2) + (32*tan(c/2 + (d*x^(1/2))/2)*(4*a^4*b^6 - 12*a^6*b^4 + 8*a^8*b^2))/(a^
7 + a^3*b^4 - 2*a^5*b^2) - (2*b*((32*(a^5*b^6 - 2*a^7*b^4 + a^9*b^2))/(a^6
+ a^2*b^4 - 2*a^4*b^2) + (32*tan(c/2 + (d*x^(1/2))/2)*(3*a^11*b - 2*a^5*b^7
 + 7*a^7*b^5 - 8*a^9*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(2*a^2 - b^2)*((a +
 b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^
2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*2i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)
 - (b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*(4*a*b^6 - 8*a^3*b^4 +
 4*a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) - (32*tan(c/2 + (d*x^(1/2))/2)*(8*
a*b^7 - 8*a^7*b - 32*a^3*b^5 + 36*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (
2*b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((32*(2*a^8*b - 2*a^6*b^3))/(
a^6 + a^2*b^4 - 2*a^4*b^2) + (32*tan(c/2 + (d*x^(1/2))/2)*(4*a^4*b^6 - 12*a
^6*b^4 + 8*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (2*b*((32*(a^5*b^6 - 2*a
^7*b^4 + a^9*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (32*tan(c/2 + (d*x^(1/2))/
2)*(3*a^11*b - 2*a^5*b^7 + 7*a^7*b^5 - 8*a^9*b^3))/(a^7 + a^3*b^4 - 2*a^5*b
^2))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4
- 3*a^6*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*2i)/(a^8 - a^2*b^6
+ 3*a^4*b^4 - 3*a^6*b^2))/((64*(8*b^5 - 16*a^2*b^3))/(a^6 + a^2*b^4 - 2*a^4
*b^2) + (64*tan(c/2 + (d*x^(1/2))/2)*(16*b^6 - 48*a^2*b^4 + 32*a^4*b^2))/(a
^7 + a^3*b^4 - 2*a^5*b^2) + (2*b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*
((32*tan(c/2 + (d*x^(1/2))/2)*(8*a*b^7 - 8*a^7*b - 32*a^3*b^5 + 36*a^5*b^3)
)/(a^7 + a^3*b^4 - 2*a^5*b^2) - (32*(4*a*b^6 - 8*a^3*b^4 + 4*a^5*b^2))/(a^6
 + a^2*b^4 - 2*a^4*b^2) + (2*b*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*((
32*(2*a^8*b - 2*a^6*b^3))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (32*tan(c/2 + (d*x^
(1/2))/2)*(4*a^4*b^6 - 12*a^6*b^4 + 8*a^8*b^2))/(a^7 + a^3*b^4 - 2*a^5*b^2)
 - (2*b*((32*(a^5*b^6 - 2*a^7*b^4 + a^9*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) +
 (32*tan(c/2 + (d*x^(1/2))/2)*(3*a^11*b - 2*a^5*b^7 + 7*a^7*b^5 - 8*a^9*b^3
))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2))/
(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^
6*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) + (2*b*(2*a^2 - b^2)*((a +
 b)^3*(a - b)^3)^(1/2)*((32*(4*a*b^6 - 8*a^3*b^4 + 4*a^5*b^2))/(a^6 + a^2*b
^4 - 2*a^4*b^2) - (32*tan(c/2 + (d*x^(1/2))/2)*(8*a*b^7 - 8*a^7*b - 32*a^3*
b^5 + 36*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (2*b*(2*a^2 - b^2)*((a + b
)^3*(a - b)^3)^(1/2)*((32*(2*a^8*b - 2*a^6*b^3))/(a^6 + a^2*b^4 - 2*a^4*b^2
) + (32*tan(c/2 + (d*x^(1/2))/2)*(4*a^4*b^6 - 12*a^6*b^4 + 8*a^8*b^2))/(a^7
 + a^3*b^4 - 2*a^5*b^2) + (2*b*((32*(a^5*b^6 - 2*a^7*b^4 + a^9*b^2))/(a^6 +
 a^2*b^4 - 2*a^4*b^2) + (32*tan(c/2 + (d*x^(1/2))/2)*(3*a^11*b - 2*a^5*b^7
+ 7*a^7*b^5 - 8*a^9*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2))*(2*a^2 - b^2)*((a +
b)^3*(a - b)^3)^(1/2))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2
*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*(
2*a^2 - b^2)*((a + b)^3*(a - b)^3)^(1/2)*4i)/(d*(a^8 - a^2*b^6 + 3*a^4*b^4
- 3*a^6*b^2))
```

3.70 $\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])^2),x]
```

```
[Out] Defer[Int][1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{3/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 50.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{3/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2} \, dx=\int \frac{1}{x^{3/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2} \, dx$$

```
[In] Integrate[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^(3/2)*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{3}{2}} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^{2}}d x$$

```
[In] int(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
[Out] int(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{3/2} \left(a+b \csc \left(c+d \sqrt{x}\right)\right)^2} \, dx=\int \frac{1}{\left(b \csc \left(d \sqrt{x}+c\right)+a\right)^{2} x^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(sqrt(x)/(b^2*x^2*csc(d*sqrt(x) + c)^2 + 2*a*b*x^2*csc(d*sqrt(x) +
c) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 5.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} \left(a+b\csc \left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{\frac{3}{2}} \left(a+b\csc \left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x**(3/2)/(a+b*csc(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**(3/2)*(a + b*csc(c + d*sqrt(x)))**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{x^{3/2} \left(a+b\csc \left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{3/2} \left(a+b\csc \left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc \left(d\sqrt{x}+c\right)+a\right)^2 x^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x^(3/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*csc(d*sqrt(x) + c) + a)^2*x^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 17.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3/2} \left(a+b\csc \left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{3/2} \left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(1/(x^(3/2)*(a + b/sin(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^(3/2)*(a + b/sin(c + d*x^(1/2)))^2), x)
```

3.71 $\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] Int[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])^2),x]
[Out] Defer[Int][1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 57.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^{5/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right)^2} \, dx = \int \frac{1}{x^{5/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right)^2} \, dx$$

```
[In] Integrate[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])^2),x]
[Out] Integrate[1/(x^(5/2)*(a + b*Csc[c + d*Sqrt[x]])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^{\frac{5}{2}} \left(a + b \csc \left(c + d\sqrt{x}\right)\right)^2} dx$$

```
[In] int(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
[Out] int(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{1}{x^{5/2} \left(a + b \csc \left(c + d\sqrt{x}\right)\right)^2} \, dx = \int \frac{1}{\left(b \csc \left(d\sqrt{x} + c\right) + a\right)^2 x^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="fricas")
[Out] integral(sqrt(x)/(b^2*x^3*csc(d*sqrt(x) + c)^2 + 2*a*b*x^3*csc(d*sqrt(x) +
c) + a^2*x^3), x)
```

Sympy [N/A]

Not integrable

Time = 39.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{\frac{5}{2}}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx$$

```
[In] integrate(1/x**(5/2)/(a+b*csc(c+d*x**(1/2)))**2,x)
[Out] Integral(1/(x**(5/2)*(a + b*csc(c + d*sqrt(x)))**2), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="maxima")
[Out] Timed out
```

Giac [N/A]

Not integrable

Time = 1.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{\left(b\csc\left(d\sqrt{x}+c\right)+a\right)^2x^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x^(5/2)/(a+b*csc(c+d*x^(1/2)))^2,x, algorithm="giac")
[Out] integrate(1/((b*csc(d*sqrt(x) + c) + a)^2*x^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 18.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{5/2}\left(a+b\csc\left(c+d\sqrt{x}\right)\right)^2}\,dx=\int \frac{1}{x^{5/2}\left(a+\frac{b}{\sin\left(c+d\sqrt{x}\right)}\right)^2}\,dx$$

```
[In] int(1/(x^(5/2)*(a + b/sin(c + d*x^(1/2)))^2),x)
[Out] int(1/(x^(5/2)*(a + b/sin(c + d*x^(1/2)))^2), x)
```

3.72 $\int (ex)^m (a+b\csc(c+dx^n))^p \, dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a+b\csc(c+dx^n))^p \, dx = x^{-m}(ex)^m \text{Int}(x^m(a+b\csc(c+dx^n))^p, x)$$

```
[Out] (e*x)^m*Unintegrable(x^m*(a+b*csc(c+d*x^n))^p,x)/(x^m)
```

Rubi [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a+b\csc(c+dx^n))^p \, dx = \int (ex)^m (a+b\csc(c+dx^n))^p \, dx$$

```
[In] Int[(e*x)^m*(a + b*Csc[c + d*x^n])^p,x]
```

```
[Out] ((e*x)^m*Defer[Int][x^m*(a + b*Csc[c + d*x^n])^p, x])/x^m
```

Rubi steps

$$\text{integral} = \left(x^{-m}(ex)^m\right) \int x^m(a+b\csc(c+dx^n))^p \, dx$$

Mathematica [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\csc(c + dx^n))^p \, dx = \int (ex)^m (a + b\csc(c + dx^n))^p \, dx$$

```
[In] Integrate[(e*x)^m*(a + b*Csc[c + d*x^n])^p,x]
[Out] Integrate[(e*x)^m*(a + b*Csc[c + d*x^n])^p, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a + b\csc(c + d\,x^n))^p dx$$

```
[In] int((e*x)^m*(a+b*csc(c+d*x^n))^p,x)
[Out] int((e*x)^m*(a+b*csc(c+d*x^n))^p,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\csc(c + dx^n))^p \, dx = \int (ex)^m (b\csc(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*csc(c+d*x^n))^p,x, algorithm="fricas")
[Out] integral((e*x)^m*(b*csc(d*x^n + c) + a)^p, x)
```

Sympy [N/A]

Not integrable

Time = 21.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a + b\csc(c + dx^n))^p \, dx = \int (ex)^m (a + b\csc(c + dx^n))^p \, dx$$

```
[In] integrate((e*x)**m*(a+b*csc(c+d*x**n))**p,x)
[Out] Integral((e*x)**m*(a + b*csc(c + d*x**n))**p, x)
```

Maxima [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\csc(c + dx^n))^p \, dx = \int (ex)^m (b\csc(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*csc(c+d*x^n))^p,x, algorithm="maxima")
[Out] integrate((e*x)^m*(b*csc(d*x^n + c) + a)^p, x)
```

Giac [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b\csc(c + dx^n))^p \, dx = \int (ex)^m (b\csc(dx^n + c) + a)^p \, dx$$

```
[In] integrate((e*x)^m*(a+b*csc(c+d*x^n))^p,x, algorithm="giac")
[Out] integrate((e*x)^m*(b*csc(d*x^n + c) + a)^p, x)
```

Mupad [N/A]

Not integrable

Time = 17.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (ex)^m (a + b\csc(c + dx^n))^p \, dx = \int \left(a + \frac{b}{\sin(c + d\,x^n)}\right)^p (e\,x)^m \, dx$$

```
[In] int((a + b/sin(c + d*x^n))^p*(e*x)^m,x)
[Out] int((a + b/sin(c + d*x^n))^p*(e*x)^m, x)
```

3.73 $\int (ex)^{-1+n} (a + b\csc(c + dx^n))\, dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int (ex)^{-1+n} (a + b\csc(c + dx^n))\, dx = \frac{a(ex)^n}{en} - \frac{bx^{-n}(ex)^n \operatorname{arctanh}(\cos(c + dx^n))}{den}$$

```
[Out] a*(e*x)^n/e/n-b*(e*x)^n*arctanh(cos(c+d*x^n))/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {14, 4294, 4290, 3855}

$$\int (ex)^{-1+n} (a + b\csc(c + dx^n))\, dx = \frac{a(ex)^n}{en} - \frac{bx^{-n}(ex)^n \operatorname{arctanh}(\cos(c + dx^n))}{den}$$

```
[In] Int[(e*x)^(-1 + n)*(a + b*Csc[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^n)/(e*n) - (b*(e*x)^n*ArcTanh[Cos[c + d*x^n]])/(d*e*n*x^n)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(a(ex)^{-1+n}+b(ex)^{-1+n}\csc\left(c+dx^n\right)\right)\,dx\\
&= \frac{a(ex)^n}{en}+b\int (ex)^{-1+n}\csc\left(c+dx^n\right)\,dx\\
&= \frac{a(ex)^n}{en}+\frac{\left(bx^{-n}(ex)^n\right)\int x^{-1+n}\csc\left(c+dx^n\right)\,dx}{e}\\
&= \frac{a(ex)^n}{en}+\frac{\left(bx^{-n}(ex)^n\right)\operatorname{Subst}\left(\int \csc(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a(ex)^n}{en}-\frac{bx^{-n}(ex)^n\operatorname{arctanh}(\cos\left(c+dx^n\right))}{den}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.36

$$\int (ex)^{-1+n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx = \frac{x^{-n}(ex)^n\left(a(c+dx^n)-b\log\left(\cos\left(\frac{1}{2}(c+dx^n)\right)\right)+b\log\left(\sin\left(\frac{1}{2}(c+dx^n)\right)\right)\right)}{den}$$

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Csc[c + d*x^n]),x]
[Out] ((e*x)^n*(a*(c + d*x^n) - b*Log[Cos[(c + d*x^n)/2]] + b*Log[Sin[(c + d*x^n)
/2]]))/(d*e*n*x^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.01 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.51

method	result
risch	$\frac{ax\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2\pi-i\,\mathrm{csgn}(iex)^3\pi+2\ln(e)+2\ln(x)\right)}{2}}}{n}-\frac{2\,\mathrm{arctanh}\left(\mathrm{e}^{i(c\cdots}\right.}{\cdots}$

[In] int((e*x)^(-1+n)*(a+b*csc(c+d*x^n)),x,method=_RETURNVERBOSE)

[Out] a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*ln(x)))-2*arctanh(exp(I*(c+d*x^n)))/d/e*e^n/n*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

$$\int (ex)^{-1+n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx = \frac{2\,ade^{n-1}x^n-be^{n-1}\log\left(\frac{1}{2}\,\cos\left(dx^n+c\right)+\frac{1}{2}\right)+be^{n-1}\log\left(-\frac{1}{2}\,\cos\left(dx^n+c\right)+\frac{1}{2}\right)}{2\,dn}$$

[In] integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n)),x, algorithm="fricas")

[Out] 1/2*(2*a*d*e^(n - 1)*x^n - b*e^(n - 1)*log(1/2*cos(d*x^n + c) + 1/2) + b*e^(n - 1)*log(-1/2*cos(d*x^n + c) + 1/2))/(d*n)

Sympy [F]

$$\int (ex)^{-1+n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx=\int (ex)^{n-1}\left(a+b\csc\left(c+dx^n\right)\right)\,dx$$

[In] integrate((e*x)**(-1+n)*(a+b*csc(c+d*x**n)),x)

[Out] Integral((e*x)**(n - 1)*(a + b*csc(c + d*x**n)), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(45) = 90.

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.84

$$\int (ex)^{-1+n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx=\frac{(ex)^n a}{en}-\frac{\left(e^n\log\left(\cos\left(dx^n\right)^2+2\,\cos\left(dx^n\right)\cos\left(c\right)+\cos\left(c\right)^2+\sin\left(dx^n\right)^2-2\,\sin\left(dx^n\right)\sin\left(c\right)+\sin\left(c\right)^2\right)-e^n\log\left(\right.\right.}{2\,den}$$

```
[In] integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n)),x, algorithm="maxima")
```

```
[Out] (e*x)^n*a/(e*n) - 1/2*(e^n*log(cos(d*x^n)^2 + 2*cos(d*x^n)*cos(c) + cos(c)^
2 + sin(d*x^n)^2 - 2*sin(d*x^n)*sin(c) + sin(c)^2) - e^n*log(cos(d*x^n)^2 -
 2*cos(d*x^n)*cos(c) + cos(c)^2 + sin(d*x^n)^2 + 2*sin(d*x^n)*sin(c) + sin(
c)^2))*b/(d*e*n)
```

Giac [F]

$$\int (ex)^{-1+n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx=\int (b\csc\left(dx^n+c\right)+a)(ex)^{n-1}\,dx$$

```
[In] integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^n + c) + a)*(e*x)^(n - 1), x)
```

Mupad [B] (verification not implemented)

Time = 20.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.36

$$\int (ex)^{-1+n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx=\frac{(e\,x)^n\left(a\,d\,x^n+b\,\ln\left(b\,(e\,x)^{n-1}\,2\mathrm{i}-b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\,2\mathrm{i}\right)-b\,\ln\left(-b\,(e\,x)^{n-1}\,2\mathrm{i}-b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\,2\mathrm{i}\right)\right.}{d\,e\,n\,x^n}$$

```
[In] int((a + b/sin(c + d*x^n))*(e*x)^(n - 1),x)
```

```
[Out] ((e*x)^n*(b*log(b*(e*x)^(n - 1)*2i - b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1
)*2i) - b*log(- b*(e*x)^(n - 1)*2i - b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1
)*2i) + a*d*x^n))/(d*e*n*x^n)
```

3.74 $\int (ex)^{-1+2n} (a + b\csc(c + dx^n))\, dx$

Optimal result

Integrand size = 22, antiderivative size = 141

$$\int (ex)^{-1+2n} (a + b\csc(c + dx^n))\, dx = \frac{a(ex)^{2n}}{2en} - \frac{2bx^{-n}(ex)^{2n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} + \frac{ibx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2, -e^{i(c+dx^n)}\right)}{d^2en} - \frac{ibx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2, e^{i(c+dx^n)}\right)}{d^2en}$$

```
[Out] 1/2*a*(e*x)^(2*n)/e/n-2*b*(e*x)^(2*n)*arctanh(exp(I*(c+d*x^n)))/d/e/n/(x^n)
+I*b*(e*x)^(2*n)*polylog(2,-exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-I*b*(e*x)^(
2*n)*polylog(2,exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {14, 4294, 4290, 4268, 2317, 2438}

$$\int (ex)^{-1+2n} (a + b\csc(c + dx^n))\, dx = \frac{a(ex)^{2n}}{2en} - \frac{2bx^{-n}(ex)^{2n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} + \frac{ibx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2, -e^{i(dx^n+c)}\right)}{d^2en} - \frac{ibx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2, e^{i(dx^n+c)}\right)}{d^2en}$$

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Csc[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^(2*n))/(2*e*n) - (2*b*(e*x)^(2*n)*ArcTanh[E^(I*(c + d*x^n))])/(d*e
*n*x^n) + (I*b*(e*x)^(2*n)*PolyLog[2, -E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*n)
) - (I*b*(e*x)^(2*n)*PolyLog[2, E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \int \left(a(ex)^{-1+2n} + b(ex)^{-1+2n}\csc\left(c+dx^n\right)\right)\, dx$$

$$
\begin{aligned}
&= \frac{a(ex)^{2n}}{2en} + b\int (ex)^{-1+2n}\csc\left(c+dx^n\right)\,dx \\
&= \frac{a(ex)^{2n}}{2en} + \frac{\left(bx^{-2n}(ex)^{2n}\right)\int x^{-1+2n}\csc\left(c+dx^n\right)\,dx}{e} \\
&= \frac{a(ex)^{2n}}{2en} + \frac{\left(bx^{-2n}(ex)^{2n}\right)\mathrm{Subst}\left(\int x\csc(c+dx)\,dx, x, x^n\right)}{en} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2bx^{-n}(ex)^{2n}\mathrm{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad - \frac{\left(bx^{-2n}(ex)^{2n}\right)\mathrm{Subst}\left(\int \log\left(1-e^{i(c+dx)}\right)\,dx, x, x^n\right)}{den} \\
&\quad + \frac{\left(bx^{-2n}(ex)^{2n}\right)\mathrm{Subst}\left(\int \log\left(1+e^{i(c+dx)}\right)\,dx, x, x^n\right)}{den} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2bx^{-n}(ex)^{2n}\mathrm{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad + \frac{\left(ibx^{-2n}(ex)^{2n}\right)\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{i(c+dx^n)}\right)}{d^2en} \\
&\quad - \frac{\left(ibx^{-2n}(ex)^{2n}\right)\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{i(c+dx^n)}\right)}{d^2en} \\
&= \frac{a(ex)^{2n}}{2en} - \frac{2bx^{-n}(ex)^{2n}\mathrm{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad + \frac{ibx^{-2n}(ex)^{2n}\,\mathrm{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} - \frac{ibx^{-2n}(ex)^{2n}\,\mathrm{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.31

$$
\int (ex)^{-1+2n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx
$$

$$
= \frac{x^{-2n}(ex)^{2n}\left(ad^2x^{2n} + 2bc\log\left(1-e^{i(c+dx^n)}\right) + 2bdx^n\log\left(1-e^{i(c+dx^n)}\right) - 2bc\log\left(1+e^{i(c+dx^n)}\right) - 2bdx^n\,\mathrm{l}\right.}{2d^2}
$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Csc[c + d*x^n]),x]
```

```
[Out] ((e*x)^(2*n)*(a*d^2*x^(2*n) + 2*b*c*Log[1 - E^(I*(c + d*x^n))] + 2*b*d*x^n*
Log[1 - E^(I*(c + d*x^n))] - 2*b*c*Log[1 + E^(I*(c + d*x^n))] - 2*b*d*x^n*L
og[1 + E^(I*(c + d*x^n))] - 2*b*c*Log[Tan[(c + d*x^n)/2]] + (2*I)*b*PolyLog
[2, -E^(I*(c + d*x^n))] - (2*I)*b*PolyLog[2, E^(I*(c + d*x^n))]))/(2*d^2*e*
n*x^(2*n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.17 (sec) , antiderivative size = 699, normalized size of antiderivative = 4.96

method	result	size
risch	Expression too large to display	699

```
[In] int((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a/n*x*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*
e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)
+2*ln(x)))+1/d/n/e*(e^n)^2*b*ln(1-exp(I*(c+d*x^n)))*x^n*(-1)^(1/2*csgn(I*e)
*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e*x)^2*n+2*csgn
(I*e)*csgn(I*e*x)*n+2*csgn(I*x)*csgn(I*e*x)*n-2*csgn(I*e)*csgn(I*x)*n+csgn(
I*e*x)^2-csgn(I*e)*csgn(I*e*x)-csgn(I*x)*csgn(I*e*x)))-1/d/n/e*(e^n)^2*b*ln
(exp(I*(c+d*x^n))+1)*x^n*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2
*I*Pi*csgn(I*e*x)*(-2*csgn(I*e*x)^2*n+2*csgn(I*e)*csgn(I*e*x)*n+2*csgn(I*x)
*csgn(I*e*x)*n-2*csgn(I*e)*csgn(I*x)*n+csgn(I*e*x)^2-csgn(I*e)*csgn(I*e*x)-
csgn(I*x)*csgn(I*e*x)))-I/d^2/n/e*(e^n)^2*b*dilog(1-exp(I*(c+d*x^n)))*(-1)^
(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e
*x)^2*n+2*csgn(I*e)*csgn(I*e*x)*n+2*csgn(I*x)*csgn(I*e*x)*n-2*csgn(I*e)*csg
n(I*x)*n+csgn(I*e*x)^2-csgn(I*e)*csgn(I*e*x)-csgn(I*x)*csgn(I*e*x)))+I/d^2/
n/e*(e^n)^2*b*dilog(exp(I*(c+d*x^n))+1)*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(
I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e*x)^2*n+2*csgn(I*e)*csgn(I*e*x
)*n+2*csgn(I*x)*csgn(I*e*x)*n-2*csgn(I*e)*csgn(I*x)*n+csgn(I*e*x)^2-csgn(I*
e)*csgn(I*e*x)-csgn(I*x)*csgn(I*e*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(131) = 262.

Time = 0.27 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.72

$$\int (ex)^{-1+2n} (a + b\csc(c + dx^n)) \, dx$$
$$= \frac{ad^2e^{2\,n-1}x^{2\,n} - bde^{2\,n-1}x^n \log(\cos(dx^n + c) + i\,\sin(dx^n + c) + 1) - bde^{2\,n-1}x^n \log(\cos(dx^n + c) - i\,\sin(c\cdots}{}$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d^2*e^(2*n - 1)*x^(2*n) - b*d*e^(2*n - 1)*x^n*log(cos(d*x^n + c) + I
*sin(d*x^n + c) + 1) - b*d*e^(2*n - 1)*x^n*log(cos(d*x^n + c) - I*sin(d*x^n
 + c) + 1) - b*c*e^(2*n - 1)*log(-1/2*cos(d*x^n + c) + 1/2*I*sin(d*x^n + c)
 + 1/2) - b*c*e^(2*n - 1)*log(-1/2*cos(d*x^n + c) - 1/2*I*sin(d*x^n + c) +
```

```
1/2) - I*b*e^(2*n - 1)*dilog(cos(d*x^n + c) + I*sin(d*x^n + c)) + I*b*e^(2*
n - 1)*dilog(cos(d*x^n + c) - I*sin(d*x^n + c)) - I*b*e^(2*n - 1)*dilog(-co
s(d*x^n + c) + I*sin(d*x^n + c)) + I*b*e^(2*n - 1)*dilog(-cos(d*x^n + c) -
I*sin(d*x^n + c)) + (b*d*e^(2*n - 1)*x^n + b*c*e^(2*n - 1))*log(-cos(d*x^n
+ c) + I*sin(d*x^n + c) + 1) + (b*d*e^(2*n - 1)*x^n + b*c*e^(2*n - 1))*log(
-cos(d*x^n + c) - I*sin(d*x^n + c) + 1))/(d^2*n)
```

Sympy [F]

$$\int (ex)^{-1+2n}(a+b\csc(c+dx^n))\,dx=\int (ex)^{2n-1}(a+b\csc(c+dx^n))\,dx$$

```
[In] integrate((e*x)**(-1+2*n)*(a+b*csc(c+d*x**n)),x)
```

```
[Out] Integral((e*x)**(2*n - 1)*(a + b*csc(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+2n}(a+b\csc(c+dx^n))\,dx=\int (b\csc(dx^n+c)+a)(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n)),x, algorithm="maxima")
```

```
[Out] (e^(2*n + 1)*integrate(x^(2*n)*sin(d*x^n + c)/(e^2*x*cos(d*x^n + c)^2 + e^2
*x*sin(d*x^n + c)^2 + 2*e^2*x*cos(d*x^n + c) + e^2*x), x) + e^(2*n + 1)*int
egrate(x^(2*n)*sin(d*x^n + c)/(e^2*x*cos(d*x^n + c)^2 + e^2*x*sin(d*x^n + c
)^2 - 2*e^2*x*cos(d*x^n + c) + e^2*x), x))*b + 1/2*(e*x)^(2*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+2n}(a+b\csc(c+dx^n))\,dx=\int (b\csc(dx^n+c)+a)(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n)),x, algorithm="giac")
```

```
[Out] integrate((b*csc(d*x^n + c) + a)*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}(a+b\csc(c+dx^n))\,dx=\int \left(a+\frac{b}{\sin(c+d\,x^n)}\right)(e\,x)^{2\,n-1}\,dx$$

```
[In] int((a + b/sin(c + d*x^n))*(e*x)^(2*n - 1),x)
[Out] int((a + b/sin(c + d*x^n))*(e*x)^(2*n - 1), x)
```

3.75 $\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx$

Optimal result

Integrand size = 22, antiderivative size = 221

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx = \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} + \frac{2ibx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en} - \frac{2bx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-e^{i(c+dx^n)}\right)}{d^3en} + \frac{2bx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,e^{i(c+dx^n)}\right)}{d^3en}$$

```
[Out] 1/3*a*(e*x)^(3*n)/e/n-2*b*(e*x)^(3*n)*arctanh(exp(I*(c+d*x^n)))/d/e/n/(x^n)
+2*I*b*(e*x)^(3*n)*polylog(2,-exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-2*I*b*(e*
x)^(3*n)*polylog(2,exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-2*b*(e*x)^(3*n)*poly
log(3,-exp(I*(c+d*x^n)))/d^3/e/n/(x^(3*n))+2*b*(e*x)^(3*n)*polylog(3,exp(I*
(c+d*x^n)))/d^3/e/n/(x^(3*n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used

$= \{14, 4294, 4290, 4268, 2611, 2320, 6724\}$

$$\int (ex)^{-1+3n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx = \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\operatorname{arctanh}\left(e^{i(c+dx^n)}\right)}{den} - \frac{2bx^{-3n}(ex)^{3n}\,\operatorname{PolyLog}\left(3,-e^{i(dx^n+c)}\right)}{d^3en} + \frac{2bx^{-3n}(ex)^{3n}\,\operatorname{PolyLog}\left(3,e^{i(dx^n+c)}\right)}{d^3en} + \frac{2ibx^{-2n}(ex)^{3n}\,\operatorname{PolyLog}\left(2,-e^{i(dx^n+c)}\right)}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\,\operatorname{PolyLog}\left(2,e^{i(dx^n+c)}\right)}{d^2en}$$

```
[In] Int[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n]),x]
```

```
[Out] (a*(e*x)^(3*n))/(3*e*n) - (2*b*(e*x)^(3*n)*ArcTanh[E^(I*(c + d*x^n))])/(d*e
*n*x^n) + ((2*I)*b*(e*x)^(3*n)*PolyLog[2, -E^(I*(c + d*x^n))])/(d^2*e*n*x^(
2*n)) - ((2*I)*b*(e*x)^(3*n)*PolyLog[2, E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*n
)) - (2*b*(e*x)^(3*n)*PolyLog[3, -E^(I*(c + d*x^n))])/(d^3*e*n*x^(3*n)) + (
2*b*(e*x)^(3*n)*PolyLog[3, E^(I*(c + d*x^n))])/(d^3*e*n*x^(3*n))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(a(ex)^{-1+3n} + b(ex)^{-1+3n}\csc\left(c+dx^n\right)\right)\,dx \\
&= \frac{a(ex)^{3n}}{3en} + b\int (ex)^{-1+3n}\csc\left(c+dx^n\right)\,dx \\
&= \frac{a(ex)^{3n}}{3en} + \frac{\left(bx^{-3n}(ex)^{3n}\right)\int x^{-1+3n}\csc\left(c+dx^n\right)\,dx}{e} \\
&= \frac{a(ex)^{3n}}{3en} + \frac{\left(bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x^2\csc(c+dx)\,dx,x,x^n\right)}{en} \\
&= \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad - \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1-e^{i(c+dx)}\right)\,dx,x,x^n\right)}{den} \\
&\quad + \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1+e^{i(c+dx)}\right)\,dx,x,x^n\right)}{den}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad + \frac{2ibx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en} \\
&\quad - \frac{(2ibx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{i(c+dx)}\right)\,dx,x,x^n\right)}{d^2en} \\
&\quad + \frac{(2ibx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \text{PolyLog}\left(2,e^{i(c+dx)}\right)\,dx,x,x^n\right)}{d^2en} \\
&= \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad + \frac{2ibx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en} \\
&\quad - \frac{(2bx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{d^3en} \\
&\quad + \frac{(2bx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{d^3en} \\
&= \frac{a(ex)^{3n}}{3en} - \frac{2bx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad + \frac{2ibx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} - \frac{2ibx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en} \\
&\quad - \frac{2bx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-e^{i(c+dx^n)}\right)}{d^3en} + \frac{2bx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,e^{i(c+dx^n)}\right)}{d^3en}
\end{aligned}
$$

Mathematica [F]

$$
\int (ex)^{-1+3n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx = \int (ex)^{-1+3n}\left(a+b\csc\left(c+dx^n\right)\right)\,dx
$$

```
[In] Integrate[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n]),x]
[Out] Integrate[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n]), x]
```

Maple [F]

$$\int (ex)^{-1+3n} (a + b\csc (c + d\,x^{n}))\,dx$$

```
[In] int((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n)),x)
```

```
[Out] int((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n)),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(211) = 422.

Time = 0.28 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.52

$$\int (ex)^{-1+3n} (a + b\csc (c + dx^{n}))\ dx$$
$$= \frac{2\,ad^{3}e^{3\,n-1}x^{3\,n} - 3\,bd^{2}e^{3\,n-1}x^{2\,n}\log\left(\cos\left(dx^{n}+c\right)+i\,\sin\left(dx^{n}+c\right)+1\right) - 3\,bd^{2}e^{3\,n-1}x^{2\,n}\log\left(\cos\left(dx^{n}+c\right.\right.}{}$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*d^3*e^(3*n - 1)*x^(3*n) - 3*b*d^2*e^(3*n - 1)*x^(2*n)*log(cos(d*x^
n + c) + I*sin(d*x^n + c) + 1) - 3*b*d^2*e^(3*n - 1)*x^(2*n)*log(cos(d*x^n
+ c) - I*sin(d*x^n + c) + 1) - 6*I*b*d*e^(3*n - 1)*x^n*dilog(cos(d*x^n + c)
 + I*sin(d*x^n + c)) + 6*I*b*d*e^(3*n - 1)*x^n*dilog(cos(d*x^n + c) - I*sin
(d*x^n + c)) - 6*I*b*d*e^(3*n - 1)*x^n*dilog(-cos(d*x^n + c) + I*sin(d*x^n
+ c)) + 6*I*b*d*e^(3*n - 1)*x^n*dilog(-cos(d*x^n + c) - I*sin(d*x^n + c)) +
 3*b*c^2*e^(3*n - 1)*log(-1/2*cos(d*x^n + c) + 1/2*I*sin(d*x^n + c) + 1/2)
+ 3*b*c^2*e^(3*n - 1)*log(-1/2*cos(d*x^n + c) - 1/2*I*sin(d*x^n + c) + 1/2)
 + 6*b*e^(3*n - 1)*polylog(3, cos(d*x^n + c) + I*sin(d*x^n + c)) + 6*b*e^(3
*n - 1)*polylog(3, cos(d*x^n + c) - I*sin(d*x^n + c)) - 6*b*e^(3*n - 1)*pol
ylog(3, -cos(d*x^n + c) + I*sin(d*x^n + c)) - 6*b*e^(3*n - 1)*polylog(3, -c
os(d*x^n + c) - I*sin(d*x^n + c)) + 3*(b*d^2*e^(3*n - 1)*x^(2*n) - b*c^2*e^
(3*n - 1))*log(-cos(d*x^n + c) + I*sin(d*x^n + c) + 1) + 3*(b*d^2*e^(3*n -
1)*x^(2*n) - b*c^2*e^(3*n - 1))*log(-cos(d*x^n + c) - I*sin(d*x^n + c) + 1)
)/(d^3*n)
```

Sympy [F]

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx=\int (ex)^{3n-1}(a+b\csc(c+dx^n))\,dx$$

```
[In] integrate((e*x)**(-1+3*n)*(a+b*csc(c+d*x**n)),x)
[Out] Integral((e*x)**(3*n - 1)*(a + b*csc(c + d*x**n)), x)
```

Maxima [F]

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx=\int (b\csc(dx^n+c)+a)(ex)^{3n-1}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n)),x, algorithm="maxima")
[Out] (e^(3*n + 1)*integrate(x^(3*n)*sin(d*x^n + c)/(e^2*x*cos(d*x^n + c)^2 + e^2
*x*sin(d*x^n + c)^2 + 2*e^2*x*cos(d*x^n + c) + e^2*x), x) + e^(3*n + 1)*int
egrate(x^(3*n)*sin(d*x^n + c)/(e^2*x*cos(d*x^n + c)^2 + e^2*x*sin(d*x^n + c
)^2 - 2*e^2*x*cos(d*x^n + c) + e^2*x), x))*b + 1/3*(e*x)^(3*n)*a/(e*n)
```

Giac [F]

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx=\int (b\csc(dx^n+c)+a)(ex)^{3n-1}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n)),x, algorithm="giac")
[Out] integrate((b*csc(d*x^n + c) + a)*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))\,dx=\int \left(a+\frac{b}{\sin(c+d\,x^n)}\right)(e\,x)^{3\,n-1}\,dx$$

```
[In] int((a + b/sin(c + d*x^n))*(e*x)^(3*n - 1),x)
[Out] int((a + b/sin(c + d*x^n))*(e*x)^(3*n - 1), x)
```

3.76 $\int (ex)^{-1+n} (a + b \csc (c + dx^n))^2 \, dx$

Optimal result

Integrand size = 22, antiderivative size = 80

$$\int (ex)^{-1+n} (a + b \csc (c + dx^n))^2 \, dx = \frac{a^2 (ex)^n}{en} - \frac{2abx^{-n}(ex)^n \text{arctanh}(\cos (c + dx^n))}{den} - \frac{b^2 x^{-n} (ex)^n \cot (c + dx^n)}{den}$$

```
[Out] a^2*(e*x)^n/e/n-2*a*b*(e*x)^n*arctanh(cos(c+d*x^n))/d/e/n/(x^n)-b^2*(e*x)^n
*cot(c+d*x^n)/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4294, 4290, 3858, 3855, 3852, 8}

$$\int (ex)^{-1+n} (a + b \csc (c + dx^n))^2 \, dx = \frac{a^2 (ex)^n}{en} - \frac{2abx^{-n}(ex)^n \text{arctanh}(\cos (c + dx^n))}{den} - \frac{b^2 x^{-n} (ex)^n \cot (c + dx^n)}{den}$$

```
[In] Int[(e*x)^(-1 + n)*(a + b*Csc[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^n)/(e*n) - (2*a*b*(e*x)^n*ArcTanh[Cos[c + d*x^n]])/(d*e*n*x^n) -
 (b^2*(e*x)^n*Cot[c + d*x^n])/(d*e*n*x^n)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3858

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[a^2*x, x] +
 (Dist[2*a*b, Int[Csc[c + d*x], x], x] + Dist[b^2, Int[Csc[c + d*x]^2, x],
x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(x^{-n}(ex)^n)\int x^{-1+n}(a+b\csc(c+dx^n))^2\,dx}{e} \\
&= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int (a+b\csc(c+dx))^2\,dx,x,x^n\right)}{en} \\
&= \frac{a^2(ex)^n}{en}+\frac{(2abx^{-n}(ex)^n)\,\text{Subst}\left(\int \csc(c+dx)\,dx,x,x^n\right)}{en} \\
&\quad+\frac{(b^2x^{-n}(ex)^n)\,\text{Subst}\left(\int \csc^2(c+dx)\,dx,x,x^n\right)}{en} \\
&= \frac{a^2(ex)^n}{en}-\frac{2abx^{-n}(ex)^n\text{arctanh}(\cos(c+dx^n))}{den}-\frac{(b^2x^{-n}(ex)^n)\,\text{Subst}\left(\int 1\,dx,x,\cot(c+dx^n)\right)}{den} \\
&= \frac{a^2(ex)^n}{en}-\frac{2abx^{-n}(ex)^n\text{arctanh}(\cos(c+dx^n))}{den}-\frac{b^2x^{-n}(ex)^n\cot(c+dx^n)}{den}
\end{aligned}$$

Mathematica [A] (verified)

Time = 2.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.28

$$\int (ex)^{-1+n} (a + b\csc (c + dx^n))^2 \, dx$$
$$= \frac{x^{-n}(ex)^n \left(-b^2 \cot \left(\frac{1}{2}(c + dx^n)\right) + 2a\left(ac + adx^n - 2b\log \left(\cos \left(\frac{1}{2}(c + dx^n)\right)\right) + 2b\log \left(\sin \left(\frac{1}{2}(c + dx^n)\right)\right)\right)}{2den}$$

```
[In] Integrate[(e*x)^(-1 + n)*(a + b*Csc[c + d*x^n])^2,x]
```

```
[Out] ((e*x)^n*(-(b^2*Cot[(c + d*x^n)/2]) + 2*a*(a*c + a*d*x^n - 2*b*Log[Cos[(c +
 d*x^n)/2]] + 2*b*Log[Sin[(c + d*x^n)/2]]) + b^2*Tan[(c + d*x^n)/2]))/(2*d*
e*n*x^n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.42 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.44

method	result
risch	$\frac{a^2 x \,\mathrm{e}^{\frac{(-1+n)\left(-i \,\mathrm{csgn}(ie) \,\mathrm{csgn}(ix) \,\mathrm{csgn}(iex)\pi + i \,\mathrm{csgn}(ie) \,\mathrm{csgn}(iex)^2\pi + i \,\mathrm{csgn}(ix) \,\mathrm{csgn}(iex)^2\pi - i \,\mathrm{csgn}(iex)^3\pi + 2\ln(e) + 2\ln(x)\right)}{2}}}{n} - 2ix\,b^2 \mathrm{e}^{\frac{(-1+n)}{}}$

```
[In] int((e*x)^(-1+n)*(a+b*csc(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^2/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*c
sgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*l
n(x)))-2*I*x*b^2*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*cs
gn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*
ln(e)+2*ln(x)))/d/n/(x^n)/(exp(2*I*(c+d*x^n))-1)-4*arctanh(exp(I*(c+d*x^n))
)/d/e*e^n/n*a*b*exp(1/2*I*Pi*csgn(I*e*x)*(-1+n)*(csgn(I*e*x)-csgn(I*x))*(-c
sgn(I*e*x)+csgn(I*e)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.45

$$\int (ex)^{-1+n} (a + b\csc (c + dx^n))^2 \, dx$$
$$= \frac{a^2de^{n-1}x^n \sin (dx^n + c) - abe^{n-1} \log \left(\frac{1}{2}\, \cos (dx^n + c) + \frac{1}{2}\right) \sin (dx^n + c) + abe^{n-1} \log \left(-\frac{1}{2}\, \cos (dx^n + c) + \right.}{dn \sin (dx^n + c)}$$

```
[In] integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] (a^2*d*e^(n - 1)*x^n*sin(d*x^n + c) - a*b*e^(n - 1)*log(1/2*cos(d*x^n + c)
+ 1/2)*sin(d*x^n + c) + a*b*e^(n - 1)*log(-1/2*cos(d*x^n + c) + 1/2)*sin(d*
x^n + c) - b^2*e^(n - 1)*cos(d*x^n + c))/(d*n*sin(d*x^n + c))
```

Sympy [F]

$$\int (ex)^{-1+n} (a + b\csc (c + dx^n))^2 \, dx = \int (ex)^{n-1} (a + b\csc (c + dx^n))^2 \, dx$$

```
[In] integrate((e*x)**(-1+n)*(a+b*csc(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(n - 1)*(a + b*csc(c + d*x**n))**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(80) = 160$.

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.59

$$\int (ex)^{-1+n} (a + b\csc (c + dx^n))^2 \, dx = -\frac{2\, b^2 e^n \sin (2\, dx^n + 2\, c)}{den \cos (2\, dx^n + 2\, c)^2 + den \sin (2\, dx^n + 2\, c)^2 - 2\, den \cos (2\, dx^n + 2\, c) + den} + \frac{(ex)^n\, a^2}{en} - \frac{\left(e^n \log \left(\cos (dx^n)^2 + 2\, \cos (dx^n) \cos (c) + \cos (c)^2 + \sin (dx^n)^2 - 2\, \sin (dx^n) \sin (c) + \sin (c)^2\right) - e^n \log (\cdots}{den}$$

```
[In] integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] -2*b^2*e^n*sin(2*d*x^n + 2*c)/(d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x
^n + 2*c)^2 - 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n) + (e*x)^n*a^2/(e*n) - (e^
n*log(cos(d*x^n)^2 + 2*cos(d*x^n)*cos(c) + cos(c)^2 + sin(d*x^n)^2 - 2*sin(
d*x^n)*sin(c) + sin(c)^2) - e^n*log(cos(d*x^n)^2 - 2*cos(d*x^n)*cos(c) + co
s(c)^2 + sin(d*x^n)^2 + 2*sin(d*x^n)*sin(c) + sin(c)^2))*a*b/(d*e*n)
```

Giac [F]

$$\int (ex)^{-1+n}(a+b\csc(c+dx^n))^2\,dx = \int (b\csc(dx^n+c)+a)^2(ex)^{n-1}\,dx$$

[In] integrate((e*x)^(-1+n)*(a+b*csc(c+d*x^n))^2,x, algorithm="giac")

[Out] integrate((b*csc(d*x^n + c) + a)^2*(e*x)^(n - 1), x)

Mupad [B] (verification not implemented)

Time = 20.61 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.28

$$\begin{aligned}&\int (ex)^{-1+n}(a+b\csc(c+dx^n))^2\,dx\\&=\frac{a^2\,x\,(e\,x)^{n-1}}{n}-\frac{b^2\,x\,(e\,x)^{n-1}\,2\mathrm{i}}{d\,n\,x^n\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x^n\,2\mathrm{i}}-1\right)}\\&\quad-\frac{2\,a\,b\,x\,\ln\left(-a\,b\,(e\,x)^{n-1}\,4\mathrm{i}-a\,b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\,4\mathrm{i}\right)\,(e\,x)^{n-1}}{d\,n\,x^n}\\&\quad+\frac{2\,a\,b\,x\,\ln\left(a\,b\,(e\,x)^{n-1}\,4\mathrm{i}-a\,b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\,4\mathrm{i}\right)\,(e\,x)^{n-1}}{d\,n\,x^n}\end{aligned}$$

[In] int((a + b/sin(c + d*x^n))^2*(e*x)^(n - 1),x)

[Out] (a^2*x*(e*x)^(n - 1))/n - (b^2*x*(e*x)^(n - 1)*2i)/(d*n*x^n*(exp(c*2i + d*x^n*2i) - 1)) - (2*a*b*x*log(- a*b*(e*x)^(n - 1)*4i - a*b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1)*4i)*(e*x)^(n - 1))/(d*n*x^n) + (2*a*b*x*log(a*b*(e*x)^(n - 1)*4i - a*b*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1)*4i)*(e*x)^(n - 1))/(d*n*x^n)

3.77 $\int (ex)^{-1+2n}(a+b\csc(c+dx^n))^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 214

$$\int (ex)^{-1+2n}(a+b\csc(c+dx^n))^2\,dx = \frac{a^2(ex)^{2n}}{2en} - \frac{4abx^{-n}(ex)^{2n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} - \frac{b^2x^{-n}(ex)^{2n}\cot(c+dx^n)}{den} + \frac{b^2x^{-2n}(ex)^{2n}\log(\sin(c+dx^n))}{d^2en} + \frac{2iabx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en}$$

```
[Out] 1/2*a^2*(e*x)^(2*n)/e/n-4*a*b*(e*x)^(2*n)*arctanh(exp(I*(c+d*x^n)))/d/e/n/(
x^n)-b^2*(e*x)^(2*n)*cot(c+d*x^n)/d/e/n/(x^n)+b^2*(e*x)^(2*n)*ln(sin(c+d*x^
n))/d^2/e/n/(x^(2*n))+2*I*a*b*(e*x)^(2*n)*polylog(2,-exp(I*(c+d*x^n)))/d^2/
e/n/(x^(2*n))-2*I*a*b*(e*x)^(2*n)*polylog(2,exp(I*(c+d*x^n)))/d^2/e/n/(x^(2
*n))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used

= {4294, 4290, 4275, 4268, 2317, 2438, 4269, 3556}

$$\int (ex)^{-1+2n} (a + b\csc(c + dx^n))^2 \, dx = \frac{a^2(ex)^{2n}}{2en} - \frac{4abx^{-n}(ex)^{2n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} + \frac{2iabx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2, -e^{i(dx^n+c)}\right)}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2, e^{i(dx^n+c)}\right)}{d^2en} + \frac{b^2x^{-2n}(ex)^{2n}\log(\sin(c + dx^n))}{d^2en} - \frac{b^2x^{-n}(ex)^{2n}\cot(c + dx^n)}{den}$$

```
[In] Int[(e*x)^(-1 + 2*n)*(a + b*Csc[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^(2*n))/(2*e*n) - (4*a*b*(e*x)^(2*n)*ArcTanh[E^(I*(c + d*x^n))])/
(d*e*n*x^n) - (b^2*(e*x)^(2*n)*Cot[c + d*x^n])/(d*e*n*x^n) + (b^2*(e*x)^(2*
n)*Log[Sin[c + d*x^n]])/(d^2*e*n*x^(2*n)) + ((2*I)*a*b*(e*x)^(2*n)*PolyLog[
2, -E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*n)) - ((2*I)*a*b*(e*x)^(2*n)*PolyLog[
2, E^(I*(c + d*x^n))])/(d^2*e*n*x^(2*n))
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^{-2n}(ex)^{2n}\right)\int x^{-1+2n}(a+b\csc\left(c+dx^n\right))^2\,dx}{e}\\
&= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int x(a+b\csc(c+dx))^2\,dx,x,x^n\right)}{en}\\
&= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\left(a^2x+2abx\csc(c+dx)+b^2x\csc^2(c+dx)\right)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^{2n}}{2en}+\frac{\left(2abx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int x\csc(c+dx)\,dx,x,x^n\right)}{en}\\
&\quad+\frac{\left(b^2x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int x\csc^2(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^{2n}}{2en}-\frac{4abx^{-n}(ex)^{2n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den}-\frac{b^2x^{-n}(ex)^{2n}\cot\left(c+dx^n\right)}{den}\\
&\quad-\frac{\left(2abx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\log\left(1-e^{i(c+dx)}\right)\,dx,x,x^n\right)}{den}\\
&\quad+\frac{\left(2abx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\log\left(1+e^{i(c+dx)}\right)\,dx,x,x^n\right)}{den}\\
&\quad+\frac{\left(b^2x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int\cot(c+dx)\,dx,x,x^n\right)}{den}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{a^2(ex)^{2n}}{2en} - \frac{4abx^{-n}(ex)^{2n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad - \frac{b^2x^{-n}(ex)^{2n}\cot(c+dx^n)}{den} + \frac{b^2x^{-2n}(ex)^{2n}\log(\sin(c+dx^n))}{d^2en} \\
&\quad + \frac{(2iabx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{i(c+dx^n)}\right)}{d^2en} \\
&\quad - \frac{(2iabx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{i(c+dx^n)}\right)}{d^2en} \\
&= \frac{a^2(ex)^{2n}}{2en} - \frac{4abx^{-n}(ex)^{2n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad - \frac{b^2x^{-n}(ex)^{2n}\cot(c+dx^n)}{den} + \frac{b^2x^{-2n}(ex)^{2n}\log(\sin(c+dx^n))}{d^2en} \\
&\quad + \frac{2iabx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2, -e^{i(c+dx^n)}\right)}{d^2en} - \frac{2iabx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2, e^{i(c+dx^n)}\right)}{d^2en}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 7.77 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.34

$$
\int (ex)^{-1+2n}\left(a+b\csc(c+dx^n)\right)^2\,dx
$$

$$
= \frac{x^{-2n}(ex)^{2n}\left(2b^2dx^n\cot(c)+dx^n(a^2dx^n-2b^2\cot(c))-2b^2(dx^n\cot(c)-\log(\sin(c+dx^n)))+4ab\left(2\,\text{arc}\right.\right.}{}
$$

```
[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Csc[c + d*x^n])^2,x]
```

```
[Out] ((e*x)^(2*n)*(2*b^2*d*x^n*Cot[c] + d*x^n*(a^2*d*x^n - 2*b^2*Cot[c]) - 2*b^2
*(d*x^n*Cot[c] - Log[Sin[c + d*x^n]]) + 4*a*b*(2*ArcTan[Tan[c]]*ArcTanh[Cos
[c] - Sin[c]*Tan[(d*x^n)/2]] + (((d*x^n + ArcTan[Tan[c]])*(Log[1 - E^(I*(d*
x^n + ArcTan[Tan[c]]))] - Log[1 + E^(I*(d*x^n + ArcTan[Tan[c]]))]) + I*Poly
Log[2, -E^(I*(d*x^n + ArcTan[Tan[c]]))] - I*PolyLog[2, E^(I*(d*x^n + ArcTan
[Tan[c]]))])*Sec[c])/Sqrt[Sec[c]^2]) + b^2*d*x^n*Csc[c/2]*Csc[(c + d*x^n)/2
]*Sin[(d*x^n)/2] + b^2*d*x^n*Sec[c/2]*Sec[(c + d*x^n)/2]*Sin[(d*x^n)/2]))/(
2*d^2*e*n*x^(2*n))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.80 (sec) , antiderivative size = 970, normalized size of antiderivative = 4.53

method	result	size
risch	Expression too large to display	970

```
[In] int((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^2/n*x*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(
I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(
e)+2*ln(x)))-2*I*x*b^2*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)
*Pi+I*csgn(I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)
^3*Pi+2*ln(e)+2*ln(x)))/d/n/(x^n)/(exp(2*I*(c+d*x^n))-1)+1/n/d^2*b^2*(e^n)^
2/e*exp(1/2*I*Pi*csgn(I*e*x)*(-1+2*n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)
+csgn(I*e)))*ln(exp(2*I*(c+d*x^n))-1)-2/n/d^2*b^2*(e^n)^2/e*exp(1/2*I*Pi*cs
gn(I*e*x)*(-1+2*n)*(csgn(I*e*x)-csgn(I*x))*(-csgn(I*e*x)+csgn(I*e)))*ln(exp
(I*x^n*d))+2/n/d*b*a/e*(e^n)^2*ln(1-exp(I*(c+d*x^n)))*x^n*(-1)^(1/2*csgn(I*
e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e*x)^2*n+2*cs
gn(I*e)*csgn(I*e*x)*n+2*csgn(I*x)*csgn(I*e*x)*n-2*csgn(I*e)*csgn(I*x)*n+csg
n(I*e*x)^2-csgn(I*e)*csgn(I*e*x)-csgn(I*x)*csgn(I*e*x)))-2/n/d*b*a/e*(e^n)^
2*ln(exp(I*(c+d*x^n))+1)*x^n*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp
(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e*x)^2*n+2*csgn(I*e)*csgn(I*e*x)*n+2*csgn(
I*x)*csgn(I*e*x)*n-2*csgn(I*e)*csgn(I*x)*n+csgn(I*e*x)^2-csgn(I*e)*csgn(I*e
*x)-csgn(I*x)*csgn(I*e*x)))-2*I/n/d^2*b*a/e*(e^n)^2*dilog(1-exp(I*(c+d*x^n)
))*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*
csgn(I*e*x)^2*n+2*csgn(I*e)*csgn(I*e*x)*n+2*csgn(I*x)*csgn(I*e*x)*n-2*csgn(
I*e)*csgn(I*x)*n+csgn(I*e*x)^2-csgn(I*e)*csgn(I*e*x)-csgn(I*x)*csgn(I*e*x))
)+2*I/n/d^2*b*a/e*(e^n)^2*dilog(exp(I*(c+d*x^n))+1)*(-1)^(1/2*csgn(I*e)*csg
n(I*x)*csgn(I*e*x))*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e*x)^2*n+2*csgn(I*e
)*csgn(I*e*x)*n+2*csgn(I*x)*csgn(I*e*x)*n-2*csgn(I*e)*csgn(I*x)*n+csgn(I*e*
x)^2-csgn(I*e)*csgn(I*e*x)-csgn(I*x)*csgn(I*e*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 568 vs. 2(206) = 412.

Time = 0.28 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.65

$$\int (ex)^{-1+2n} (a+b\csc (c+dx^n))^2 \, dx$$

$$= \frac{a^2d^2e^{2n-1}x^{2n}\sin (dx^n+c) - 2b^2de^{2n-1}x^n\cos (dx^n+c) - 2i\,abe^{2n-1}\mathrm{Li}_2(\cos (dx^n+c)+i\,\sin (dx^n+c))\,\mathrm{si}}{}$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*d^2*e^(2*n - 1)*x^(2*n)*sin(d*x^n + c) - 2*b^2*d*e^(2*n - 1)*x^n*c
os(d*x^n + c) - 2*I*a*b*e^(2*n - 1)*dilog(cos(d*x^n + c) + I*sin(d*x^n + c)
)*sin(d*x^n + c) + 2*I*a*b*e^(2*n - 1)*dilog(cos(d*x^n + c) - I*sin(d*x^n +
 c))*sin(d*x^n + c) - 2*I*a*b*e^(2*n - 1)*dilog(-cos(d*x^n + c) + I*sin(d*x
^n + c))*sin(d*x^n + c) + 2*I*a*b*e^(2*n - 1)*dilog(-cos(d*x^n + c) - I*sin
(d*x^n + c))*sin(d*x^n + c) - (2*a*b*c - b^2)*e^(2*n - 1)*log(-1/2*cos(d*x^
n + c) + 1/2*I*sin(d*x^n + c) + 1/2)*sin(d*x^n + c) - (2*a*b*c - b^2)*e^(2*
n - 1)*log(-1/2*cos(d*x^n + c) - 1/2*I*sin(d*x^n + c) + 1/2)*sin(d*x^n + c)
 - (2*a*b*d*e^(2*n - 1)*x^n - b^2*e^(2*n - 1))*log(cos(d*x^n + c) + I*sin(d
*x^n + c) + 1)*sin(d*x^n + c) - (2*a*b*d*e^(2*n - 1)*x^n - b^2*e^(2*n - 1))
*log(cos(d*x^n + c) - I*sin(d*x^n + c) + 1)*sin(d*x^n + c) + 2*(a*b*d*e^(2*
n - 1)*x^n + a*b*c*e^(2*n - 1))*log(-cos(d*x^n + c) + I*sin(d*x^n + c) + 1)
*sin(d*x^n + c) + 2*(a*b*d*e^(2*n - 1)*x^n + a*b*c*e^(2*n - 1))*log(-cos(d*
x^n + c) - I*sin(d*x^n + c) + 1)*sin(d*x^n + c))/(d^2*n*sin(d*x^n + c))
```

Sympy [F]

$$\int (ex)^{-1+2n} (a + b\csc(c + dx^n))^2 \, dx = \int (ex)^{2n-1} (a + b\csc(c + dx^n))^2 \, dx$$

```
[In] integrate((e*x)**(-1+2*n)*(a+b*csc(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(2*n - 1)*(a + b*csc(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+2n} (a + b\csc(c + dx^n))^2 \, dx = \int (b\csc(dx^n + c) + a)^2 (ex)^{2n-1} \, dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e*x)^(2*n)*a^2/(e*n) - (2*b^2*e^(2*n)*x^n*sin(2*d*x^n + 2*c) - (d*e*n*
cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 - 2*d*e*n*cos(2*d*x^n + 2
*c) + d*e*n)*integrate((2*a*b*d*e^(2*n)*x^(2*n) - b^2*e^(2*n)*x^n)*sin(d*x^
n + c)/(d*e*x*cos(d*x^n + c)^2 + d*e*x*sin(d*x^n + c)^2 + 2*d*e*x*cos(d*x^n
 + c) + d*e*x), x) - (d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)
^2 - 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n)*integrate((2*a*b*d*e^(2*n)*x^(2*n)
 + b^2*e^(2*n)*x^n)*sin(d*x^n + c)/(d*e*x*cos(d*x^n + c)^2 + d*e*x*sin(d*x^
n + c)^2 - 2*d*e*x*cos(d*x^n + c) + d*e*x), x))/(d*e*n*cos(2*d*x^n + 2*c)^2
 + d*e*n*sin(2*d*x^n + 2*c)^2 - 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n)
```

Giac [F]

$$\int (ex)^{-1+2n}\left(a+b\csc\left(c+dx^n\right)\right)^2\,dx=\int \left(b\csc\left(dx^n+c\right)+a\right)^2(ex)^{2n-1}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((b*csc(d*x^n + c) + a)^2*(e*x)^(2*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+2n}\left(a+b\csc\left(c+dx^n\right)\right)^2\,dx=\int \left(a+\frac{b}{\sin\left(c+d\,x^n\right)}\right)^2(e\,x)^{2\,n-1}\,dx$$

```
[In] int((a + b/sin(c + d*x^n))^2*(e*x)^(2*n - 1),x)
[Out] int((a + b/sin(c + d*x^n))^2*(e*x)^(2*n - 1), x)
```

3.78 $\int (ex)^{-1+3n} (a+b\csc(c+dx^n))^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 377

$$\begin{aligned}\int (ex)^{-1+3n} (a+b\csc(c+dx^n))^2 \, dx = {} & \frac{a^2(ex)^{3n}}{3en} - \frac{ib^2x^{-n}(ex)^{3n}}{den} \\ & - \frac{4abx^{-n}(ex)^{3n}\operatorname{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\ & - \frac{b^2x^{-n}(ex)^{3n}\cot(c+dx^n)}{den} \\ & + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1-e^{2i(c+dx^n)}\right)}{d^2en} \\ & + \frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} \\ & - \frac{4iabx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en} \\ & - \frac{ib^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,e^{2i(c+dx^n)}\right)}{d^3en} \\ & - \frac{4abx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,-e^{i(c+dx^n)}\right)}{d^3en} \\ & + \frac{4abx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,e^{i(c+dx^n)}\right)}{d^3en}\end{aligned}$$

```
[Out] 1/3*a^2*(e*x)^(3*n)/e/n-I*b^2*(e*x)^(3*n)/d/e/n/(x^n)-4*a*b*(e*x)^(3*n)*arc
tanh(exp(I*(c+d*x^n)))/d/e/n/(x^n)-b^2*(e*x)^(3*n)*cot(c+d*x^n)/d/e/n/(x^n)
+2*b^2*(e*x)^(3*n)*ln(1-exp(2*I*(c+d*x^n)))/d^2/e/n/(x^(2*n))+4*I*a*b*(e*x)
^(3*n)*polylog(2,-exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-4*I*a*b*(e*x)^(3*n)*p
olylog(2,exp(I*(c+d*x^n)))/d^2/e/n/(x^(2*n))-I*b^2*(e*x)^(3*n)*polylog(2,ex
p(2*I*(c+d*x^n)))/d^3/e/n/(x^(3*n))-4*a*b*(e*x)^(3*n)*polylog(3,-exp(I*(c+d
*x^n)))/d^3/e/n/(x^(3*n))+4*a*b*(e*x)^(3*n)*polylog(3,exp(I*(c+d*x^n)))/d^3
/e/n/(x^(3*n))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4294, 4290, 4275, 4268, 2611, 2320, 6724, 4269, 3798, 2221, 2317, 2438}

$$\int (ex)^{-1+3n}(a+b\csc(c+dx^n))^2\,dx = \frac{a^2(ex)^{3n}}{3en} - \frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} - \frac{4abx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,-e^{i(dx^n+c)}\right)}{d^3en} + \frac{4abx^{-3n}(ex)^{3n}\,\text{PolyLog}\left(3,e^{i(dx^n+c)}\right)}{d^3en} + \frac{4iabx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,-e^{i(dx^n+c)}\right)}{d^2en} - \frac{4iabx^{-2n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{i(dx^n+c)}\right)}{d^2en} - \frac{ib^2x^{-3n}(ex)^{3n}\,\text{PolyLog}\left(2,e^{2i(dx^n+c)}\right)}{d^3en} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1-e^{2i(c+dx^n)}\right)}{d^2en} - \frac{b^2x^{-n}(ex)^{3n}\cot(c+dx^n)}{den} - \frac{ib^2x^{-n}(ex)^{3n}}{den}$$

```
[In] Int[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n])^2,x]
```

```
[Out] (a^2*(e*x)^(3*n))/(3*e*n) - (I*b^2*(e*x)^(3*n))/(d*e*n*x^n) - (4*a*b*(e*x)^
(3*n)*ArcTanh[E^(I*(c + d*x^n))])/(d*e*n*x^n) - (b^2*(e*x)^(3*n)*Cot[c + d*
x^n])/(d*e*n*x^n) + (2*b^2*(e*x)^(3*n)*Log[1 - E^((2*I)*(c + d*x^n))])/(d^2
*e*n*x^(2*n)) + ((4*I)*a*b*(e*x)^(3*n)*PolyLog[2, -E^(I*(c + d*x^n))])/(d^2
*e*n*x^(2*n)) - ((4*I)*a*b*(e*x)^(3*n)*PolyLog[2, E^(I*(c + d*x^n))])/(d^2*
e*n*x^(2*n)) - (I*b^2*(e*x)^(3*n)*PolyLog[2, E^((2*I)*(c + d*x^n))])/(d^3*e
*n*x^(3*n)) - (4*a*b*(e*x)^(3*n)*PolyLog[3, -E^(I*(c + d*x^n))])/(d^3*e*n*x
^(3*n)) + (4*a*b*(e*x)^(3*n)*PolyLog[3, E^(I*(c + d*x^n))])/(d^3*e*n*x^(3*n
))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m
*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4268

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Dist[d*(m/f), Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[d*(m/f), Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4275

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Csc[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{-3n}(ex)^{3n}\right)\int x^{-1+3n}\left(a+b\csc\left(c+dx^n\right)\right)^2\,dx}{e}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x^2(a+b\csc(c+dx))^2\,dx,x,x^n\right)}{en}\\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int\left(a^2x^2+2abx^2\csc(c+dx)+b^2x^2\csc^2(c+dx)\right)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^{3n}}{3en}+\frac{\left(2abx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x^2\csc(c+dx)\,dx,x,x^n\right)}{en}\\
&\quad+\frac{\left(b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x^2\csc^2(c+dx)\,dx,x,x^n\right)}{en}\\
&= \frac{a^2(ex)^{3n}}{3en}-\frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den}-\frac{b^2x^{-n}(ex)^{3n}\cot\left(c+dx^n\right)}{den}\\
&\quad-\frac{\left(4abx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1-e^{i(c+dx)}\right)\,dx,x,x^n\right)}{den}\\
&\quad+\frac{\left(4abx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1+e^{i(c+dx)}\right)\,dx,x,x^n\right)}{den}\\
&\quad+\frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\cot(c+dx)\,dx,x,x^n\right)}{den}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a^2(ex)^{3n}}{3en} - \frac{ib^2x^{-n}(ex)^{3n}}{den} - \frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad - \frac{b^2x^{-n}(ex)^{3n}\cot(c+dx^n)}{den} + \frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} \\
&\quad - \frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en} \\
&\quad - \frac{(4iabx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{i(c+dx)}\right)\,dx,x,x^n\right)}{d^2en} \\
&\quad + \frac{(4iabx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \text{PolyLog}\left(2,e^{i(c+dx)}\right)\,dx,x,x^n\right)}{d^2en} \\
&\quad - \frac{(4ib^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{2i(c+dx)}x}{1-e^{2i(c+dx)}}\,dx,x,x^n\right)}{den}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a^2(ex)^{3n}}{3en} - \frac{ib^2x^{-n}(ex)^{3n}}{den} - \frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} \\
&\quad - \frac{b^2x^{-n}(ex)^{3n}\cot(c+dx^n)}{den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1-e^{2i(c+dx^n)}\right)}{d^2en} \\
&\quad + \frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} - \frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en} \\
&\quad - \frac{(4abx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{d^3en} \\
&\quad + \frac{(4abx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{\text{PolyLog}(2,x)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{d^3en} \\
&\quad - \frac{(2b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \log\left(1-e^{2i(c+dx)}\right)\,dx,x,x^n\right)}{d^2en}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a^2(ex)^{3n}}{3en} - \frac{ib^2x^{-n}(ex)^{3n}}{den} - \frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} - \frac{b^2x^{-n}(ex)^{3n}\cot(c+dx^n)}{den} \\
&\quad + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1-e^{2i(c+dx^n)}\right)}{d^2en} + \frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} \\
&\quad - \frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en} - \frac{4abx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,-e^{i(c+dx^n)}\right)}{d^3en} \\
&\quad + \frac{4abx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,e^{i(c+dx^n)}\right)}{d^3en} + \frac{(ib^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2i(c+dx^n)}\right)}{d^3en}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{a^2(ex)^{3n}}{3en} - \frac{ib^2x^{-n}(ex)^{3n}}{den} - \frac{4abx^{-n}(ex)^{3n}\text{arctanh}\left(e^{i(c+dx^n)}\right)}{den} - \frac{b^2x^{-n}(ex)^{3n}\cot(c+dx^n)}{den} \\
&\quad + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1-e^{2i(c+dx^n)}\right)}{d^2en} + \frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,-e^{i(c+dx^n)}\right)}{d^2en} \\
&\quad - \frac{4iabx^{-2n}(ex)^{3n}\text{PolyLog}\left(2,e^{i(c+dx^n)}\right)}{d^2en} - \frac{ib^2x^{-3n}(ex)^{3n}\text{PolyLog}\left(2,e^{2i(c+dx^n)}\right)}{d^3en} \\
&\quad - \frac{4abx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,-e^{i(c+dx^n)}\right)}{d^3en} + \frac{4abx^{-3n}(ex)^{3n}\text{PolyLog}\left(3,e^{i(c+dx^n)}\right)}{d^3en}
\end{aligned}
$$

Mathematica [F]

$$\int (ex)^{-1+3n} \left(a+b\csc\left(c+dx^n\right)\right)^2\,dx=\int (ex)^{-1+3n} \left(a+b\csc\left(c+dx^n\right)\right)^2\,dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n])^2,x]
[Out] Integrate[(e*x)^(-1 + 3*n)*(a + b*Csc[c + d*x^n])^2, x]
```

Maple [F]

$$\int (ex)^{-1+3n} \left(a+b\csc\left(c+d\,x^n\right)\right)^2dx$$

```
[In] int((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n))^2,x)
[Out] int((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 890 vs. $2(362) = 724$.

Time = 0.30 (sec) , antiderivative size = 890, normalized size of antiderivative = 2.36

$$\int (ex)^{-1+3n} \left(a+b\csc\left(c+dx^n\right)\right)^2\,dx$$
$$=\frac{a^2d^3e^{3\,n-1}x^{3\,n}\sin\left(dx^n+c\right)-3\,b^2d^2e^{3\,n-1}x^{2\,n}\cos\left(dx^n+c\right)+6\,abe^{3\,n-1}\mathrm{polylog}(3,\cos\left(dx^n+c\right)+i\,\sin\left(dx^n\right.}{}$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
[Out] 1/3*(a^2*d^3*e^(3*n - 1)*x^(3*n)*sin(d*x^n + c) - 3*b^2*d^2*e^(3*n - 1)*x^(
2*n)*cos(d*x^n + c) + 6*a*b*e^(3*n - 1)*polylog(3, cos(d*x^n + c) + I*sin(d
*x^n + c))*sin(d*x^n + c) + 6*a*b*e^(3*n - 1)*polylog(3, cos(d*x^n + c) - I
*sin(d*x^n + c))*sin(d*x^n + c) - 6*a*b*e^(3*n - 1)*polylog(3, -cos(d*x^n +
 c) + I*sin(d*x^n + c))*sin(d*x^n + c) - 6*a*b*e^(3*n - 1)*polylog(3, -cos(
d*x^n + c) - I*sin(d*x^n + c))*sin(d*x^n + c) + 3*(a*b*c^2 - b^2*c)*e^(3*n
- 1)*log(-1/2*cos(d*x^n + c) + 1/2*I*sin(d*x^n + c) + 1/2)*sin(d*x^n + c) +
 3*(a*b*c^2 - b^2*c)*e^(3*n - 1)*log(-1/2*cos(d*x^n + c) - 1/2*I*sin(d*x^n
+ c) + 1/2)*sin(d*x^n + c) - 3*(2*I*a*b*d*e^(3*n - 1)*x^n + I*b^2*e^(3*n -
1))*dilog(cos(d*x^n + c) + I*sin(d*x^n + c))*sin(d*x^n + c) - 3*(-2*I*a*b*d
*e^(3*n - 1)*x^n - I*b^2*e^(3*n - 1))*dilog(cos(d*x^n + c) - I*sin(d*x^n +
c))*sin(d*x^n + c) - 3*(2*I*a*b*d*e^(3*n - 1)*x^n - I*b^2*e^(3*n - 1))*dilo
g(-cos(d*x^n + c) + I*sin(d*x^n + c))*sin(d*x^n + c) - 3*(-2*I*a*b*d*e^(3*n
```

```
 - 1)*x^n + I*b^2*e^(3*n - 1))*dilog(-cos(d*x^n + c) - I*sin(d*x^n + c))*si
n(d*x^n + c) - 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) - b^2*d*e^(3*n - 1)*x^n)*log(
cos(d*x^n + c) + I*sin(d*x^n + c) + 1)*sin(d*x^n + c) - 3*(a*b*d^2*e^(3*n -
 1)*x^(2*n) - b^2*d*e^(3*n - 1)*x^n)*log(cos(d*x^n + c) - I*sin(d*x^n + c)
+ 1)*sin(d*x^n + c) + 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) + b^2*d*e^(3*n - 1)*x^
n - (a*b*c^2 - b^2*c)*e^(3*n - 1))*log(-cos(d*x^n + c) + I*sin(d*x^n + c) +
 1)*sin(d*x^n + c) + 3*(a*b*d^2*e^(3*n - 1)*x^(2*n) + b^2*d*e^(3*n - 1)*x^n
 - (a*b*c^2 - b^2*c)*e^(3*n - 1))*log(-cos(d*x^n + c) - I*sin(d*x^n + c) +
1)*sin(d*x^n + c))/(d^3*n*sin(d*x^n + c))
```

Sympy [F]

$$\int (ex)^{-1+3n} (a + b\csc(c + dx^n))^2 \, dx = \int (ex)^{3n-1} (a + b\csc(c + dx^n))^2 \, dx$$

```
[In] integrate((e*x)**(-1+3*n)*(a+b*csc(c+d*x**n))**2,x)
[Out] Integral((e*x)**(3*n - 1)*(a + b*csc(c + d*x**n))**2, x)
```

Maxima [F]

$$\int (ex)^{-1+3n} (a + b\csc(c + dx^n))^2 \, dx = \int (b\csc(dx^n + c) + a)^2 (ex)^{3n-1} \, dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
[Out] 1/3*(e*x)^(3*n)*a^2/(e*n) - (2*b^2*e^(3*n)*x^(2*n)*sin(2*d*x^n + 2*c) - (d*
e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 - 2*d*e*n*cos(2*d*x^n
 + 2*c) + d*e*n)*integrate(2*(a*b*d*e^(3*n)*x^(3*n) - b^2*e^(3*n)*x^(2*n))*
sin(d*x^n + c)/(d*e*x*cos(d*x^n + c)^2 + d*e*x*sin(d*x^n + c)^2 + 2*d*e*x*c
os(d*x^n + c) + d*e*x), x) - (d*e*n*cos(2*d*x^n + 2*c)^2 + d*e*n*sin(2*d*x^
n + 2*c)^2 - 2*d*e*n*cos(2*d*x^n + 2*c) + d*e*n)*integrate(2*(a*b*d*e^(3*n)
*x^(3*n) + b^2*e^(3*n)*x^(2*n))*sin(d*x^n + c)/(d*e*x*cos(d*x^n + c)^2 + d*
e*x*sin(d*x^n + c)^2 - 2*d*e*x*cos(d*x^n + c) + d*e*x), x))/(d*e*n*cos(2*d*
x^n + 2*c)^2 + d*e*n*sin(2*d*x^n + 2*c)^2 - 2*d*e*n*cos(2*d*x^n + 2*c) + d*
e*n)
```

Giac [F]

$$\int (ex)^{-1+3n} (a + b\csc(c + dx^n))^2 \, dx = \int (b\csc(dx^n + c) + a)^2 (ex)^{3n-1} \, dx$$

```
[In] integrate((e*x)^(-1+3*n)*(a+b*csc(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((b*csc(d*x^n + c) + a)^2*(e*x)^(3*n - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+3n} (a + b\csc(c + dx^n))^2 \, dx = \int \left(a + \frac{b}{\sin(c + d\,x^n)}\right)^2 (e\,x)^{3\,n-1} \, dx$$

```
[In] int((a + b/sin(c + d*x^n))^2*(e*x)^(3*n - 1),x)
[Out] int((a + b/sin(c + d*x^n))^2*(e*x)^(3*n - 1), x)
```

3.79 $\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx = \frac{(ex)^n}{aen} + \frac{2bx^{-n}(ex)^n\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}den}$$

```
[Out] (e*x)^n/a/e/n+2*b*(e*x)^n*arctanh((a+b*tan(1/2*c+1/2*d*x^n))/(a^2-b^2)^(1/2
))/a/d/e/n/(x^n)/(a^2-b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4294, 4290, 3868, 2739, 632, 212}

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx = \frac{2bx^{-n}(ex)^n\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)}{aden\sqrt{a^2-b^2}} + \frac{(ex)^n}{aen}$$

```
[In] Int[(e*x)^(-1 + n)/(a + b*Csc[c + d*x^n]),x]
```

```
[Out] (e*x)^n/(a*e*n) + (2*b*(e*x)^n*ArcTanh[(a + b*Tan[(c + d*x^n)/2])/Sqrt[a^2
- b^2]])/(a*Sqrt[a^2 - b^2]*d*e*n*x^n)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{-n}(ex)^n\right)\int \frac{x^{-1+n}}{a+b\csc(c+dx^n)}\,dx}{e}\\
&= \frac{\left(x^{-n}(ex)^n\right)\operatorname{Subst}\left(\int \frac{1}{a+b\csc(c+dx)}\,dx,x,x^n\right)}{en}\\
&= \frac{(ex)^n}{aen}-\frac{\left(x^{-n}(ex)^n\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{a\sin(c+dx)}{b}}\,dx,x,x^n\right)}{aen}\\
&= \frac{(ex)^n}{aen}-\frac{\left(2x^{-n}(ex)^n\right)\operatorname{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx^n)\right)\right)}{aden}
\end{aligned}
$$

$$= \frac{(ex)^n}{aen} + \frac{(4x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b} + 2\tan\left(\frac{1}{2}(c+dx^n)\right)\right)}{aden}$$

$$= \frac{(ex)^n}{aen} + \frac{2bx^{-n}(ex)^n\text{arctanh}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{1}{2}(c+dx^n)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}den}$$

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx = \frac{(ex)^n\left(d+cx^{-n}-\frac{2bx^{-n}\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)}{aden}$$

```
[In] Integrate[(e*x)^(-1 + n)/(a + b*Csc[c + d*x^n]),x]
```

```
[Out] ((e*x)^n*(d + c/x^n - (2*b*ArcTan[(a + b*Tan[(c + d*x^n)/2])/Sqrt[-a^2 + b^
2]])/(Sqrt[-a^2 + b^2]*x^n)))/(a*d*e*n)
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.09 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.71

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\text{csgn}(ie)\,\text{csgn}(ix)\,\text{csgn}(iex)\pi+i\,\text{csgn}(ie)\,\text{csgn}(iex)^2\pi+i\,\text{csgn}(ix)\,\text{csgn}(iex)^2\pi-i\,\text{csgn}(iex)^3\pi+2\ln(e)+2\ln(x)\right)}{2}}}{an} - \frac{2i\arctan\left(\frac{2ia\,\mathrm{e}^{i(}}{2\sqrt{}}\right.}{}$

```
[In] int((e*x)^(-1+n)/(a+b*csc(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)*c
sgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2*l
n(x)))-2*I*arctan(1/2*(2*I*a*exp(I*(d*x^n+2*c))-2*exp(I*c)*b)/(a^2*exp(2*I*
c)-exp(2*I*c)*b^2)^(1/2))/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/d/e*e^n/n*b
/a*exp(1/2*I*(-Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*n*csgn(I*e)*csgn(I*e
*x)^2+Pi*n*csgn(I*x)*csgn(I*e*x)^2-Pi*n*csgn(I*e*x)^3+Pi*csgn(I*e)*csgn(I*x
)*csgn(I*e*x)-Pi*csgn(I*e)*csgn(I*e*x)^2-Pi*csgn(I*x)*csgn(I*e*x)^2+Pi*csgn
(I*e*x)^3+2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 301, normalized size of antiderivative = 3.54

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx$$
$$= \left[\frac{2\,(a^2-b^2)de^{n-1}x^n+\sqrt{a^2-b^2}be^{n-1}\log\left(\frac{(a^2-2\,b^2)\cos(dx^n+c)^2+2\,\sqrt{a^2-b^2}a\cos(dx^n+c)+a^2+b^2+2\left(\sqrt{a^2-b^2}b\cos(dx^n+c)+ab\right.}{a^2\cos(dx^n+c)^2-2\,ab\sin(dx^n+c)-a^2-b^2}\right.}{2\,(a^3-ab^2)dn}\right.$$

```
[In] integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2 - b^2)*d*e^(n - 1)*x^n + sqrt(a^2 - b^2)*b*e^(n - 1)*log(((a^2
 - 2*b^2)*cos(d*x^n + c)^2 + 2*sqrt(a^2 - b^2)*a*cos(d*x^n + c) + a^2 + b^2
 + 2*(sqrt(a^2 - b^2)*b*cos(d*x^n + c) + a*b)*sin(d*x^n + c))/(a^2*cos(d*x^
n + c)^2 - 2*a*b*sin(d*x^n + c) - a^2 - b^2)))/((a^3 - a*b^2)*d*n), ((a^2 -
 b^2)*d*e^(n - 1)*x^n + sqrt(-a^2 + b^2)*b*e^(n - 1)*arctan(-(sqrt(-a^2 + b
^2)*b*sin(d*x^n + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*cos(d*x^n + c))))/(
(a^3 - a*b^2)*d*n)]
```

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx = \int \frac{(ex)^{n-1}}{a+b\csc(c+dx^n)}\,dx$$

```
[In] integrate((e*x)**(-1+n)/(a+b*csc(c+d*x**n)),x)
```

```
[Out] Integral((e*x)**(n - 1)/(a + b*csc(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx = \int \frac{(ex)^{n-1}}{b\csc(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n)),x, algorithm="maxima")
```

```
[Out] -(2*a*b*e^(n + 1)*n*integrate((2*b*x^n*cos(d*x^n + c)^2 + a*x^n*cos(d*x^n +
 c)*sin(2*d*x^n + 2*c) - a*x^n*cos(2*d*x^n + 2*c)*sin(d*x^n + c) + 2*b*x^n*
sin(d*x^n + c)^2 + a*x^n*sin(d*x^n + c))/(a^3*e*x*cos(2*d*x^n + 2*c)^2 + 4*
a*b^2*e*x*cos(d*x^n + c)^2 + 4*a^2*b*e*x*cos(d*x^n + c)*sin(2*d*x^n + 2*c)
+ a^3*e*x*sin(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*sin(d*x^n + c)^2 + 4*a^2*b*e*x
*sin(d*x^n + c) + a^3*e*x - 2*(2*a^2*b*e*x*sin(d*x^n + c) + a^3*e*x)*cos(2*
d*x^n + 2*c)), x) - e^n*x^n)/(a*e*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx = \int \frac{(ex)^{n-1}}{b\csc(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n)),x, algorithm="giac")
```

```
[Out] integrate((e*x)^(n - 1)/(b*csc(d*x^n + c) + a), x)
```

Mupad [B] (verification not implemented)

Time = 19.77 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.69

$$\int \frac{(ex)^{-1+n}}{a+b\csc(c+dx^n)}\,dx = \frac{x\,(e\,x)^{n-1}}{a\,n} - \frac{b\,x\,\ln\left(b\,x\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\,2\mathrm{i} - \frac{2\,b\,x\,(e\,x)^{n-1}\,\left(a\,1\mathrm{i}+b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)\,(e\,x)^{n-1}}{a\,d\,n\,x^n\,\sqrt{a+b}\,\sqrt{a-b}} + \frac{b\,x\,\ln\left(b\,x\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\,(e\,x)^{n-1}\,2\mathrm{i} + \frac{2\,b\,x\,(e\,x)^{n-1}\,\left(a\,1\mathrm{i}+b\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x^n\,1\mathrm{i}}\right)}{\sqrt{a+b}\,\sqrt{a-b}}\right)\,(e\,x)^{n-1}}{a\,d\,n\,x^n\,\sqrt{a+b}\,\sqrt{a-b}}$$

```
[In] int((e*x)^(n - 1)/(a + b/sin(c + d*x^n)),x)
```

```
[Out] (x*(e*x)^(n - 1))/(a*n) - (b*x*log(b*x*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1
)*2i - (2*b*x*(e*x)^(n - 1)*(a*1i + b*exp(c*1i)*exp(d*x^n*1i)))/((a + b)^(1
/2)*(a - b)^(1/2)))*(e*x)^(n - 1))/(a*d*n*x^n*(a + b)^(1/2)*(a - b)^(1/2))
+ (b*x*log(b*x*exp(c*1i)*exp(d*x^n*1i)*(e*x)^(n - 1)*2i + (2*b*x*(e*x)^(n -
 1)*(a*1i + b*exp(c*1i)*exp(d*x^n*1i)))/((a + b)^(1/2)*(a - b)^(1/2)))*(e*x
)^(n - 1))/(a*d*n*x^n*(a + b)^(1/2)*(a - b)^(1/2))
```

3.80 $\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 338

$$\begin{aligned}\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx &= \frac{(ex)^{2n}}{2aen} + \frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\ &\quad - \frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\ &\quad + \frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} \\ &\quad - \frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}\end{aligned}$$

```
[Out] 1/2*(e*x)^(2*n)/a/e/n+I*b*(e*x)^(2*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^
2)^(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)-I*b*(e*x)^(2*n)*ln(1-I*a*exp(I*(c
+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)+b*(e*x)^(2*n)
*polylog(2,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(2*n))/(
-a^2+b^2)^(1/2)-b*(e*x)^(2*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^
(1/2)))/a/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4294, 4290, 4276, 3404, 2296, 2221, 2317, 2438}

$$\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx = \frac{bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} - \frac{bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} + \frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{aden\sqrt{b^2-a^2}} - \frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{aden\sqrt{b^2-a^2}} + \frac{(ex)^{2n}}{2aen}$$

```
[In] Int[(e*x)^(-1 + 2*n)/(a + b*Csc[c + d*x^n]),x]
```

```
[Out] (e*x)^(2*n)/(2*a*e*n) + (I*b*(e*x)^(2*n)*Log[1 - (I*a*E^(I*(c + d*x^n)))/(b
 - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d*e*n*x^n) - (I*b*(e*x)^(2*n)*Lo
g[1 - (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*
d*e*n*x^n) + (b*(e*x)^(2*n)*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a
^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) - (b*(e*x)^(2*n)*PolyLog[
2, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2
*e*n*x^(2*n))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(x^{-2n}(ex)^{2n}\right)\int \frac{x^{-1+2n}}{a+b\csc(c+dx^n)}\,dx}{e}\\ &= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{x}{a+b\csc(c+dx)}\,dx, x, x^n\right)}{en}\\ &= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \left(\frac{x}{a}-\frac{bx}{a(b+a\sin(c+dx))}\right)\,dx, x, x^n\right)}{en}\end{aligned}$$

$$=\frac{(ex)^{2n}}{2aen}-\frac{(bx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\frac{x}{b+a\sin(c+dx)}\,dx,x,x^n\right)}{aen}$$

$$=\frac{(ex)^{2n}}{2aen}-\frac{(2bx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\frac{e^{i(c+dx)}x}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx,x,x^n\right)}{aen}$$

$$=\frac{(ex)^{2n}}{2aen}+\frac{(2ibx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx,x,x^n\right)}{\sqrt{-a^2+b^2}en}-\frac{(2ibx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx,x,x^n\right)}{\sqrt{-a^2+b^2}en}$$

$$=\frac{(ex)^{2n}}{2aen}+\frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}-\frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}-\frac{(ibx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a\sqrt{-a^2+b^2}den}+\frac{(ibx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a\sqrt{-a^2+b^2}den}$$

$$=\frac{(ex)^{2n}}{2aen}+\frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}-\frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}-\frac{(bx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{2iax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{a\sqrt{-a^2+b^2}d^2en}+\frac{(bx^{-2n}(ex)^{2n})\operatorname{Subst}\left(\int\frac{\log\left(1-\frac{2iax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}\,dx,x,e^{i(c+dx^n)}\right)}{a\sqrt{-a^2+b^2}d^2en}$$

$$=\frac{(ex)^{2n}}{2aen}+\frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}-\frac{ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den}+\frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}-\frac{bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1003 vs. $2(338) = 676$.

Time = 6.35 (sec) , antiderivative size = 1003, normalized size of antiderivative = 2.97

$$\int \frac{(ex)^{-1+2n}}{a + b\csc\left(c + dx^n\right)}\,dx$$
$$= \frac{(ex)^{2n}\csc\left(c+dx^n\right)\left(1 - \frac{2bx^{-2n}\left(\frac{\pi\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + \frac{2\left(c-\arccos\left(-\frac{b}{a}\right)\right)\operatorname{arctanh}\left(\frac{(a-b)\cot\left(\frac{1}{4}(2c+\pi+2dx^n)\right)}{\sqrt{a^2-b^2}}\right) + (-2c+\pi-2\cdots}{\cdots}\right)}{\cdots}\right)}{\cdots}$$

```
[In] Integrate[(e*x)^(-1 + 2*n)/(a + b*Csc[c + d*x^n]),x]
```

```
[Out] ((e*x)^(2*n)*Csc[c + d*x^n]*(1 - (2*b*((Pi*ArcTan[(a + b*Tan[(c + d*x^n)/2]
)/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(c - ArcCos[-(b/a)])*ArcTanh[((a
 - b)*Cot[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2 - b^2]] + (-2*c + Pi - 2*d*x^n)
*ArcTanh[((a + b)*Tan[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2 - b^2]] - (ArcCos[-
(b/a)] - (2*I)*ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2 - b^2
]])*Log[((a + b)*(a - b - I*Sqrt[a^2 - b^2])*(1 + I*Cot[(2*c + Pi + 2*d*x^n
)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi + 2*d*x^n)/4]))] + (ArcCos
[-(b/a)] + (2*I)*(-ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2 -
 b^2]] + ArcTanh[((a + b)*Tan[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2 - b^2]]))*L
og[((-1)^(1/4)*Sqrt[a^2 - b^2])/(Sqrt[2]*Sqrt[a]*E^((I/2)*(c + d*x^n))*Sqrt
[b + a*Sin[c + d*x^n]])] + (ArcCos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Cot[(2*
c + Pi + 2*d*x^n)/4])/Sqrt[a^2 - b^2]] - (2*I)*ArcTanh[((a + b)*Tan[(2*c +
Pi + 2*d*x^n)/4])/Sqrt[a^2 - b^2]])*Log[-(((-1)^(3/4)*Sqrt[a^2 - b^2]*E^((I
/2)*(c + d*x^n)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Sin[c + d*x^n]]))] - (ArcCos[
-(b/a)] + (2*I)*ArcTanh[((a - b)*Cot[(2*c + Pi + 2*d*x^n)/4])/Sqrt[a^2 - b^
2]])*Log[1 + (I*(I*b + Sqrt[a^2 - b^2])*(a + b + Sqrt[a^2 - b^2]*Tan[(2*c -
 Pi + 2*d*x^n)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi + 2*d*x^n)/4]
))] + I*(PolyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b + Sqrt[a^2 - b^2]*Tan[(
2*c - Pi + 2*d*x^n)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi + 2*d*x^
n)/4]))] - PolyLog[2, ((b + I*Sqrt[a^2 - b^2])*(a + b + Sqrt[a^2 - b^2]*Tan
[(2*c - Pi + 2*d*x^n)/4]))/(a*(a + b + Sqrt[a^2 - b^2]*Cot[(2*c + Pi + 2*d*
x^n)/4]))]))/Sqrt[a^2 - b^2]))/(d^2*x^(2*n)))*(b + a*Sin[c + d*x^n]))/(2*a*
e*n*(a + b*Csc[c + d*x^n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.34 (sec) , antiderivative size = 769, normalized size of antiderivative = 2.28

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(-1+2n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^{2}\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^{2}\pi-i\,\mathrm{csgn}(iex)^{3}\pi+2\ln(e)+2\ln(x)\right)}{2}}}{2an}+\frac{i\left(ix^{n}d\ln\left(\frac{-ie^{i}}{}\right.\right.}{}$

```
[In] int((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a/n*x*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*
e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)
+2*ln(x)))+I/(a^2-b^2)*(I*x^n*d*ln((-I*exp(I*c)*b-a*exp(I*(d*x^n+2*c))+(a^2
*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))/(-I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c
)*b^2)^(1/2)))-I*x^n*d*ln((I*exp(I*c)*b+a*exp(I*(d*x^n+2*c))+(a^2*exp(2*I*c
)-exp(2*I*c)*b^2)^(1/2))/(I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2
)))+dilog(-I/(-I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*exp(I*c)
*b-a/(-I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*exp(I*(d*x^n+2*c
))+1/(-I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*(a^2*exp(2*I*c)-
exp(2*I*c)*b^2)^(1/2))-dilog(I/(I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2
)^(1/2))*exp(I*c)*b+a/(I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*
exp(I*(d*x^n+2*c))+1/(I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*(
a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)))*(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2
)/d^2/n/e*(e^n)^2*b/a*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*
Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e
*x)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn
(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1235 vs. 2(302) = 604.

Time = 0.44 (sec) , antiderivative size = 1235, normalized size of antiderivative = 3.65

$$\int \frac{(ex)^{-1+2n}}{a+b\csc\left(c+dx^{n}\right)}\,dx=\text{Too large to display}$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) + 2*I*
a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) + a*b*c*e^(2*n - 1)*s
qrt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2*a*sq
rt((a^2 - b^2)/a^2) - 2*I*b) - a*b*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*log(
```

```
-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*
I*b) - a*b*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*log(-2*a*cos(d*x^n + c) - 2*
I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b) - (a^2 - b^2)*d^2*e
^(2*n - 1)*x^(2*n) - I*a*b*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*dilog(((a*sqrt
((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) + (I*a*sqrt((a^2 - b^2)/a^2) - b)*s
in(d*x^n + c) - a)/a + 1) - I*a*b*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*dilog(-
((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) - (I*a*sqrt((a^2 - b^2)/a^2
) - b)*sin(d*x^n + c) + a)/a + 1) + I*a*b*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)
*dilog(((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) + (-I*a*sqrt((a^2 -
b^2)/a^2) - b)*sin(d*x^n + c) - a)/a + 1) + I*a*b*e^(2*n - 1)*sqrt((a^2 - b
^2)/a^2)*dilog(-((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) - (-I*a*sqr
t((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) + a)/a + 1) + (a*b*d*e^(2*n - 1)*x^n
*sqrt((a^2 - b^2)/a^2) + a*b*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*log(-((a*
sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) + (I*a*sqrt((a^2 - b^2)/a^2) -
b)*sin(d*x^n + c) - a)/a) - (a*b*d*e^(2*n - 1)*x^n*sqrt((a^2 - b^2)/a^2) +
a*b*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*log(((a*sqrt((a^2 - b^2)/a^2) + I*
b)*cos(d*x^n + c) - (I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) + a)/a)
+ (a*b*d*e^(2*n - 1)*x^n*sqrt((a^2 - b^2)/a^2) + a*b*c*e^(2*n - 1)*sqrt((a^
2 - b^2)/a^2))*log(-((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) + (-I*a
*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a) - (a*b*d*e^(2*n - 1)*x^n
*sqrt((a^2 - b^2)/a^2) + a*b*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*log(((a*s
qrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) - (-I*a*sqrt((a^2 - b^2)/a^2) -
b)*sin(d*x^n + c) + a)/a))/((a^3 - a*b^2)*d^2*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{a + b\csc(c + dx^n)}\,dx = \int \frac{(ex)^{2n-1}}{a + b\csc(c + dx^n)}\,dx$$

```
[In] integrate((e*x)**(-1+2*n)/(a+b*csc(c+d*x**n)),x)
[Out] Integral((e*x)**(2*n - 1)/(a + b*csc(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{a + b\csc(c + dx^n)}\,dx = \int \frac{(ex)^{2n-1}}{b\csc(dx^n + c) + a}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n)),x, algorithm="maxima")
[Out] -1/2*(4*a*b*e^(2*n + 1)*n*integrate((2*b*x^(2*n)*cos(d*x^n + c)^2 + a*x^(2*
n)*cos(d*x^n + c)*sin(2*d*x^n + 2*c) - a*x^(2*n)*cos(2*d*x^n + 2*c)*sin(d*x
```

```
^n + c) + 2*b*x^(2*n)*sin(d*x^n + c)^2 + a*x^(2*n)*sin(d*x^n + c))/(a^3*e*x
*cos(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*cos(d*x^n + c)^2 + 4*a^2*b*e*x*cos(d*x^
n + c)*sin(2*d*x^n + 2*c) + a^3*e*x*sin(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*sin(
d*x^n + c)^2 + 4*a^2*b*e*x*sin(d*x^n + c) + a^3*e*x - 2*(2*a^2*b*e*x*sin(d*
x^n + c) + a^3*e*x)*cos(2*d*x^n + 2*c)), x) - e^(2*n)*x^(2*n))/(a*e*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(ex)^{2n-1}}{b\csc(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n)),x, algorithm="giac")
[Out] integrate((e*x)^(2*n - 1)/(b*csc(d*x^n + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{a+b\csc(c+dx^n)}\,dx=\int \frac{(e\,x)^{2\,n-1}}{a+\frac{b}{\sin(c+d\,x^n)}}\,dx$$

```
[In] int((e*x)^(2*n - 1)/(a + b/sin(c + d*x^n)),x)
[Out] int((e*x)^(2*n - 1)/(a + b/sin(c + d*x^n)), x)
```

3.81 $\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 499

$$\begin{aligned}\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx = {} & \frac{(ex)^{3n}}{3aen}+\frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\ & -\frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} \\ & +\frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} \\ & -\frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} \\ & +\frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en} \\ & -\frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}\end{aligned}$$

```
[Out] 1/3*(e*x)^(3*n)/a/e/n+I*b*(e*x)^(3*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^
2)^(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)-I*b*(e*x)^(3*n)*ln(1-I*a*exp(I*(c
+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a/d/e/n/(x^n)/(-a^2+b^2)^(1/2)+2*b*(e*x)^(3*
n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a/d^2/e/n/(x^(2*n))
/(-a^2+b^2)^(1/2)-2*b*(e*x)^(3*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b+(-a^2+b
^2)^(1/2)))/a/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)+2*I*b*(e*x)^(3*n)*polylog(
3,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a/d^3/e/n/(x^(3*n))/(-a^2+b^2)
^(1/2)-2*I*b*(e*x)^(3*n)*polylog(3,I*a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)
))/a/d^3/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4294, 4290, 4276, 3404, 2296, 2221, 2611, 2320, 6724}

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx = \frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} - \frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^3en\sqrt{b^2-a^2}} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{ad^2en\sqrt{b^2-a^2}} + \frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{aden\sqrt{b^2-a^2}} - \frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{aden\sqrt{b^2-a^2}} + \frac{(ex)^{3n}}{3aen}$$

```
[In] Int[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n]),x]
```

```
[Out] (e*x)^(3*n)/(3*a*e*n) + (I*b*(e*x)^(3*n)*Log[1 - (I*a*E^(I*(c + d*x^n)))/(b
 - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d*e*n*x^n) - (I*b*(e*x)^(3*n)*Lo
g[1 - (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*
d*e*n*x^n) + (2*b*(e*x)^(3*n)*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[
-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) - (2*b*(e*x)^(3*n)*Poly
Log[2, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]
*d^2*e*n*x^(2*n)) + ((2*I)*b*(e*x)^(3*n)*PolyLog[3, (I*a*E^(I*(c + d*x^n)))
/(b - Sqrt[-a^2 + b^2])])/(a*Sqrt[-a^2 + b^2]*d^3*e*n*x^(3*n)) - ((2*I)*b*(
e*x)^(3*n)*PolyLog[3, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a*S
qrt[-a^2 + b^2]*d^3*e*n*x^(3*n))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
```

```
+ b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = \frac{\left(x^{-3n}(ex)^{3n}\right)\int \frac{x^{-1+3n}}{a+b\csc(c+dx^n)}\,dx}{e}$$

$$= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{a+b\csc(c+dx)}\,dx, x, x^n\right)}{en}$$

$$= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \left(\frac{x^2}{a}-\frac{bx^2}{a(b+a\sin(c+dx))}\right)\,dx, x, x^n\right)}{en}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{\left(bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{b+a\sin(c+dx)}\,dx, x, x^n\right)}{aen}$$

$$= \frac{(ex)^{3n}}{3aen} - \frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, x^n\right)}{aen}$$

$$= \frac{(ex)^{3n}}{3aen} + \frac{\left(2ibx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^n\right)}{\sqrt{-a^2+b^2}en} - \frac{\left(2ibx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^n\right)}{\sqrt{-a^2+b^2}en}$$

$$= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{\left(2ibx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}den} + \frac{\left(2ibx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}den}$$

$$= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} - \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a\sqrt{-a^2+b^2}d^2en} + \frac{(2bx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\operatorname{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a\sqrt{-a^2+b^2}d^2en}$$

$$= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} + \frac{(2ibx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a\sqrt{-a^2+b^2}d^3en} - \frac{(2ibx^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a\sqrt{-a^2+b^2}d^3en}$$

$$= \frac{(ex)^{3n}}{3aen} + \frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} - \frac{ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}den} + \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} - \frac{2bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^2en} + \frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en} - \frac{2ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}d^3en}$$

Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx = \int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n]),x]
[Out] Integrate[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n]), x]
```

Maple [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\csc\left(c+d\,x^{n}\right)}dx$$

[In] int((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n)),x)

[Out] int((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n)),x)

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1663 vs. $2(447) = 894$.

Time = 0.45 (sec) , antiderivative size = 1663, normalized size of antiderivative = 3.33

$$\int \frac{(ex)^{-1+3n}}{a+b\csc\left(c+dx^{n}\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n)),x, algorithm="fricas")
```

```
[Out] 1/6*(6*I*a*b*d*e^(3*n - 1)*x^n*sqrt((a^2 - b^2)/a^2)*dilog(((a*sqrt((a^2 -
b^2)/a^2) + I*b)*cos(d*x^n + c) + (I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n
 + c) - a)/a + 1) + 6*I*a*b*d*e^(3*n - 1)*x^n*sqrt((a^2 - b^2)/a^2)*dilog(-
((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) - (I*a*sqrt((a^2 - b^2)/a^2
) - b)*sin(d*x^n + c) + a)/a + 1) - 6*I*a*b*d*e^(3*n - 1)*x^n*sqrt((a^2 - b
^2)/a^2)*dilog(((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) + (-I*a*sqrt
((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a + 1) - 6*I*a*b*d*e^(3*n - 1)*x
^n*sqrt((a^2 - b^2)/a^2)*dilog(-((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n
+ c) - (-I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) + a)/a + 1) + 3*a*b*
c^2*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) + 2*I*a*sin(d*
x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*I*b) + 3*a*b*c^2*e^(3*n - 1)*sqrt(
(a^2 - b^2)/a^2)*log(2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt((
a^2 - b^2)/a^2) - 2*I*b) - 3*a*b*c^2*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2)*log(
-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2*
I*b) - 3*a*b*c^2*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2)*log(-2*a*cos(d*x^n + c)
- 2*I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b) + 2*(a^2 - b^2)
*d^3*e^(3*n - 1)*x^(3*n) + 6*a*b*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2)*polylog(
3, -((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) + (-I*a*sqrt((a^2 - b^2
)/a^2) + b)*sin(d*x^n + c))/a) - 6*a*b*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2)*po
lylog(3, ((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) - (-I*a*sqrt((a^2
- b^2)/a^2) + b)*sin(d*x^n + c))/a) + 6*a*b*e^(3*n - 1)*sqrt((a^2 - b^2)/a^
2)*polylog(3, -((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) + (I*a*sqrt(
(a^2 - b^2)/a^2) + b)*sin(d*x^n + c))/a) - 6*a*b*e^(3*n - 1)*sqrt((a^2 - b^
2)/a^2)*polylog(3, ((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) - (I*a*s
qrt((a^2 - b^2)/a^2) + b)*sin(d*x^n + c))/a) - 3*(a*b*d^2*e^(3*n - 1)*x^(2*
```

```
n)*sqrt((a^2 - b^2)/a^2) - a*b*c^2*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2))*log(-
((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) + (I*a*sqrt((a^2 - b^2)/a^2
) - b)*sin(d*x^n + c) - a)/a) + 3*(a*b*d^2*e^(3*n - 1)*x^(2*n)*sqrt((a^2 -
b^2)/a^2) - a*b*c^2*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2))*log(((a*sqrt((a^2 -
b^2)/a^2) + I*b)*cos(d*x^n + c) - (I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n
 + c) + a)/a) - 3*(a*b*d^2*e^(3*n - 1)*x^(2*n)*sqrt((a^2 - b^2)/a^2) - a*b*
c^2*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2))*log(-((a*sqrt((a^2 - b^2)/a^2) - I*b
)*cos(d*x^n + c) + (-I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a)
+ 3*(a*b*d^2*e^(3*n - 1)*x^(2*n)*sqrt((a^2 - b^2)/a^2) - a*b*c^2*e^(3*n - 1
)*sqrt((a^2 - b^2)/a^2))*log(((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c
) - (-I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) + a)/a))/((a^3 - a*b^2)
*d^3*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx = \int \frac{(ex)^{3n-1}}{a+b\csc(c+dx^n)}\,dx$$

```
[In] integrate((e*x)**(-1+3*n)/(a+b*csc(c+d*x**n)),x)
[Out] Integral((e*x)**(3*n - 1)/(a + b*csc(c + d*x**n)), x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx = \int \frac{(ex)^{3n-1}}{b\csc(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n)),x, algorithm="maxima")
[Out] -1/3*(6*a*b*e^(3*n + 1)*n*integrate((2*b*x^(3*n)*cos(d*x^n + c)^2 + a*x^(3*
n)*cos(d*x^n + c)*sin(2*d*x^n + 2*c) - a*x^(3*n)*cos(2*d*x^n + 2*c)*sin(d*x
^n + c) + 2*b*x^(3*n)*sin(d*x^n + c)^2 + a*x^(3*n)*sin(d*x^n + c))/(a^3*e*x
*cos(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*cos(d*x^n + c)^2 + 4*a^2*b*e*x*cos(d*x^
n + c)*sin(2*d*x^n + 2*c) + a^3*e*x*sin(2*d*x^n + 2*c)^2 + 4*a*b^2*e*x*sin(
d*x^n + c)^2 + 4*a^2*b*e*x*sin(d*x^n + c) + a^3*e*x - 2*(2*a^2*b*e*x*sin(d*
x^n + c) + a^3*e*x)*cos(2*d*x^n + 2*c)), x) - e^(3*n)*x^(3*n))/(a*e*n)
```

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx = \int \frac{(ex)^{3n-1}}{b\csc(dx^n+c)+a}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n)),x, algorithm="giac")
[Out] integrate((e*x)^(3*n - 1)/(b*csc(d*x^n + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{a+b\csc(c+dx^n)}\,dx = \int \frac{(e\,x)^{3\,n-1}}{a+\frac{b}{\sin(c+d\,x^n)}}\,dx$$

```
[In] int((e*x)^(3*n - 1)/(a + b/sin(c + d*x^n)),x)
[Out] int((e*x)^(3*n - 1)/(a + b/sin(c + d*x^n)), x)
```

3.82 $\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \frac{(ex)^n}{a^2en} + \frac{2b(2a^2-b^2)\,x^{-n}(ex)^n\text{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\,(a^2-b^2)^{3/2}\,den} - \frac{b^2x^{-n}(ex)^n\cot(c+dx^n)}{a\,(a^2-b^2)\,den\,(a+b\csc(c+dx^n))}$$

```
[Out] (e*x)^n/a^2/e/n+2*b*(2*a^2-b^2)*(e*x)^n*arctanh((a+b*tan(1/2*c+1/2*d*x^n))/
(a^2-b^2)^(1/2))/a^2/(a^2-b^2)^(3/2)/d/e/n/(x^n)-b^2*(e*x)^n*cot(c+d*x^n)/a
/(a^2-b^2)/d/e/n/(x^n)/(a+b*csc(c+d*x^n))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4294, 4290, 3870, 4004, 3916, 2739, 632, 212}

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \frac{2b(2a^2-b^2)\,x^{-n}(ex)^n\text{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{a^2-b^2}}\right)}{a^2den\,(a^2-b^2)^{3/2}} - \frac{b^2x^{-n}(ex)^n\cot(c+dx^n)}{aden\,(a^2-b^2)\,(a+b\csc(c+dx^n))} + \frac{(ex)^n}{a^2en}$$

```
[In] Int[(e*x)^(-1 + n)/(a + b*Csc[c + d*x^n])^2,x]
```

```
[Out] (e*x)^n/(a^2*e*n) + (2*b*(2*a^2 - b^2)*(e*x)^n*ArcTanh[(a + b*Tan[(c + d*x^
n)/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(3/2)*d*e*n*x^n) - (b^2*(e*x)^n*C
ot[c + d*x^n])/(a*(a^2 - b^2)*d*e*n*x^n*(a + b*Csc[c + d*x^n]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2739

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3870

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[b^2*Cot[
c + d*x]*((a + b*Csc[c + d*x])^(n + 1)/(a*d*(n + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
```

```
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\text{integral} = \frac{(x^{-n}(ex)^n)\int \frac{x^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx}{e}$$

$$= \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{(a+b\csc(c+dx))^2}\,dx, x, x^n\right)}{en}$$

$$= -\frac{b^2x^{-n}(ex)^n\cot(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\csc(c+dx^n))} - \frac{(x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{-a^2+b^2+ab\csc(c+dx)}{a+b\csc(c+dx)}\,dx, x, x^n\right)}{a(a^2-b^2)\,en}$$

$$= \frac{(ex)^n}{a^2en} - \frac{b^2x^{-n}(ex)^n\cot(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\csc(c+dx^n))} + \frac{((-a^2b+b(-a^2+b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{\csc(c+dx)}{a+b\csc(c+dx)}\,dx, x, x^n\right)}{a^2(a^2-b^2)\,en}$$

$$= \frac{(ex)^n}{a^2en} - \frac{b^2x^{-n}(ex)^n\cot(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\csc(c+dx^n))} + \frac{((-a^2b+b(-a^2+b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{1+\frac{a\sin(c+dx)}{b}}\,dx, x, x^n\right)}{a^2b(a^2-b^2)\,en}$$

$$= \frac{(ex)^n}{a^2en} - \frac{b^2x^{-n}(ex)^n\cot(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\csc(c+dx^n))} + \frac{(2(-a^2b+b(-a^2+b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{1+\frac{2ax}{b}+x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx^n)\right)\right)}{a^2b(a^2-b^2)\,den}$$

$$= \frac{(ex)^n}{a^2en} - \frac{b^2x^{-n}(ex)^n\cot(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\csc(c+dx^n))} - \frac{(4(-a^2b+b(-a^2+b^2))\,x^{-n}(ex)^n)\,\text{Subst}\left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2}\,dx, x, \frac{2a}{b}+2\tan\left(\frac{1}{2}(c+dx^n)\right)\right)}{a^2b(a^2-b^2)\,den}$$

$$= \frac{(ex)^n}{a^2en} + \frac{2b(2a^2-b^2)\,x^{-n}(ex)^n\text{arctanh}\left(\frac{b\left(\frac{a}{b}+\tan\left(\frac{1}{2}(c+dx^n)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{3/2}\,den} - \frac{b^2x^{-n}(ex)^n\cot(c+dx^n)}{a(a^2-b^2)\,den\,(a+b\csc(c+dx^n))}$$

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.13

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \frac{x^{-n}(ex)^n\left(2b(-2a^2+b^2)\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(c+dx^n)\right)}{\sqrt{-a^2+b^2}}\right)(a+b\csc(c+dx^n))+\sqrt{-a^2+b^2}(-ab^2\cot(c+dx^n)\right.}{a^2(a-b)(a+b)\sqrt{-a^2+b^2}den\,(a+b\csc(c+dx^n))}$$

```
[In] Integrate[(e*x)^(-1 + n)/(a + b*Csc[c + d*x^n])^2,x]
```

```
[Out] ((e*x)^n*(2*b*(-2*a^2 + b^2)*ArcTan[(a + b*Tan[(c + d*x^n)/2])/Sqrt[-a^2 +
b^2]]*(a + b*Csc[c + d*x^n]) + Sqrt[-a^2 + b^2]*(-(a*b^2*Cot[c + d*x^n]) +
(a^2 - b^2)*(c + d*x^n)*(a + b*Csc[c + d*x^n]))))/(a^2*(a - b)*(a + b)*Sqrt
[-a^2 + b^2]*d*e*n*x^n*(a + b*Csc[c + d*x^n]))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 6.79 (sec) , antiderivative size = 646, normalized size of antiderivative = 4.14

method	result
risch	$\frac{x\,\mathrm{e}^{\frac{(-1+n)\left(-i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)\pi+i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(iex)^2\pi+i\,\mathrm{csgn}(ix)\,\mathrm{csgn}(iex)^2\pi-i\,\mathrm{csgn}(iex)^3\pi+2\ln(e)+2\ln(x)\right)}{2}}}{a^2 n} - \frac{2ib^2e^n(-1)^{\mathrm{csgn}(}}{}$

```
[In] int((e*x)^(-1+n)/(a+b*csc(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/n*x*exp(1/2*(-1+n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(I*e)
*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(e)+2
*ln(x)))-2*I*b^2/a^2/(-a^2+b^2)/d/n/(2*b*exp(I*(c+d*x^n))-I*a*exp(2*I*(c+d*
x^n))+I*a)*e^n*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(b*exp(1/2*I*(-Pi
*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*n*csgn(I*e)*csgn(I*e*x)^2+Pi*n*csgn(I
*x)*csgn(I*e*x)^2-Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*e*x)^2-Pi*csgn(I*x
)*csgn(I*e*x)^2+Pi*csgn(I*e*x)^3+2*d*x^n+2*c))+I*exp(1/2*I*Pi*csgn(I*e*x)*(
-csgn(I*e)*csgn(I*x)*n+csgn(I*e)*csgn(I*e*x)*n+csgn(I*x)*csgn(I*e*x)*n-csgn
(I*e*x)^2*n-csgn(I*e)*csgn(I*e*x)-csgn(I*x)*csgn(I*e*x)+csgn(I*e*x)^2))*a)/
e-2*I*arctan(1/2*(2*I*a*exp(I*(d*x^n+2*c))-2*exp(I*c)*b)/(a^2*exp(2*I*c)-ex
p(2*I*c)*b^2)^(1/2))/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/d/e*e^n/n/a^2/(-
a^2+b^2)*(-2*a^2+b^2)*b*exp(1/2*I*(-Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi
*n*csgn(I*e)*csgn(I*e*x)^2+Pi*n*csgn(I*x)*csgn(I*e*x)^2-Pi*n*csgn(I*e*x)^3+
Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-Pi*csgn(I*e)*csgn(I*e*x)^2-Pi*csgn(I*x)*
csgn(I*e*x)^2+Pi*csgn(I*e*x)^3+2*c))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 630, normalized size of antiderivative = 4.04

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx$$
$$= \left[\frac{2\left(a^5-2\,a^3b^2+ab^4\right)de^{n-1}x^n\sin\left(dx^n+c\right)+2\left(a^4b-2\,a^2b^3+b^5\right)de^{n-1}x^n-2\left(a^3b^2-ab^4\right)e^{n-1}\cos\left(dx^n+\right.}{2\left(\left(a^7\right.\right.}\right.$$

```
[In] integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*e^(n - 1)*x^n*sin(d*x^n + c) + 2*(a^4*b
 - 2*a^2*b^3 + b^5)*d*e^(n - 1)*x^n - 2*(a^3*b^2 - a*b^4)*e^(n - 1)*cos(d*x
^n + c) + ((2*a^3*b - a*b^3)*sqrt(a^2 - b^2)*e^(n - 1)*sin(d*x^n + c) + (2*
a^2*b^2 - b^4)*sqrt(a^2 - b^2)*e^(n - 1))*log(((a^2 - 2*b^2)*cos(d*x^n + c)
^2 + 2*sqrt(a^2 - b^2)*a*cos(d*x^n + c) + a^2 + b^2 + 2*(sqrt(a^2 - b^2)*b*
cos(d*x^n + c) + a*b)*sin(d*x^n + c))/(a^2*cos(d*x^n + c)^2 - 2*a*b*sin(d*x
^n + c) - a^2 - b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*sin(d*x^n + c) + (a
^6*b - 2*a^4*b^3 + a^2*b^5)*d*n), ((a^5 - 2*a^3*b^2 + a*b^4)*d*e^(n - 1)*x^
n*sin(d*x^n + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*e^(n - 1)*x^n - (a^3*b^2 - a
*b^4)*e^(n - 1)*cos(d*x^n + c) + ((2*a^3*b - a*b^3)*sqrt(-a^2 + b^2)*e^(n -
 1)*sin(d*x^n + c) + (2*a^2*b^2 - b^4)*sqrt(-a^2 + b^2)*e^(n - 1))*arctan(-
(sqrt(-a^2 + b^2)*b*sin(d*x^n + c) + sqrt(-a^2 + b^2)*a)/((a^2 - b^2)*cos(d
*x^n + c))))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*n*sin(d*x^n + c) + (a^6*b - 2*a
^4*b^3 + a^2*b^5)*d*n)]
```

Sympy [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(a+b\csc(c+dx^n))^2}\,dx$$

```
[In] integrate((e*x)**(-1+n)/(a+b*csc(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(n - 1)/(a + b*csc(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] ((a^4 - a^2*b^2)*d*e^n*x^n*cos(2*d*x^n + 2*c)^2 - 2*a*b^3*e^n*cos(d*x^n + c
) + 4*(a^2*b^2 - b^4)*d*e^n*x^n*cos(d*x^n + c)^2 + (a^4 - a^2*b^2)*d*e^n*x^
n*sin(2*d*x^n + 2*c)^2 + 4*(a^2*b^2 - b^4)*d*e^n*x^n*sin(d*x^n + c)^2 + 4*(
a^3*b - a*b^3)*d*e^n*x^n*sin(d*x^n + c) + (a^4 - a^2*b^2)*d*e^n*x^n - 2*(a*
b^3*e^n*cos(d*x^n + c) + 2*(a^3*b - a*b^3)*d*e^n*x^n*sin(d*x^n + c) + (a^4
- a^2*b^2)*d*e^n*x^n)*cos(2*d*x^n + 2*c) + 2*((2*a^8*b - 3*a^6*b^3 + a^4*b^
5)*d*e^(n + 1)*n*cos(2*d*x^n + 2*c)^2*sin(c) + 4*(2*a^6*b^3 - 3*a^4*b^5 + a
^2*b^7)*d*e^(n + 1)*n*cos(d*x^n + c)^2*sin(c) + 4*(2*a^7*b^2 - 3*a^5*b^4 +
a^3*b^6)*d*e^(n + 1)*n*cos(d*x^n + c)*sin(2*d*x^n + 2*c)*sin(c) + (2*a^8*b
- 3*a^6*b^3 + a^4*b^5)*d*e^(n + 1)*n*sin(2*d*x^n + 2*c)^2*sin(c) + 4*(2*a^6
*b^3 - 3*a^4*b^5 + a^2*b^7)*d*e^(n + 1)*n*sin(d*x^n + c)^2*sin(c) + 4*(2*a^
7*b^2 - 3*a^5*b^4 + a^3*b^6)*d*e^(n + 1)*n*sin(d*x^n + c)*sin(c) + (2*a^8*b
 - 3*a^6*b^3 + a^4*b^5)*d*e^(n + 1)*n*sin(c) - 2*(2*(2*a^7*b^2 - 3*a^5*b^4
+ a^3*b^6)*d*e^(n + 1)*n*sin(d*x^n + c)*sin(c) + (2*a^8*b - 3*a^6*b^3 + a^4
*b^5)*d*e^(n + 1)*n*sin(c))*cos(2*d*x^n + 2*c))*integrate((a^3*x^n*cos(2*d*
x^n + 2*c)*cos(d*x^n) + a^3*x^n*sin(2*d*x^n + 2*c)*sin(d*x^n) - 2*(a^2*b -
b^3)*x^n*cos(d*x^n)^2*sin(c) - 2*(a^2*b - b^3)*x^n*sin(d*x^n)^2*sin(c) - (a
^3 - a*b^2)*x^n*cos(d*x^n) - (a*b^2*x^n*cos(d*x^n)*cos(2*c) + a*b^2*x^n*sin
(d*x^n)*sin(2*c))*cos(2*d*x^n) - (a*b^2*x^n*cos(2*c)*sin(d*x^n) - a*b^2*x^n
*cos(d*x^n)*sin(2*c))*sin(2*d*x^n))/(a^8*e*x*cos(2*d*x^n + 2*c)^2 + a^8*e*x
*sin(2*d*x^n + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*e*x*cos(2
*d*x^n)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*
b^4 + a^2*b^6)*sin(c)^2)*e*x*cos(d*x^n)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*s
in(2*c)^2)*e*x*sin(2*d*x^n)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*e*x*cos(c)*
sin(d*x^n) + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4
*b^4 + a^2*b^6)*sin(c)^2)*e*x*sin(d*x^n)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5
)*e*x*cos(d*x^n)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*e*x - 2*(2*((a^5*b^3
- a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*e*x*cos(d
*x^n) - (a^6*b^2 - a^4*b^4)*e*x*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*
cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*e*x*sin(d*x^n))*cos(2*d*x^n)
- 2*(a^6*b^2*e*x*cos(2*d*x^n)*cos(2*c) - a^6*b^2*e*x*sin(2*d*x^n)*sin(2*c)
+ 2*(a^7*b - a^5*b^3)*e*x*cos(c)*sin(d*x^n) + 2*(a^7*b - a^5*b^3)*e*x*cos(d
*x^n)*sin(c) + (a^8 - a^6*b^2)*e*x)*cos(2*d*x^n + 2*c) - 2*(2*((a^5*b^3 - a
^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*e*x*cos(d*x^
n) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*
sin(c))*e*x*sin(d*x^n) + (a^6*b^2 - a^4*b^4)*e*x*sin(2*c))*sin(2*d*x^n) - 2
*(a^6*b^2*e*x*cos(2*c)*sin(2*d*x^n) + a^6*b^2*e*x*cos(2*d*x^n)*sin(2*c) - 2
```

```
*(a^7*b - a^5*b^3)*e*x*cos(d*x^n)*cos(c) + 2*(a^7*b - a^5*b^3)*e*x*sin(d*x^
n)*sin(c))*sin(2*d*x^n + 2*c)), x) - 2*((2*a^8*b - 3*a^6*b^3 + a^4*b^5)*d*e
^(n + 1)*n*cos(2*d*x^n + 2*c)^2*cos(c) + 4*(2*a^6*b^3 - 3*a^4*b^5 + a^2*b^7
)*d*e^(n + 1)*n*cos(d*x^n + c)^2*cos(c) + 4*(2*a^7*b^2 - 3*a^5*b^4 + a^3*b^
6)*d*e^(n + 1)*n*cos(d*x^n + c)*cos(c)*sin(2*d*x^n + 2*c) + (2*a^8*b - 3*a^
6*b^3 + a^4*b^5)*d*e^(n + 1)*n*cos(c)*sin(2*d*x^n + 2*c)^2 + 4*(2*a^6*b^3 -
 3*a^4*b^5 + a^2*b^7)*d*e^(n + 1)*n*cos(c)*sin(d*x^n + c)^2 + 4*(2*a^7*b^2
- 3*a^5*b^4 + a^3*b^6)*d*e^(n + 1)*n*cos(c)*sin(d*x^n + c) + (2*a^8*b - 3*a
^6*b^3 + a^4*b^5)*d*e^(n + 1)*n*cos(c) - 2*(2*(2*a^7*b^2 - 3*a^5*b^4 + a^3*
b^6)*d*e^(n + 1)*n*cos(c)*sin(d*x^n + c) + (2*a^8*b - 3*a^6*b^3 + a^4*b^5)*
d*e^(n + 1)*n*cos(c))*cos(2*d*x^n + 2*c))*integrate((a^3*x^n*cos(d*x^n)*sin
(2*d*x^n + 2*c) - a^3*x^n*cos(2*d*x^n + 2*c)*sin(d*x^n) + 2*(a^2*b - b^3)*x
^n*cos(d*x^n)^2*cos(c) + 2*(a^2*b - b^3)*x^n*cos(c)*sin(d*x^n)^2 + (a^3 - a
*b^2)*x^n*sin(d*x^n) + (a*b^2*x^n*cos(2*c)*sin(d*x^n) - a*b^2*x^n*cos(d*x^n
)*sin(2*c))*cos(2*d*x^n) - (a*b^2*x^n*cos(d*x^n)*cos(2*c) + a*b^2*x^n*sin(d
*x^n)*sin(2*c))*sin(2*d*x^n))/(a^8*e*x*cos(2*d*x^n + 2*c)^2 + a^8*e*x*sin(2
*d*x^n + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*e*x*cos(2*d*x^n
)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 +
a^2*b^6)*sin(c)^2)*e*x*cos(d*x^n)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c
)^2)*e*x*sin(2*d*x^n)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*e*x*cos(c)*sin(d*
x^n) + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 +
 a^2*b^6)*sin(c)^2)*e*x*sin(d*x^n)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*e*x*
cos(d*x^n)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*e*x - 2*(2*((a^5*b^3 - a^3*
b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*e*x*cos(d*x^n)
- (a^6*b^2 - a^4*b^4)*e*x*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c)
 + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*e*x*sin(d*x^n))*cos(2*d*x^n) - 2*(a
^6*b^2*e*x*cos(2*d*x^n)*cos(2*c) - a^6*b^2*e*x*sin(2*d*x^n)*sin(2*c) + 2*(a
^7*b - a^5*b^3)*e*x*cos(c)*sin(d*x^n) + 2*(a^7*b - a^5*b^3)*e*x*cos(d*x^n)*
sin(c) + (a^8 - a^6*b^2)*e*x)*cos(2*d*x^n + 2*c) - 2*(2*((a^5*b^3 - a^3*b^5
)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*e*x*cos(d*x^n) + 2
*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c)
)*e*x*sin(d*x^n) + (a^6*b^2 - a^4*b^4)*e*x*sin(2*c))*sin(2*d*x^n) - 2*(a^6*
b^2*e*x*cos(2*c)*sin(2*d*x^n) + a^6*b^2*e*x*cos(2*d*x^n)*sin(2*c) - 2*(a^7*
b - a^5*b^3)*e*x*cos(d*x^n)*cos(c) + 2*(a^7*b - a^5*b^3)*e*x*sin(d*x^n)*sin
(c))*sin(2*d*x^n + 2*c)), x) - 2*(a*b^3*e^n*sin(d*x^n + c) + a^2*b^2*e^n -
2*(a^3*b - a*b^3)*d*e^n*x^n*cos(d*x^n + c))*sin(2*d*x^n + 2*c))/((a^6 - a^4
*b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n +
c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c)*sin(2*d*x^n + 2*c) + (a^6 -
 a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*x^
n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n
 - 2*(2*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n)*cos
(2*d*x^n + 2*c))
```

Giac [F]

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+n)/(a+b*csc(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((e*x)^(n - 1)/(b*csc(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(e\,x)^{n-1}}{\left(a+\frac{b}{\sin(c+d\,x^n)}\right)^2}\,dx$$

```
[In] int((e*x)^(n - 1)/(a + b/sin(c + d*x^n))^2,x)
[Out] int((e*x)^(n - 1)/(a + b/sin(c + d*x^n))^2, x)
```

3.83 $\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 778

$$\begin{aligned}
\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx &= \frac{(ex)^{2n}}{2a^2en} - \frac{ib^3x^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} \\
&+ \frac{2ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} \\
&+ \frac{ib^3x^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den} \\
&- \frac{2ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} \\
&+ \frac{b^2x^{-2n}(ex)^{2n}\log(b+a\sin(c+dx^n))}{a^2(a^2-b^2)d^2en} \\
&- \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2en} \\
&+ \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} \\
&+ \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}d^2en} \\
&- \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} \\
&- \frac{b^2x^{-n}(ex)^{2n}\cos(c+dx^n)}{a(a^2-b^2)den(b+a\sin(c+dx^n))}
\end{aligned}$$

```
[Out] 1/2*(e*x)^(2*n)/a^2/e/n+b^2*(e*x)^(2*n)*ln(b+a*sin(c+d*x^n))/a^2/(a^2-b^2)/
d^2/e/n/(x^(2*n))-I*b^3*(e*x)^(2*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)
^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)+I*b^3*(e*x)^(2*n)*ln(1-I*a*exp(I*
(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)-b^3*(e*x)
^(2*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^
(3/2)/d^2/e/n/(x^(2*n))+b^3*(e*x)^(2*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b+(
-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))-b^2*(e*x)^(2*n)*co
s(c+d*x^n)/a/(a^2-b^2)/d/e/n/(x^n)/(b+a*sin(c+d*x^n))+2*I*b*(e*x)^(2*n)*ln(
1-I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/d/e/n/(x^n)/(-a^2+b^2)^(1/
2)-2*I*b*(e*x)^(2*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/d/
e/n/(x^n)/(-a^2+b^2)^(1/2)+2*b*(e*x)^(2*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(
b-(-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)-2*b*(e*x)^(2*n)
*polylog(2,I*a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))
/(-a^2+b^2)^(1/2)
```

Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 778, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules

used = {4294, 4290, 4276, 3405, 3404, 2296, 2221, 2317, 2438, 2747, 31}

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx = \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} - \frac{2bx^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2en\sqrt{b^2-a^2}} + \frac{b^2x^{-2n}(ex)^{2n}\log(a\sin(c+dx^n)+b)}{a^2d^2en(a^2-b^2)} + \frac{2ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{a^2den\sqrt{b^2-a^2}} - \frac{2ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{a^2den\sqrt{b^2-a^2}} - \frac{b^2x^{-n}(ex)^{2n}\cos(c+dx^n)}{aden(a^2-b^2)(a\sin(c+dx^n)+b)} - \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)}{a^2d^2en(b^2-a^2)^{3/2}} + \frac{b^3x^{-2n}(ex)^{2n}\operatorname{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)}{a^2d^2en(b^2-a^2)^{3/2}} - \frac{ib^3x^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{b^2-a^2}}\right)}{a^2den(b^2-a^2)^{3/2}} + \frac{ib^3x^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{\sqrt{b^2-a^2}+b}\right)}{a^2den(b^2-a^2)^{3/2}} + \frac{(ex)^{2n}}{2a^2en}$$

```
[In] Int[(e*x)^(-1 + 2*n)/(a + b*Csc[c + d*x^n])^2,x]
```

```
[Out] (e*x)^(2*n)/(2*a^2*e*n) - (I*b^3*(e*x)^(2*n)*Log[1 - (I*a*E^(I*(c + d*x^n))
)/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) + ((2*I)*b*(e
*x)^(2*n)*Log[1 - (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqr
t[-a^2 + b^2]*d*e*n*x^n) + (I*b^3*(e*x)^(2*n)*Log[1 - (I*a*E^(I*(c + d*x^n)
))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) - ((2*I)*b*(
e*x)^(2*n)*Log[1 - (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sq
rt[-a^2 + b^2]*d*e*n*x^n) + (b^2*(e*x)^(2*n)*Log[b + a*Sin[c + d*x^n]])/(a^
2*(a^2 - b^2)*d^2*e*n*x^(2*n)) - (b^3*(e*x)^(2*n)*PolyLog[2, (I*a*E^(I*(c +
 d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2*e*n*x^(2*n))
 + (2*b*(e*x)^(2*n)*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2
])])/(a^2*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) + (b^3*(e*x)^(2*n)*PolyLog[2, (
I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2
*e*n*x^(2*n)) - (2*b*(e*x)^(2*n)*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b + Sq
```

```
rt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) - (b^2*(e*x)^(2*n)
*Cos[c + d*x^n])/(a*(a^2 - b^2)*d*e*n*x^n*(b + a*Sin[c + d*x^n]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
```

```
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(x^{-2n}(ex)^{2n}\right)\int \frac{x^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx}{e} \\
&= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{x}{(a+b\csc(c+dx))^2}\,dx,x,x^n\right)}{en} \\
&= \frac{\left(x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \left(\frac{x}{a^2}+\frac{b^2x}{a^2(b+a\sin(c+dx))^2}-\frac{2bx}{a^2(b+a\sin(c+dx))}\right)dx,x,x^n\right)}{en} \\
&= \frac{(ex)^{2n}}{2a^2en}-\frac{\left(2bx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{x}{b+a\sin(c+dx)}\,dx,x,x^n\right)}{a^2en} \\
&\quad+\frac{\left(b^2x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{x}{(b+a\sin(c+dx))^2}\,dx,x,x^n\right)}{a^2en}
\end{aligned}$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{b^2x^{-n}(ex)^{2n}\cos(c+dx^n)}{a(a^2-b^2)den(b+a\sin(c+dx^n))} - \frac{(4bx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx,x,x^n\right)}{a^2en} - \frac{(b^3x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{x}{b+a\sin(c+dx)}\,dx,x,x^n\right)}{a^2(a^2-b^2)en} + \frac{(b^2x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{\cos(c+dx)}{b+a\sin(c+dx)}\,dx,x,x^n\right)}{a(a^2-b^2)den}$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{b^2x^{-n}(ex)^{2n}\cos(c+dx^n)}{a(a^2-b^2)den(b+a\sin(c+dx^n))} - \frac{(2b^3x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx,x,x^n\right)}{a^2(a^2-b^2)en} + \frac{(4ibx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx,x,x^n\right)}{a\sqrt{-a^2+b^2}en} - \frac{(4ibx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx,x,x^n\right)}{a\sqrt{-a^2+b^2}en} + \frac{(b^2x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{1}{b+x}\,dx,x,a\sin(c+dx^n)\right)}{a^2(a^2-b^2)d^2en}$$

$$= \frac{(ex)^{2n}}{2a^2en} + \frac{2ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{b^2x^{-2n}(ex)^{2n}\log(b+a\sin(c+dx^n))}{a^2(a^2-b^2)d^2en} - \frac{b^2x^{-n}(ex)^{2n}\cos(c+dx^n)}{a(a^2-b^2)den(b+a\sin(c+dx^n))} + \frac{(2ib^3x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx,x,x^n\right)}{a(a^2-b^2)\sqrt{-a^2+b^2}en} - \frac{(2ib^3x^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx,x,x^n\right)}{a(a^2-b^2)\sqrt{-a^2+b^2}en} - \frac{(2ibx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{(2ibx^{-2n}(ex)^{2n})\,\text{Subst}\left(\int \log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}den}$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{ib^3x^{-n}(ex)^{2n}\log\left(1 - \frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} + \frac{2ibx^{-n}(ex)^{2n}\log\left(1 - \frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$+ \frac{ib^3x^{-n}(ex)^{2n}\log\left(1 - \frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} - \frac{2ibx^{-n}(ex)^{2n}\log\left(1 - \frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$+ \frac{b^2x^{-2n}(ex)^{2n}\log\left(b + a\sin\left(c+dx^n\right)\right)}{a^2\left(a^2-b^2\right)d^2en} - \frac{b^2x^{-n}(ex)^{2n}\cos\left(c+dx^n\right)}{a\left(a^2-b^2\right)den\left(b+a\sin\left(c+dx^n\right)\right)}$$
$$- \frac{\left(2bx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{2iax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+dx^n)}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$
$$+ \frac{\left(2bx^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{2iax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+dx^n)}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$
$$- \frac{\left(ib^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \log\left(1 - \frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a^2\left(a^2-b^2\right)\sqrt{-a^2+b^2}den}$$
$$+ \frac{\left(ib^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \log\left(1 - \frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx, x, x^n\right)}{a^2\left(a^2-b^2\right)\sqrt{-a^2+b^2}den}$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{ib^3x^{-n}(ex)^{2n}\log\left(1 - \frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} + \frac{2ibx^{-n}(ex)^{2n}\log\left(1 - \frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$+ \frac{ib^3x^{-n}(ex)^{2n}\log\left(1 - \frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\left(-a^2+b^2\right)^{3/2}den} - \frac{2ibx^{-n}(ex)^{2n}\log\left(1 - \frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$
$$+ \frac{b^2x^{-2n}(ex)^{2n}\log\left(b + a\sin\left(c+dx^n\right)\right)}{a^2\left(a^2-b^2\right)d^2en} + \frac{2bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2, \frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$
$$- \frac{2bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2, \frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} - \frac{b^2x^{-n}(ex)^{2n}\cos\left(c+dx^n\right)}{a\left(a^2-b^2\right)den\left(b+a\sin\left(c+dx^n\right)\right)}$$
$$- \frac{\left(b^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{2iax}{2b-2\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+dx^n)}\right)}{a^2\left(a^2-b^2\right)\sqrt{-a^2+b^2}d^2en}$$
$$+ \frac{\left(b^3x^{-2n}(ex)^{2n}\right)\text{Subst}\left(\int \frac{\log\left(1-\frac{2iax}{2b+2\sqrt{-a^2+b^2}}\right)}{x}dx, x, e^{i(c+dx^n)}\right)}{a^2\left(a^2-b^2\right)\sqrt{-a^2+b^2}d^2en}$$

$$= \frac{(ex)^{2n}}{2a^2en} - \frac{ib^3x^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} + \frac{2ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{ib^3x^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} - \frac{2ibx^{-n}(ex)^{2n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{b^2x^{-2n}(ex)^{2n}\log(b+a\sin(c+dx^n))}{a^2(a^2-b^2)\,d^2en} - \frac{b^3x^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} + \frac{2bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{b^3x^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} - \frac{2bx^{-2n}(ex)^{2n}\,\text{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} - \frac{b^2x^{-n}(ex)^{2n}\cos(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\sin(c+dx^n))}$$

Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2839 vs. 2(778) = 1556.

Time = 11.17 (sec) , antiderivative size = 2839, normalized size of antiderivative = 3.65

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(e*x)^(-1 + 2*n)/(a + b*Csc[c + d*x^n])^2,x]
```

```
[Out] -1/2*(b^2*x^(1 - n)*(e*x)^(-1 + 2*n)*Csc[c/2]*Csc[c + d*x^n]^2*Sec[c/2]*(b*
Cos[c] + a*Sin[d*x^n])*(b + a*Sin[c + d*x^n]))/(a^2*(-a + b)*(a + b)*d*n*(a
 + b*Csc[c + d*x^n])^2) - (b^2*x^(1 - n)*(e*x)^(-1 + 2*n)*Cot[c]*Csc[c + d*
x^n]^2*(b + a*Sin[c + d*x^n])^2)/(a^2*(-a^2 + b^2)*d*n*(a + b*Csc[c + d*x^n
])^2) + (2*b^3*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*ArcTanh[(a*Cos[c + d*x^n] + I*(
b + a*Sin[c + d*x^n]))/Sqrt[a^2 - b^2]]*Cot[c]*Csc[c + d*x^n]^2*(b + a*Sin[
c + d*x^n])^2)/(a^2*(a^2 - b^2)^(3/2)*d^2*n*(a + b*Csc[c + d*x^n])^2) - (2*
b*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*Csc[c + d*x^n]^2*((Pi*ArcTan[(a + b*Tan[(c +
 d*x^n)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(-c + Pi/2 - d*x^n)*Ar
cTanh[((a + b)*Cot[(-c + Pi/2 - d*x^n)/2]])/Sqrt[a^2 - b^2]] - 2*(-c + ArcCo
s[-(b/a)])*ArcTanh[((a - b)*Tan[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2 - b^2]] +
(ArcCos[-(b/a)] - (2*I)*(ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x^n)/2])/Sqrt[
a^2 - b^2]] - ArcTanh[((a - b)*Tan[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2 - b^2]]
))*Log[Sqrt[a^2 - b^2]/(Sqrt[2]*Sqrt[a]*E^((I/2)*(-c + Pi/2 - d*x^n))*Sqrt[
b + a*Sin[c + d*x^n]])] + (ArcCos[-(b/a)] + (2*I)*(ArcTanh[((a + b)*Cot[(-c
 + Pi/2 - d*x^n)/2])/Sqrt[a^2 - b^2]] - ArcTanh[((a - b)*Tan[(-c + Pi/2 - d
*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^2]*E^((I/2)*(-c + Pi/2 - d*x
^n)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Sin[c + d*x^n]])] - (ArcCos[-(b/a)] + (2*
```

```
I)*ArcTanh[((a - b)*Tan[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2 - b^2]])*Log[1 - (
(b - I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(-c + Pi/2 - d*x^n)/2]
))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(-c + Pi/2 - d*x^n)/2]))] + (-ArcCos[-(b
/a)] + (2*I)*ArcTanh[((a - b)*Tan[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2 - b^2]])
*Log[1 - ((b + I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(-c + Pi/2 -
 d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(-c + Pi/2 - d*x^n)/2]))] + I*
(PolyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(-c + Pi/
2 - d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(-c + Pi/2 - d*x^n)/2]))] -
 PolyLog[2, ((b + I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(-c + Pi/
2 - d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(-c + Pi/2 - d*x^n)/2]))]))
/Sqrt[a^2 - b^2])*(b + a*Sin[c + d*x^n])^2)/((a^2 - b^2)*d^2*n*(a + b*Csc[c
 + d*x^n])^2) + (b^3*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*Csc[c + d*x^n]^2*((Pi*Arc
Tan[(a + b*Tan[(c + d*x^n)/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (2*(-c
 + Pi/2 - d*x^n)*ArcTanh[((a + b)*Cot[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2 - b^
2]] - 2*(-c + ArcCos[-(b/a)])*ArcTanh[((a - b)*Tan[(-c + Pi/2 - d*x^n)/2])/
Sqrt[a^2 - b^2]] + (ArcCos[-(b/a)] - (2*I)*(ArcTanh[((a + b)*Cot[(-c + Pi/2
 - d*x^n)/2])/Sqrt[a^2 - b^2]] - ArcTanh[((a - b)*Tan[(-c + Pi/2 - d*x^n)/2
])/Sqrt[a^2 - b^2]]))*Log[Sqrt[a^2 - b^2]/(Sqrt[2]*Sqrt[a]*E^((I/2)*(-c + P
i/2 - d*x^n))*Sqrt[b + a*Sin[c + d*x^n]])] + (ArcCos[-(b/a)] + (2*I)*(ArcTa
nh[((a + b)*Cot[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2 - b^2]] - ArcTanh[((a - b)
*Tan[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2 - b^2]]))*Log[(Sqrt[a^2 - b^2]*E^((I/
2)*(-c + Pi/2 - d*x^n)))/(Sqrt[2]*Sqrt[a]*Sqrt[b + a*Sin[c + d*x^n]])] - (A
rcCos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Tan[(-c + Pi/2 - d*x^n)/2])/Sqrt[a^2
 - b^2]])*Log[1 - ((b - I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^2]*Tan[(-c
 + Pi/2 - d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(-c + Pi/2 - d*x^n)/2
]))] + (-ArcCos[-(b/a)] + (2*I)*ArcTanh[((a - b)*Tan[(-c + Pi/2 - d*x^n)/2]
)/Sqrt[a^2 - b^2]])*Log[1 - ((b + I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 - b^
2]*Tan[(-c + Pi/2 - d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(-c + Pi/2
- d*x^n)/2]))] + I*(PolyLog[2, ((b - I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 -
 b^2]*Tan[(-c + Pi/2 - d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(-c + Pi
/2 - d*x^n)/2]))] - PolyLog[2, ((b + I*Sqrt[a^2 - b^2])*(a + b - Sqrt[a^2 -
 b^2]*Tan[(-c + Pi/2 - d*x^n)/2]))/(a*(a + b + Sqrt[a^2 - b^2]*Tan[(-c + Pi
/2 - d*x^n)/2]))]))/Sqrt[a^2 - b^2])*(b + a*Sin[c + d*x^n])^2)/(a^2*(a^2 -
b^2)*d^2*n*(a + b*Csc[c + d*x^n])^2) + (x^(1 - n)*(e*x)^(-1 + 2*n)*Csc[c/2]
*Csc[c + d*x^n]^2*Sec[c/2]*(-2*b^2*Cos[c] + a^2*d*x^n*Sin[c] - b^2*d*x^n*Si
n[c])*(b + a*Sin[c + d*x^n])^2)/(4*a^2*(a - b)*(a + b)*d*n*(a + b*Csc[c + d
*x^n])^2) + (b^2*x^(1 - 2*n)*(e*x)^(-1 + 2*n)*Csc[c]*Csc[c + d*x^n]^2*(-(a*
d*x^n*Cos[c]) + a*Log[b + a*Cos[d*x^n]*Sin[c] + a*Cos[c]*Sin[d*x^n]]*Sin[c]
 + ((2*I)*a*b*ArcTan[(I*a*Cos[c] - I*(-b + a*Sin[c])*Tan[(d*x^n)/2])/Sqrt[-
b^2 + a^2*Cos[c]^2 + a^2*Sin[c]^2]]*Cos[c])/Sqrt[-b^2 + a^2*Cos[c]^2 + a^2*
Sin[c]^2])*(b + a*Sin[c + d*x^n])^2)/(a*(a^2 - b^2)*d^2*n*(a + b*Csc[c + d*
x^n])^2*(a^2*Cos[c]^2 + a^2*Sin[c]^2))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.14 (sec) , antiderivative size = 3867, normalized size of antiderivative = 4.97

method	result	size
risch	Expression too large to display	3867

```
[In] int((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2/n*x*exp(1/2*(-1+2*n)*(-I*csgn(I*e)*csgn(I*x)*csgn(I*e*x)*Pi+I*csgn(
I*e)*csgn(I*e*x)^2*Pi+I*csgn(I*x)*csgn(I*e*x)^2*Pi-I*csgn(I*e*x)^3*Pi+2*ln(
e)+2*ln(x)))-2*I*b^2/a^2/(-a^2+b^2)/d/n*x^n/(2*b*exp(I*(c+d*x^n))-I*a*exp(2
*I*(c+d*x^n))+I*a)*(e^n)^2*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*(b*ex
p(1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e
*x)-2*Pi*n*csgn(I*e*x)^3+2*Pi*n*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e)*csgn(I
*e*x)^2+Pi*csgn(I*e*x)^3-Pi*csgn(I*x)*csgn(I*e*x)^2+2*d*x^n+2*c))+I*exp(1/2
*I*Pi*csgn(I*e*x)*(-2*csgn(I*e*x)^2*n+2*csgn(I*e)*csgn(I*e*x)*n+2*csgn(I*x)
*csgn(I*e*x)*n-2*csgn(I*e)*csgn(I*x)*n+csgn(I*e*x)^2-csgn(I*e)*csgn(I*e*x)-
csgn(I*x)*csgn(I*e*x)))*a)/e-2/d/(a^2-b^2)^2*b*(a^2*exp(2*I*c)-exp(2*I*c)*b
^2)^(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*
Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e
*x)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn
(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*x^n*ln((-I*exp(I*c)*b-a*exp(I*(d
*x^n+2*c))+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))/(-I*exp(I*c)*b+(a^2*exp(2
*I*c)-exp(2*I*c)*b^2)^(1/2)))+1/a^2/d/(a^2-b^2)^2*b^3*(a^2*exp(2*I*c)-exp(2
*I*c)*b^2)^(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*
e*x)-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*c
sgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+
Pi*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*x^n*ln((-I*exp(I*c)*b-a*e
xp(I*(d*x^n+2*c))+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))/(-I*exp(I*c)*b+(a^
2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)))+2/d/(a^2-b^2)^2*b*(a^2*exp(2*I*c)-exp(
2*I*c)*b^2)^(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I
*e*x)-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*
csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2
+Pi*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*x^n*ln((I*exp(I*c)*b+a*e
xp(I*(d*x^n+2*c))+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))/(I*exp(I*c)*b+(a^2
*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)))-1/a^2/d/(a^2-b^2)^2*b^3*(a^2*exp(2*I*c)
-exp(2*I*c)*b^2)^(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*c
sgn(I*e*x)-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*
Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e
*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*x^n*ln((I*exp(I*c)*
b+a*exp(I*(d*x^n+2*c))+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))/(I*exp(I*c)*b
+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)))+2*I/d^2/(a^2-b^2)^2*b*(a^2*exp(2*I
*c)-exp(2*I*c)*b^2)^(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x
```

```
)*csgn(I*e*x)-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2
+2*Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(
I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*dilog(-I/(-I*exp
(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*exp(I*c)*b-a/(-I*exp(I*c)*b+
(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*exp(I*(d*x^n+2*c))+1/(-I*exp(I*c)*b+
(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2
))-I/a^2/d^2/(a^2-b^2)^2*b^3*(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/n/e*(e^n
)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*n*csgn(I*e)*csg
n(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)
*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^
2-Pi*csgn(I*e*x)^3+2*c))*dilog(-I/(-I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)
*b^2)^(1/2))*exp(I*c)*b-a/(-I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1
/2))*exp(I*(d*x^n+2*c))+1/(-I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1
/2))*(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))-2*I/d^2/(a^2-b^2)^2*b*(a^2*exp(
2*I*c)-exp(2*I*c)*b^2)^(1/2)/n/e*(e^n)^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(
I*x)*csgn(I*e*x)-2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x
)^2+2*Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*cs
gn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2-Pi*csgn(I*e*x)^3+2*c))*dilog(I/(I*ex
p(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*exp(I*c)*b+a/(I*exp(I*c)*b+
(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*exp(I*(d*x^n+2*c))+1/(I*exp(I*c)*b+(
a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)
)+I/a^2/d^2/(a^2-b^2)^2*b^3*(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/n/e*(e^n)
^2*exp(-1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)-2*Pi*n*csgn(I*e)*csgn
(I*e*x)^2-2*Pi*n*csgn(I*x)*csgn(I*e*x)^2+2*Pi*n*csgn(I*e*x)^3-Pi*csgn(I*e)*
csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e)*csgn(I*e*x)^2+Pi*csgn(I*x)*csgn(I*e*x)^2
-Pi*csgn(I*e*x)^3+2*c))*dilog(I/(I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^
2)^(1/2))*exp(I*c)*b+a/(I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))
*exp(I*(d*x^n+2*c))+1/(I*exp(I*c)*b+(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))*
(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2))+2/a^2/d^2/(a^2-b^2)*b^3/n*arctan(1/2
*(2*I*a*exp(I*(d*x^n+2*c))-2*exp(I*c)*b)/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1
/2))/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)/e*(e^n)^2*exp(1/2*I*(-2*Pi*n*csg
n(I*e*x)^3+2*Pi*n*csgn(I*e)*csgn(I*e*x)^2+2*Pi*n*csgn(I*x)*csgn(I*e*x)^2-2*
Pi*n*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+Pi*csgn(I*e*x)^3-Pi*csgn(I*e)*csgn(I*e
*x)^2-Pi*csgn(I*x)*csgn(I*e*x)^2+Pi*csgn(I*e)*csgn(I*x)*csgn(I*e*x)+2*c))-2
/a^2/d^2/(a^2-b^2)*b^3/n/e*(e^n)^2*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x
))/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)*arctan(I*a/(a^2*exp(2*I*c)-exp(2*I
*c)*b^2)^(1/2)*exp(I*(d*x^n+2*c))-1/(a^2*exp(2*I*c)-exp(2*I*c)*b^2)^(1/2)*e
xp(I*c)*b)*exp(1/2*I*(2*Pi*n*csgn(I*e)*csgn(I*e*x)^2-2*Pi*n*csgn(I*e)*csgn(
I*x)*csgn(I*e*x)-2*Pi*n*csgn(I*e*x)^3+2*Pi*n*csgn(I*x)*csgn(I*e*x)^2-Pi*csg
n(I*e)*csgn(I*e*x)^2+Pi*csgn(I*e*x)^3-Pi*csgn(I*x)*csgn(I*e*x)^2+2*c))+2/a^
2/d^2/(a^2-b^2)*b^2/n/e*(e^n)^2*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*
Im(d*x^n)*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e*x)^2*n+2*csgn(I*e)*csgn(I*e
*x)*n+2*csgn(I*x)*csgn(I*e*x)*n-2*csgn(I*e)*csgn(I*x)*n+csgn(I*e*x)^2-csgn(
I*e)*csgn(I*e*x)-csgn(I*x)*csgn(I*e*x)))-2/a^2/d^2/(a^2-b^2)*b^2/n/e*(e^n)^
2*(-1)^(1/2*csgn(I*e)*csgn(I*x)*csgn(I*e*x))*ln(exp(I*Re(d*x^n)))*exp(1/2*I
```

```
*Pi*csgn(I*e*x)*(-2*csgn(I*e*x)^2*n+2*csgn(I*e)*csgn(I*e*x)*n+2*csgn(I*x)*c
sgn(I*e*x)*n-2*csgn(I*e)*csgn(I*x)*n+csgn(I*e*x)^2-csgn(I*e)*csgn(I*e*x)-cs
gn(I*x)*csgn(I*e*x)))+1/a^2/d^2/(a^2-b^2)*b^2/n/e*(e^n)^2*(-1)^(1/2*csgn(I*
e)*csgn(I*x)*csgn(I*e*x))*ln(-2*b*exp(I*(c+d*x^n))+I*a*exp(2*I*(c+d*x^n))-I
*a)*exp(1/2*I*Pi*csgn(I*e*x)*(-2*csgn(I*e*x)^2*n+2*csgn(I*e)*csgn(I*e*x)*n+
2*csgn(I*x)*csgn(I*e*x)*n-2*csgn(I*e)*csgn(I*x)*n+csgn(I*e*x)^2-csgn(I*e)*c
sgn(I*e*x)-csgn(I*x)*csgn(I*e*x)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2435 vs. 2(710) = 1420.

Time = 0.52 (sec) , antiderivative size = 2435, normalized size of antiderivative = 3.13

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^5 - 2*a^3*b^2 + a*b^4)*d^2*e^(2*n - 1)*x^(2*n)*sin(d*x^n + c) + (a^
4*b - 2*a^2*b^3 + b^5)*d^2*e^(2*n - 1)*x^(2*n) - 2*(a^3*b^2 - a*b^4)*d*e^(2
*n - 1)*x^n*cos(d*x^n + c) + ((2*I*a^4*b - I*a^2*b^3)*e^(2*n - 1)*sqrt((a^2
 - b^2)/a^2)*sin(d*x^n + c) + (2*I*a^3*b^2 - I*a*b^4)*e^(2*n - 1)*sqrt((a^2
 - b^2)/a^2))*dilog(((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) + (I*a*
sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a + 1) + ((2*I*a^4*b - I*a^2
*b^3)*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*sin(d*x^n + c) + (2*I*a^3*b^2 - I*a
*b^4)*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*dilog(-((a*sqrt((a^2 - b^2)/a^2) +
 I*b)*cos(d*x^n + c) - (I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) + a)/
a + 1) + ((-2*I*a^4*b + I*a^2*b^3)*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)*sin(d*
x^n + c) + (-2*I*a^3*b^2 + I*a*b^4)*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*dilo
g(((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) + (-I*a*sqrt((a^2 - b^2)/
a^2) - b)*sin(d*x^n + c) - a)/a + 1) + ((-2*I*a^4*b + I*a^2*b^3)*e^(2*n - 1
)*sqrt((a^2 - b^2)/a^2)*sin(d*x^n + c) + (-2*I*a^3*b^2 + I*a*b^4)*e^(2*n -
1)*sqrt((a^2 - b^2)/a^2))*dilog(-((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n
 + c) - (-I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) + a)/a + 1) + ((a^3
*b^2 - a*b^4 - (2*a^4*b - a^2*b^3)*c*sqrt((a^2 - b^2)/a^2))*e^(2*n - 1)*sin
(d*x^n + c) + (a^2*b^3 - b^5 - (2*a^3*b^2 - a*b^4)*c*sqrt((a^2 - b^2)/a^2))
*e^(2*n - 1))*log(2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt((a^2
 - b^2)/a^2) + 2*I*b) + ((a^3*b^2 - a*b^4 - (2*a^4*b - a^2*b^3)*c*sqrt((a^2
 - b^2)/a^2))*e^(2*n - 1)*sin(d*x^n + c) + (a^2*b^3 - b^5 - (2*a^3*b^2 - a*
b^4)*c*sqrt((a^2 - b^2)/a^2))*e^(2*n - 1))*log(2*a*cos(d*x^n + c) - 2*I*a*s
in(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b) + ((a^3*b^2 - a*b^4 + (2
*a^4*b - a^2*b^3)*c*sqrt((a^2 - b^2)/a^2))*e^(2*n - 1)*sin(d*x^n + c) + (a^
2*b^3 - b^5 + (2*a^3*b^2 - a*b^4)*c*sqrt((a^2 - b^2)/a^2))*e^(2*n - 1))*log
```

```
(-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) + 2
*I*b) + ((a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3)*c*sqrt((a^2 - b^2)/a^2))*e^
(2*n - 1)*sin(d*x^n + c) + (a^2*b^3 - b^5 + (2*a^3*b^2 - a*b^4)*c*sqrt((a^2
 - b^2)/a^2))*e^(2*n - 1))*log(-2*a*cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) +
 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b) - ((2*a^3*b^2 - a*b^4)*d*e^(2*n - 1)*x^
n*sqrt((a^2 - b^2)/a^2) + (2*a^3*b^2 - a*b^4)*c*e^(2*n - 1)*sqrt((a^2 - b^2
)/a^2) + ((2*a^4*b - a^2*b^3)*d*e^(2*n - 1)*x^n*sqrt((a^2 - b^2)/a^2) + (2*
a^4*b - a^2*b^3)*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*sin(d*x^n + c))*log(-
((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) + (I*a*sqrt((a^2 - b^2)/a^2
) - b)*sin(d*x^n + c) - a)/a) + ((2*a^3*b^2 - a*b^4)*d*e^(2*n - 1)*x^n*sqrt
((a^2 - b^2)/a^2) + (2*a^3*b^2 - a*b^4)*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2)
 + ((2*a^4*b - a^2*b^3)*d*e^(2*n - 1)*x^n*sqrt((a^2 - b^2)/a^2) + (2*a^4*b
- a^2*b^3)*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*sin(d*x^n + c))*log(((a*sqr
t((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) - (I*a*sqrt((a^2 - b^2)/a^2) - b)*
sin(d*x^n + c) + a)/a) - ((2*a^3*b^2 - a*b^4)*d*e^(2*n - 1)*x^n*sqrt((a^2 -
 b^2)/a^2) + (2*a^3*b^2 - a*b^4)*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2) + ((2*
a^4*b - a^2*b^3)*d*e^(2*n - 1)*x^n*sqrt((a^2 - b^2)/a^2) + (2*a^4*b - a^2*b
^3)*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*sin(d*x^n + c))*log(-((a*sqrt((a^2
 - b^2)/a^2) - I*b)*cos(d*x^n + c) + (-I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d
*x^n + c) - a)/a) + ((2*a^3*b^2 - a*b^4)*d*e^(2*n - 1)*x^n*sqrt((a^2 - b^2)
/a^2) + (2*a^3*b^2 - a*b^4)*c*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2) + ((2*a^4*b
 - a^2*b^3)*d*e^(2*n - 1)*x^n*sqrt((a^2 - b^2)/a^2) + (2*a^4*b - a^2*b^3)*c
*e^(2*n - 1)*sqrt((a^2 - b^2)/a^2))*sin(d*x^n + c))*log(((a*sqrt((a^2 - b^2
)/a^2) - I*b)*cos(d*x^n + c) - (-I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n +
 c) + a)/a))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d^2*n*sin(d*x^n + c) + (a^6*b - 2
*a^4*b^3 + a^2*b^5)*d^2*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{2n-1}}{(a+b\csc(c+dx^n))^2}\,dx$$

```
[In] integrate((e*x)**(-1+2*n)/(a+b*csc(c+d*x**n))**2,x)
[Out] Integral((e*x)**(2*n - 1)/(a + b*csc(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{2n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*b^3*e^(2*n)*x^n*cos(d*x^n + c) - (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n
)*cos(2*d*x^n + 2*c)^2 - 4*(a^2*b^2 - b^4)*d*e^(2*n)*x^(2*n)*cos(d*x^n + c)
^2 - (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n)*sin(2*d*x^n + 2*c)^2 - 4*(a^2*b^2 -
b^4)*d*e^(2*n)*x^(2*n)*sin(d*x^n + c)^2 - 4*(a^3*b - a*b^3)*d*e^(2*n)*x^(2*
n)*sin(d*x^n + c) - (a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n) + 2*(2*a*b^3*e^(2*n)*
x^n*cos(d*x^n + c) + 2*(a^3*b - a*b^3)*d*e^(2*n)*x^(2*n)*sin(d*x^n + c) + (
a^4 - a^2*b^2)*d*e^(2*n)*x^(2*n))*cos(2*d*x^n + 2*c) - 2*((a^6 - a^4*b^2)*d
*e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n + c)^2 +
4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c)*sin(2*d*x^n + 2*c) + (a^6 - a^4*b^
2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*x^n + c)^
2 + 4*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n - 2*(2
*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n)*cos(2*d*x^
n + 2*c))*integrate(-2*(a^2*b^4*e^(2*n)*x^n*cos(2*c)*sin(2*d*x^n) + a^2*b^4
*e^(2*n)*x^n*cos(2*d*x^n)*sin(2*c) - 2*(a^3*b^3 - a*b^5)*e^(2*n)*x^n*cos(d*
x^n)*cos(c) + 2*(a^3*b^3 - a*b^5)*e^(2*n)*x^n*sin(d*x^n)*sin(c) - (a^3*b^3*
e^(2*n)*x^n*cos(d*x^n + c) + (2*a^5*b - a^3*b^3)*d*e^(2*n)*x^(2*n)*sin(d*x^
n + c))*cos(2*d*x^n + 2*c) + ((a^3*b^3 - a*b^5)*e^(2*n)*x^n + (a*b^5*e^(2*n
)*x^n*cos(2*c) - (2*a^3*b^3 - a*b^5)*d*e^(2*n)*x^(2*n)*sin(2*c))*cos(2*d*x^
n) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(2*n)*x^(2*n)*cos(c) + (a^2*b^4 -
 b^6)*e^(2*n)*x^n*sin(c))*cos(d*x^n) - (a*b^5*e^(2*n)*x^n*sin(2*c) + (2*a^3
*b^3 - a*b^5)*d*e^(2*n)*x^(2*n)*cos(2*c))*sin(2*d*x^n) - 2*((2*a^4*b^2 - 3*
a^2*b^4 + b^6)*d*e^(2*n)*x^(2*n)*sin(c) - (a^2*b^4 - b^6)*e^(2*n)*x^n*cos(c
))*sin(d*x^n))*cos(d*x^n + c) - (a^3*b^3*e^(2*n)*x^n*sin(d*x^n + c) + a^4*b
^2*e^(2*n)*x^n - (2*a^5*b - a^3*b^3)*d*e^(2*n)*x^(2*n)*cos(d*x^n + c))*sin(
2*d*x^n + 2*c) + ((2*a^5*b - 3*a^3*b^3 + a*b^5)*d*e^(2*n)*x^(2*n) + (a*b^5*
e^(2*n)*x^n*sin(2*c) + (2*a^3*b^3 - a*b^5)*d*e^(2*n)*x^(2*n)*cos(2*c))*cos(
2*d*x^n) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(2*n)*x^(2*n)*sin(c) - (a^2
*b^4 - b^6)*e^(2*n)*x^n*cos(c))*cos(d*x^n) + (a*b^5*e^(2*n)*x^n*cos(2*c) -
(2*a^3*b^3 - a*b^5)*d*e^(2*n)*x^(2*n)*sin(2*c))*sin(2*d*x^n) + 2*((2*a^4*b^
2 - 3*a^2*b^4 + b^6)*d*e^(2*n)*x^(2*n)*cos(c) + (a^2*b^4 - b^6)*e^(2*n)*x^n
*sin(c))*sin(d*x^n))*sin(d*x^n + c))/(a^8*d*e*x*cos(2*d*x^n + 2*c)^2 + a^8*
d*e*x*sin(2*d*x^n + 2*c)^2 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*e*
x*cos(2*d*x^n)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^6*b^2 -
 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*e*x*cos(d*x^n)^2 + (a^4*b^4*cos(2*c)^2 +
a^4*b^4*sin(2*c)^2)*d*e*x*sin(2*d*x^n)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*
d*e*x*cos(c)*sin(d*x^n) + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*cos(c)^2 + (a^
6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*e*x*sin(d*x^n)^2 + 4*(a^7*b - 2*a^
```

```
5*b^3 + a^3*b^5)*d*e*x*cos(d*x^n)*sin(c) + (a^8 - 2*a^6*b^2 + a^4*b^4)*d*e*
x - 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c
)*sin(c))*d*e*x*cos(d*x^n) - (a^6*b^2 - a^4*b^4)*d*e*x*cos(2*c) - 2*((a^5*b
^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*e*x*
sin(d*x^n))*cos(2*d*x^n) - 2*(a^6*b^2*d*e*x*cos(2*d*x^n)*cos(2*c) - a^6*b^2
*d*e*x*sin(2*d*x^n)*sin(2*c) + 2*(a^7*b - a^5*b^3)*d*e*x*cos(c)*sin(d*x^n)
+ 2*(a^7*b - a^5*b^3)*d*e*x*cos(d*x^n)*sin(c) + (a^8 - a^6*b^2)*d*e*x)*cos(
2*d*x^n + 2*c) - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3
*b^5)*sin(2*c)*sin(c))*d*e*x*cos(d*x^n) + 2*((a^5*b^3 - a^3*b^5)*cos(c)*sin
(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*e*x*sin(d*x^n) + (a^6*b^2 -
a^4*b^4)*d*e*x*sin(2*c))*sin(2*d*x^n) - 2*(a^6*b^2*d*e*x*cos(2*c)*sin(2*d*x
^n) + a^6*b^2*d*e*x*cos(2*d*x^n)*sin(2*c) - 2*(a^7*b - a^5*b^3)*d*e*x*cos(d
*x^n)*cos(c) + 2*(a^7*b - a^5*b^3)*d*e*x*sin(d*x^n)*sin(c))*sin(2*d*x^n + 2
*c)), x) + 4*(a*b^3*e^(2*n)*x^n*sin(d*x^n + c) + a^2*b^2*e^(2*n)*x^n - (a^3
*b - a*b^3)*d*e^(2*n)*x^(2*n)*cos(d*x^n + c))*sin(2*d*x^n + 2*c))/((a^6 - a
^4*b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n
+ c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c)*sin(2*d*x^n + 2*c) + (a^6
 - a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*
x^n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^4*b^2)*d*e
*n - 2*(2*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n)*c
os(2*d*x^n + 2*c))
```

Giac [F]

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx=\int \frac{(ex)^{2n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+2*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x)^(2*n - 1)/(b*csc(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+2n}}{(a+b\csc(c+dx^n))^2}\,dx=\int \frac{(e\,x)^{2\,n-1}}{\left(a+\frac{b}{\sin(c+d\,x^n)}\right)^2}\,dx$$

```
[In] int((e*x)^(2*n - 1)/(a + b/sin(c + d*x^n))^2,x)
```

```
[Out] int((e*x)^(2*n - 1)/(a + b/sin(c + d*x^n))^2, x)
```

3.84 $\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 1417

$$
\begin{aligned}
\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx &= \frac{(ex)^{3n}}{3a^2en}-\frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den}+\frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en}\\
&+\frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en}\\
&-\frac{ib^3x^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}\\
&+\frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}\\
&+\frac{ib^3x^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}\\
&-\frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}\\
&-\frac{2ib^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en}\\
&-\frac{2ib^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en}\\
&-\frac{2b^3x^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en}\\
&+\frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}\\
&+\frac{2b^3x^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en}\\
&-\frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}\\
&-\frac{2ib^3x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en}\\
&+\frac{4ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en}\\
&+\frac{2ib^3x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en}\\
&-\frac{4ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en}\\
&\quad b^2x^{-n}(ex)^{3n}\cos(c+dx^n)
\end{aligned}
$$

```
[Out] 1/3*(e*x)^(3*n)/a^2/e/n-I*b^3*(e*x)^(3*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b-(-a^
2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)+2*b^2*(e*x)^(3*n)*ln(1+a*ex
p(I*(c+d*x^n))/(I*b-(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^2/e/n/(x^(2*n))+2*b^2
*(e*x)^(3*n)*ln(1+a*exp(I*(c+d*x^n))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d
^2/e/n/(x^(2*n))-2*I*b^3*(e*x)^(3*n)*polylog(3,I*a*exp(I*(c+d*x^n))/(b-(-a^
2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3/e/n/(x^(3*n))-2*I*b^2*(e*x)^(3*n)*p
olylog(2,-a*exp(I*(c+d*x^n))/(I*b+(a^2-b^2)^(1/2)))/a^2/(a^2-b^2)/d^3/e/n/(
x^(3*n))-2*I*b^2*(e*x)^(3*n)*polylog(2,-a*exp(I*(c+d*x^n))/(I*b-(a^2-b^2)^(
1/2)))/a^2/(a^2-b^2)/d^3/e/n/(x^(3*n))+I*b^3*(e*x)^(3*n)*ln(1-I*a*exp(I*(c+
d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d/e/n/(x^n)-2*b^3*(e*x)^
(3*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(
3/2)/d^2/e/n/(x^(2*n))+2*b^3*(e*x)^(3*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b+
(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^2/e/n/(x^(2*n))+4*I*b*(e*x)^(3*n)
*polylog(3,I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)^(1/2)))/a^2/d^3/e/n/(x^(3*n))
/(-a^2+b^2)^(1/2)+2*I*b*(e*x)^(3*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b-(-a^2+b^2)
^(1/2)))/a^2/d/e/n/(x^n)/(-a^2+b^2)^(1/2)-b^2*(e*x)^(3*n)*cos(c+d*x^n)/a/(a
^2-b^2)/d/e/n/(x^n)/(b+a*sin(c+d*x^n))+2*I*b^3*(e*x)^(3*n)*polylog(3,I*a*ex
p(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/(-a^2+b^2)^(3/2)/d^3/e/n/(x^(3*n))
-2*I*b*(e*x)^(3*n)*ln(1-I*a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/d/e/
n/(x^n)/(-a^2+b^2)^(1/2)+4*b*(e*x)^(3*n)*polylog(2,I*a*exp(I*(c+d*x^n))/(b-
(-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(-a^2+b^2)^(1/2)-4*b*(e*x)^(3*n)*p
olylog(2,I*a*exp(I*(c+d*x^n))/(b+(-a^2+b^2)^(1/2)))/a^2/d^2/e/n/(x^(2*n))/(
-a^2+b^2)^(1/2)-4*I*b*(e*x)^(3*n)*polylog(3,I*a*exp(I*(c+d*x^n))/(b+(-a^2+b
^2)^(1/2)))/a^2/d^3/e/n/(x^(3*n))/(-a^2+b^2)^(1/2)-I*b^2*(e*x)^(3*n)/a^2/(a
^2-b^2)/d/e/n/(x^n)
```

Rubi [A] (verified)

Time = 2.99 (sec) , antiderivative size = 1417, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules

used = {4294, 4290, 4276, 3405, 3404, 2296, 2221, 2611, 2320, 6724, 4617, 2317, 2438}

$$
\begin{aligned}
\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx = &-\frac{2ib^2(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{ib-\sqrt{a^2-b^2}}\right)x^{-3n}}{a^2(a^2-b^2)\,d^3en} \\
&-\frac{2ib^2(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(dx^n+c)}}{ib+\sqrt{a^2-b^2}}\right)x^{-3n}}{a^2(a^2-b^2)\,d^3en} \\
&+\frac{4ib(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2\sqrt{b^2-a^2}\,d^3en} \\
&-\frac{2ib^3(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(b^2-a^2)^{3/2}\,d^3en} \\
&-\frac{4ib(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2\sqrt{b^2-a^2}\,d^3en} \\
&+\frac{2ib^3(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)x^{-3n}}{a^2(b^2-a^2)^{3/2}\,d^3en} \\
&+\frac{2b^2(ex)^{3n}\log\left(\frac{e^{i(dx^n+c)}a}{ib-\sqrt{a^2-b^2}}+1\right)x^{-2n}}{a^2(a^2-b^2)\,d^2en} \\
&+\frac{2b^2(ex)^{3n}\log\left(\frac{e^{i(dx^n+c)}a}{ib+\sqrt{a^2-b^2}}+1\right)x^{-2n}}{a^2(a^2-b^2)\,d^2en} \\
&+\frac{4b(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2\sqrt{b^2-a^2}\,d^2en} \\
&-\frac{2b^3(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2(b^2-a^2)^{3/2}\,d^2en} \\
&-\frac{4b(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2\sqrt{b^2-a^2}\,d^2en} \\
&+\frac{2b^3(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)x^{-2n}}{a^2(b^2-a^2)^{3/2}\,d^2en} \\
&-\frac{ib^2(ex)^{3n}x^{-n}}{a^2(a^2-b^2)\,den}+\frac{2ib(ex)^{3n}\log\left(1-\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)x^{-n}}{a^2\sqrt{b^2-a^2}\,den} \\
&-\frac{ib^3(ex)^{3n}\log\left(1-\frac{iae^{i(dx^n+c)}}{b-\sqrt{b^2-a^2}}\right)x^{-n}}{a^2(b^2-a^2)^{3/2}\,den} \\
&-\frac{2ib(ex)^{3n}\log\left(1-\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)x^{-n}}{a^2\sqrt{b^2-a^2}\,den} \\
&+\frac{ib^3(ex)^{3n}\log\left(1-\frac{iae^{i(dx^n+c)}}{b+\sqrt{b^2-a^2}}\right)x^{-n}}{a^2(b^2-a^2)^{3/2}\,den} \\
&-\frac{b^2(ex)^{3n}\cos(dx^n+c)\,x^{-n}}{a(a^2-b^2)\,den\,(b+a\sin(dx^n+c))}+\frac{(ex)^{3n}}{3a^2en}
\end{aligned}
$$

```
[In] Int[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n])^2,x]
```

```
[Out] (e*x)^(3*n)/(3*a^2*e*n) - (I*b^2*(e*x)^(3*n))/(a^2*(a^2 - b^2)*d*e*n*x^n) +
 (2*b^2*(e*x)^(3*n)*Log[1 + (a*E^(I*(c + d*x^n)))/(I*b - Sqrt[a^2 - b^2])])
/(a^2*(a^2 - b^2)*d^2*e*n*x^(2*n)) + (2*b^2*(e*x)^(3*n)*Log[1 + (a*E^(I*(c
+ d*x^n)))/(I*b + Sqrt[a^2 - b^2])])/(a^2*(a^2 - b^2)*d^2*e*n*x^(2*n)) - (I
*b^3*(e*x)^(3*n)*Log[1 - (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(
a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) + ((2*I)*b*(e*x)^(3*n)*Log[1 - (I*a*E^(I*
(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d*e*n*x^n) + (
I*b^3*(e*x)^(3*n)*Log[1 - (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/
(a^2*(-a^2 + b^2)^(3/2)*d*e*n*x^n) - ((2*I)*b*(e*x)^(3*n)*Log[1 - (I*a*E^(I
*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d*e*n*x^n) -
((2*I)*b^2*(e*x)^(3*n)*PolyLog[2, -((a*E^(I*(c + d*x^n)))/(I*b - Sqrt[a^2 -
 b^2]))])/(a^2*(a^2 - b^2)*d^3*e*n*x^(3*n)) - ((2*I)*b^2*(e*x)^(3*n)*PolyLo
g[2, -((a*E^(I*(c + d*x^n)))/(I*b + Sqrt[a^2 - b^2]))])/(a^2*(a^2 - b^2)*d^
3*e*n*x^(3*n)) - (2*b^3*(e*x)^(3*n)*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b -
 Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2*e*n*x^(2*n)) + (4*b*(e*x)^
(3*n)*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt
[-a^2 + b^2]*d^2*e*n*x^(2*n)) + (2*b^3*(e*x)^(3*n)*PolyLog[2, (I*a*E^(I*(c
+ d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3/2)*d^2*e*n*x^(2*n)
) - (4*b*(e*x)^(3*n)*PolyLog[2, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^
2])])/(a^2*Sqrt[-a^2 + b^2]*d^2*e*n*x^(2*n)) - ((2*I)*b^3*(e*x)^(3*n)*PolyL
og[3, (I*a*E^(I*(c + d*x^n)))/(b - Sqrt[-a^2 + b^2])])/(a^2*(-a^2 + b^2)^(3
/2)*d^3*e*n*x^(3*n)) + ((4*I)*b*(e*x)^(3*n)*PolyLog[3, (I*a*E^(I*(c + d*x^n
)))/(b - Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d^3*e*n*x^(3*n)) + ((2*I
)*b^3*(e*x)^(3*n)*PolyLog[3, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])
])/(a^2*(-a^2 + b^2)^(3/2)*d^3*e*n*x^(3*n)) - ((4*I)*b*(e*x)^(3*n)*PolyLog[
3, (I*a*E^(I*(c + d*x^n)))/(b + Sqrt[-a^2 + b^2])])/(a^2*Sqrt[-a^2 + b^2]*d
^3*e*n*x^(3*n)) - (b^2*(e*x)^(3*n)*Cos[c + d*x^n])/(a*(a^2 - b^2)*d*e*n*x^n
*(b + a*Sin[c + d*x^n]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3404

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Dist[2, Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x)
) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[
a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 4276

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, 1/(Sin[e + f*x]^n/(b + a*Si
```

```
n[e + f*x])^n), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && ILtQ[n, 0] && IGt
Q[m, 0]
```

Rule 4290

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
1)/n], 0] && IntegerQ[p]
```

Rule 4294

```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*((e_)*(x_))^(m_.), x
_Symbol] :> Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
 + b*Csc[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x]
```

Rule 4617

```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2
] + b*E^(I*(c + d*x)))), x], x] + Dist[I, Int[(e + f*x)^m*(E^(I*(c + d*x))/
(I*a + Rt[-a^2 + b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c,
d, e, f}, x] && IGtQ[m, 0] && NegQ[a^2 - b^2]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(x^{-3n}(ex)^{3n}\right)\int \frac{x^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx}{e} \\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{(a+b\csc(c+dx))^2}\,dx, x, x^n\right)}{en} \\
&= \frac{\left(x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \left(\frac{x^2}{a^2}+\frac{b^2x^2}{a^2(b+a\sin(c+dx))^2}-\frac{2bx^2}{a^2(b+a\sin(c+dx))}\right)dx, x, x^n\right)}{en} \\
&= \frac{(ex)^{3n}}{3a^2en}-\frac{\left(2bx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{b+a\sin(c+dx)}\,dx, x, x^n\right)}{a^2en} \\
&\quad+\frac{\left(b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int \frac{x^2}{(b+a\sin(c+dx))^2}\,dx, x, x^n\right)}{a^2en}
\end{aligned}
$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{b^2x^{-n}(ex)^{3n}\cos(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\sin(c+dx^n))} - \frac{(4bx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, x^n\right)}{a^2en} - \frac{(b^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{x^2}{b+a\sin(c+dx)}\,dx, x, x^n\right)}{a^2(a^2-b^2)\,en} + \frac{(2b^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{x\cos(c+dx)}{b+a\sin(c+dx)}\,dx, x, x^n\right)}{a(a^2-b^2)\,den}$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} - \frac{b^2x^{-n}(ex)^{3n}\cos(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\sin(c+dx^n))} - \frac{(2b^3x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{ia+2be^{i(c+dx)}-iae^{2i(c+dx)}}\,dx, x, x^n\right)}{a^2(a^2-b^2)\,en} + \frac{(4ibx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}\,en} - \frac{(4ibx^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx, x, x^n\right)}{a\sqrt{-a^2+b^2}\,en} + \frac{(2ib^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib-\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, x^n\right)}{a(a^2-b^2)\,den} + \frac{(2ib^2x^{-3n}(ex)^{3n})\,\text{Subst}\left(\int \frac{e^{i(c+dx)}x}{ib+\sqrt{a^2-b^2}+ae^{i(c+dx)}}\,dx, x, x^n\right)}{a(a^2-b^2)\,den}$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2\left(a^2-b^2\right)den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2en}$$

$$+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)d^2en} + \frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den}$$

$$- \frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{b^2x^{-n}(ex)^{3n}\cos\left(c+dx^n\right)}{a\left(a^2-b^2\right)den\left(b+a\sin\left(c+dx^n\right)\right)}$$

$$+ \frac{\left(2ib^3x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int\frac{e^{i(c+dx)}x^2}{2b-2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx,x,x^n\right)}{a\left(a^2-b^2\right)\sqrt{-a^2+b^2}en}$$

$$- \frac{\left(2ib^3x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int\frac{e^{i(c+dx)}x^2}{2b+2\sqrt{-a^2+b^2}-2iae^{i(c+dx)}}\,dx,x,x^n\right)}{a\left(a^2-b^2\right)\sqrt{-a^2+b^2}en}$$

$$- \frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int\log\left(1+\frac{ae^{i(c+dx)}}{ib-\sqrt{a^2-b^2}}\right)\,dx,x,x^n\right)}{a^2\left(a^2-b^2\right)d^2en}$$

$$- \frac{\left(2b^2x^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int\log\left(1+\frac{ae^{i(c+dx)}}{ib+\sqrt{a^2-b^2}}\right)\,dx,x,x^n\right)}{a^2\left(a^2-b^2\right)d^2en}$$

$$- \frac{\left(4ibx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}den}$$

$$+ \frac{\left(4ibx^{-3n}(ex)^{3n}\right)\text{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)\,dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}den}$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2en}$$
$$+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)d^2en} - \frac{ib^3x^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den}$$
$$+ \frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{ib^3x^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}den}$$
$$- \frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$
$$- \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} - \frac{b^2x^{-n}(ex)^{3n}\cos(c+dx^n)}{a(a^2-b^2)den(b+a\sin(c+dx^n))}$$
$$+ \frac{(2ib^2x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{ax}{ib-\sqrt{a^2-b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a^2(a^2-b^2)d^3en}$$
$$+ \frac{(2ib^2x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\log\left(1+\frac{ax}{ib+\sqrt{a^2-b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a^2(a^2-b^2)d^3en}$$
$$- \frac{(4bx^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$
$$+ \frac{(4bx^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a^2\sqrt{-a^2+b^2}d^2en}$$
$$- \frac{(2ib^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}den}$$
$$+ \frac{(2ib^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int x\log\left(1-\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}den}$$

$$
\begin{aligned}
&= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} \\
&+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{ib^3x^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} \\
&+ \frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{ib^3x^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} \\
&- \frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2ib^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en} \\
&- \frac{2ib^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en} \\
&- \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} \\
&+ \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} \\
&- \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} - \frac{b^2x^{-n}(ex)^{3n}\cos(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\sin(c+dx^n))} \\
&+ \frac{(4ibx^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a^2\sqrt{-a^2+b^2}d^3en} \\
&- \frac{(4ibx^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\frac{\mathrm{PolyLog}\left(2,\frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a^2\sqrt{-a^2+b^2}d^3en} \\
&- \frac{(2b^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b-2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}d^2en} \\
&+ \frac{(2b^3x^{-3n}(ex)^{3n})\,\mathrm{Subst}\left(\int\mathrm{PolyLog}\left(2,\frac{2iae^{i(c+dx)}}{2b+2\sqrt{-a^2+b^2}}\right)dx,x,x^n\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}d^2en}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} \\
&+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{ib^3x^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} \\
&+ \frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} + \frac{ib^3x^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den} \\
&- \frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}den} - \frac{2ib^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en} \\
&- \frac{2ib^2x^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en} \\
&- \frac{2b^3x^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} \\
&+ \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{2b^3x^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} \\
&- \frac{4bx^{-2n}(ex)^{3n}\operatorname{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^2en} + \frac{4ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en} \\
&- \frac{4ibx^{-3n}(ex)^{3n}\operatorname{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}d^3en} - \frac{b^2x^{-n}(ex)^{3n}\cos(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\sin(c+dx^n))} \\
&+ \frac{(2ib^3x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{iax}{b-\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}d^3en} \\
&- \frac{(2ib^3x^{-3n}(ex)^{3n})\operatorname{Subst}\left(\int\frac{\operatorname{PolyLog}\left(2,\frac{iax}{b+\sqrt{-a^2+b^2}}\right)}{x}dx,x,e^{i(c+dx^n)}\right)}{a^2(a^2-b^2)\sqrt{-a^2+b^2}d^3en}
\end{aligned}
$$

$$= \frac{(ex)^{3n}}{3a^2en} - \frac{ib^2x^{-n}(ex)^{3n}}{a^2(a^2-b^2)\,den} + \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en}$$
$$+ \frac{2b^2x^{-2n}(ex)^{3n}\log\left(1+\frac{ae^{i(c+dx^n)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^2en} - \frac{ib^3x^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}$$
$$+ \frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,den} + \frac{ib^3x^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,den}$$
$$- \frac{2ibx^{-n}(ex)^{3n}\log\left(1-\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,den} - \frac{2ib^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{ib-\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en}$$
$$- \frac{2ib^2x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,-\frac{ae^{i(c+dx^n)}}{ib+\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)\,d^3en}$$
$$- \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} + \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2en}$$
$$+ \frac{2b^3x^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^2en} - \frac{4bx^{-2n}(ex)^{3n}\,\mathrm{PolyLog}\left(2,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^2en}$$
$$- \frac{2ib^3x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en}$$
$$+ \frac{4ibx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b-\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^3en} + \frac{2ib^3x^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2(-a^2+b^2)^{3/2}\,d^3en}$$
$$- \frac{4ibx^{-3n}(ex)^{3n}\,\mathrm{PolyLog}\left(3,\frac{iae^{i(c+dx^n)}}{b+\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}\,d^3en} - \frac{b^2x^{-n}(ex)^{3n}\cos(c+dx^n)}{a(a^2-b^2)\,den\,(b+a\sin(c+dx^n))}$$

Mathematica [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx$$

```
[In] Integrate[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n])^2,x]
[Out] Integrate[(e*x)^(-1 + 3*n)/(a + b*Csc[c + d*x^n])^2, x]
```

Maple [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+d\,x^n))^2}dx$$

```
[In] int((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n))^2,x)
```

```
[Out] int((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n))^2,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3755 vs. $2(1291) = 2582$.

Time = 0.57 (sec) , antiderivative size = 3755, normalized size of antiderivative = 2.65

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx = \text{Too large to display}$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d^3*e^(3*n - 1)*x^(3*n)*sin(d*x^n + c) + 2
*(a^4*b - 2*a^2*b^3 + b^5)*d^3*e^(3*n - 1)*x^(3*n) - 6*(a^3*b^2 - a*b^4)*d^
2*e^(3*n - 1)*x^(2*n)*cos(d*x^n + c) - 6*(-I*(2*a^3*b^2 - a*b^4)*d*e^(3*n -
 1)*x^n*sqrt((a^2 - b^2)/a^2) + (I*a^2*b^3 - I*b^5)*e^(3*n - 1) + (-I*(2*a^
4*b - a^2*b^3)*d*e^(3*n - 1)*x^n*sqrt((a^2 - b^2)/a^2) + (I*a^3*b^2 - I*a*b
^4)*e^(3*n - 1))*sin(d*x^n + c))*dilog(((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos
(d*x^n + c) + (I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a + 1) -
6*(-I*(2*a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x^n*sqrt((a^2 - b^2)/a^2) + (-I*a^2
*b^3 + I*b^5)*e^(3*n - 1) + (-I*(2*a^4*b - a^2*b^3)*d*e^(3*n - 1)*x^n*sqrt(
(a^2 - b^2)/a^2) + (-I*a^3*b^2 + I*a*b^4)*e^(3*n - 1))*sin(d*x^n + c))*dilo
g(-((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) - (I*a*sqrt((a^2 - b^2)/
a^2) - b)*sin(d*x^n + c) + a)/a + 1) - 6*(I*(2*a^3*b^2 - a*b^4)*d*e^(3*n -
1)*x^n*sqrt((a^2 - b^2)/a^2) + (-I*a^2*b^3 + I*b^5)*e^(3*n - 1) + (I*(2*a^4
*b - a^2*b^3)*d*e^(3*n - 1)*x^n*sqrt((a^2 - b^2)/a^2) + (-I*a^3*b^2 + I*a*b
^4)*e^(3*n - 1))*sin(d*x^n + c))*dilog(((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos
(d*x^n + c) + (-I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a + 1) -
 6*(I*(2*a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x^n*sqrt((a^2 - b^2)/a^2) + (I*a^2*
b^3 - I*b^5)*e^(3*n - 1) + (I*(2*a^4*b - a^2*b^3)*d*e^(3*n - 1)*x^n*sqrt((a
^2 - b^2)/a^2) + (I*a^3*b^2 - I*a*b^4)*e^(3*n - 1))*sin(d*x^n + c))*dilog(-
((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) - (-I*a*sqrt((a^2 - b^2)/a^
2) - b)*sin(d*x^n + c) + a)/a + 1) + 3*(((2*a^4*b - a^2*b^3)*c^2*sqrt((a^2
- b^2)/a^2) - 2*(a^3*b^2 - a*b^4)*c)*e^(3*n - 1)*sin(d*x^n + c) + ((2*a^3*b
^2 - a*b^4)*c^2*sqrt((a^2 - b^2)/a^2) - 2*(a^2*b^3 - b^5)*c)*e^(3*n - 1))*l
og(2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) +
2*I*b) + 3*(((2*a^4*b - a^2*b^3)*c^2*sqrt((a^2 - b^2)/a^2) - 2*(a^3*b^2 - a
```

```
*b^4)*c)*e^(3*n - 1)*sin(d*x^n + c) + ((2*a^3*b^2 - a*b^4)*c^2*sqrt((a^2 -
b^2)/a^2) - 2*(a^2*b^3 - b^5)*c)*e^(3*n - 1))*log(2*a*cos(d*x^n + c) - 2*I*
a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b) - 3*(((2*a^4*b - a^2*
b^3)*c^2*sqrt((a^2 - b^2)/a^2) + 2*(a^3*b^2 - a*b^4)*c)*e^(3*n - 1)*sin(d*x
^n + c) + ((2*a^3*b^2 - a*b^4)*c^2*sqrt((a^2 - b^2)/a^2) + 2*(a^2*b^3 - b^5
)*c)*e^(3*n - 1))*log(-2*a*cos(d*x^n + c) + 2*I*a*sin(d*x^n + c) + 2*a*sqrt
((a^2 - b^2)/a^2) + 2*I*b) - 3*(((2*a^4*b - a^2*b^3)*c^2*sqrt((a^2 - b^2)/a
^2) + 2*(a^3*b^2 - a*b^4)*c)*e^(3*n - 1)*sin(d*x^n + c) + ((2*a^3*b^2 - a*b
^4)*c^2*sqrt((a^2 - b^2)/a^2) + 2*(a^2*b^3 - b^5)*c)*e^(3*n - 1))*log(-2*a*
cos(d*x^n + c) - 2*I*a*sin(d*x^n + c) + 2*a*sqrt((a^2 - b^2)/a^2) - 2*I*b)
 - 3*((2*a^3*b^2 - a*b^4)*d^2*e^(3*n - 1)*x^(2*n)*sqrt((a^2 - b^2)/a^2) - 2*
(a^2*b^3 - b^5)*d*e^(3*n - 1)*x^n - ((2*a^3*b^2 - a*b^4)*c^2*sqrt((a^2 - b^
2)/a^2) + 2*(a^2*b^3 - b^5)*c)*e^(3*n - 1) + ((2*a^4*b - a^2*b^3)*d^2*e^(3*
n - 1)*x^(2*n)*sqrt((a^2 - b^2)/a^2) - 2*(a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x^
n - ((2*a^4*b - a^2*b^3)*c^2*sqrt((a^2 - b^2)/a^2) + 2*(a^3*b^2 - a*b^4)*c)
*e^(3*n - 1))*sin(d*x^n + c))*log(-((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x
^n + c) + (I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a) + 3*((2*a^
3*b^2 - a*b^4)*d^2*e^(3*n - 1)*x^(2*n)*sqrt((a^2 - b^2)/a^2) + 2*(a^2*b^3 -
 b^5)*d*e^(3*n - 1)*x^n - ((2*a^3*b^2 - a*b^4)*c^2*sqrt((a^2 - b^2)/a^2) -
2*(a^2*b^3 - b^5)*c)*e^(3*n - 1) + ((2*a^4*b - a^2*b^3)*d^2*e^(3*n - 1)*x^(
2*n)*sqrt((a^2 - b^2)/a^2) + 2*(a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x^n - ((2*a^
4*b - a^2*b^3)*c^2*sqrt((a^2 - b^2)/a^2) - 2*(a^3*b^2 - a*b^4)*c)*e^(3*n -
1))*sin(d*x^n + c))*log(((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c) - (
I*a*sqrt((a^2 - b^2)/a^2) - b)*sin(d*x^n + c) + a)/a) - 3*((2*a^3*b^2 - a*b
^4)*d^2*e^(3*n - 1)*x^(2*n)*sqrt((a^2 - b^2)/a^2) - 2*(a^2*b^3 - b^5)*d*e^(
3*n - 1)*x^n - ((2*a^3*b^2 - a*b^4)*c^2*sqrt((a^2 - b^2)/a^2) + 2*(a^2*b^3
 - b^5)*c)*e^(3*n - 1) + ((2*a^4*b - a^2*b^3)*d^2*e^(3*n - 1)*x^(2*n)*sqrt((
a^2 - b^2)/a^2) - 2*(a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x^n - ((2*a^4*b - a^2*b
^3)*c^2*sqrt((a^2 - b^2)/a^2) + 2*(a^3*b^2 - a*b^4)*c)*e^(3*n - 1))*sin(d*x
^n + c))*log(-((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) + (-I*a*sqrt(
(a^2 - b^2)/a^2) - b)*sin(d*x^n + c) - a)/a) + 3*((2*a^3*b^2 - a*b^4)*d^2*e
^(3*n - 1)*x^(2*n)*sqrt((a^2 - b^2)/a^2) + 2*(a^2*b^3 - b^5)*d*e^(3*n - 1)*
x^n - ((2*a^3*b^2 - a*b^4)*c^2*sqrt((a^2 - b^2)/a^2) - 2*(a^2*b^3 - b^5)*c)
*e^(3*n - 1) + ((2*a^4*b - a^2*b^3)*d^2*e^(3*n - 1)*x^(2*n)*sqrt((a^2 - b^2
)/a^2) + 2*(a^3*b^2 - a*b^4)*d*e^(3*n - 1)*x^n - ((2*a^4*b - a^2*b^3)*c^2*s
qrt((a^2 - b^2)/a^2) - 2*(a^3*b^2 - a*b^4)*c)*e^(3*n - 1))*sin(d*x^n + c))*
log(((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) - (-I*a*sqrt((a^2 - b^2
)/a^2) - b)*sin(d*x^n + c) + a)/a) + 6*((2*a^4*b - a^2*b^3)*e^(3*n - 1)*sqr
t((a^2 - b^2)/a^2)*sin(d*x^n + c) + (2*a^3*b^2 - a*b^4)*e^(3*n - 1)*sqrt((a
^2 - b^2)/a^2))*polylog(3, -((a*sqrt((a^2 - b^2)/a^2) + I*b)*cos(d*x^n + c)
 + (-I*a*sqrt((a^2 - b^2)/a^2) + b)*sin(d*x^n + c))/a) - 6*((2*a^4*b - a^2*
b^3)*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2)*sin(d*x^n + c) + (2*a^3*b^2 - a*b^4)
*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2))*polylog(3, ((a*sqrt((a^2 - b^2)/a^2) +
I*b)*cos(d*x^n + c) - (-I*a*sqrt((a^2 - b^2)/a^2) + b)*sin(d*x^n + c))/a) +
 6*((2*a^4*b - a^2*b^3)*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2)*sin(d*x^n + c) +
```

```
(2*a^3*b^2 - a*b^4)*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2))*polylog(3, -((a*sqrt
((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) + (I*a*sqrt((a^2 - b^2)/a^2) + b)*s
in(d*x^n + c))/a) - 6*((2*a^4*b - a^2*b^3)*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2
)*sin(d*x^n + c) + (2*a^3*b^2 - a*b^4)*e^(3*n - 1)*sqrt((a^2 - b^2)/a^2))*p
olylog(3, ((a*sqrt((a^2 - b^2)/a^2) - I*b)*cos(d*x^n + c) - (I*a*sqrt((a^2
- b^2)/a^2) + b)*sin(d*x^n + c))/a))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d^3*n*sin
(d*x^n + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d^3*n)
```

Sympy [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx=\int \frac{(ex)^{3n-1}}{(a+b\csc(c+dx^n))^2}\,dx$$

```
[In] integrate((e*x)**(-1+3*n)/(a+b*csc(c+d*x**n))**2,x)
```

```
[Out] Integral((e*x)**(3*n - 1)/(a + b*csc(c + d*x**n))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx=\int \frac{(ex)^{3n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(6*a*b^3*e^(3*n)*x^(2*n)*cos(d*x^n + c) - (a^4 - a^2*b^2)*d*e^(3*n)*x^
(3*n)*cos(2*d*x^n + 2*c)^2 - 4*(a^2*b^2 - b^4)*d*e^(3*n)*x^(3*n)*cos(d*x^n
+ c)^2 - (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n)*sin(2*d*x^n + 2*c)^2 - 4*(a^2*b^
2 - b^4)*d*e^(3*n)*x^(3*n)*sin(d*x^n + c)^2 - 4*(a^3*b - a*b^3)*d*e^(3*n)*x
^(3*n)*sin(d*x^n + c) - (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n) + 2*(3*a*b^3*e^(3
*n)*x^(2*n)*cos(d*x^n + c) + 2*(a^3*b - a*b^3)*d*e^(3*n)*x^(3*n)*sin(d*x^n
+ c) + (a^4 - a^2*b^2)*d*e^(3*n)*x^(3*n))*cos(2*d*x^n + 2*c) - 3*((a^6 - a^
4*b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n +
 c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*cos(d*x^n + c)*sin(2*d*x^n + 2*c) + (a^6
- a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*x
^n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^4*b^2)*d*e*
n - 2*(2*(a^5*b - a^3*b^3)*d*e*n*sin(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n)*co
s(2*d*x^n + 2*c))*integrate(-2*(2*a^2*b^4*e^(3*n)*x^(2*n)*cos(2*c)*sin(2*d*
x^n) + 2*a^2*b^4*e^(3*n)*x^(2*n)*cos(2*d*x^n)*sin(2*c) - 4*(a^3*b^3 - a*b^5
)*e^(3*n)*x^(2*n)*cos(d*x^n)*cos(c) + 4*(a^3*b^3 - a*b^5)*e^(3*n)*x^(2*n)*s
in(d*x^n)*sin(c) - (2*a^3*b^3*e^(3*n)*x^(2*n)*cos(d*x^n + c) + (2*a^5*b - a
^3*b^3)*d*e^(3*n)*x^(3*n)*sin(d*x^n + c))*cos(2*d*x^n + 2*c) + (2*(a^3*b^3
- a*b^5)*e^(3*n)*x^(2*n) + (2*a*b^5*e^(3*n)*x^(2*n)*cos(2*c) - (2*a^3*b^3 -
```

```
 a*b^5)*d*e^(3*n)*x^(3*n)*sin(2*c))*cos(2*d*x^n) + 2*((2*a^4*b^2 - 3*a^2*b^
4 + b^6)*d*e^(3*n)*x^(3*n)*cos(c) + 2*(a^2*b^4 - b^6)*e^(3*n)*x^(2*n)*sin(c
))*cos(d*x^n) - (2*a*b^5*e^(3*n)*x^(2*n)*sin(2*c) + (2*a^3*b^3 - a*b^5)*d*e
^(3*n)*x^(3*n)*cos(2*c))*sin(2*d*x^n) - 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*
e^(3*n)*x^(3*n)*sin(c) - 2*(a^2*b^4 - b^6)*e^(3*n)*x^(2*n)*cos(c))*sin(d*x^
n))*cos(d*x^n + c) - (2*a^3*b^3*e^(3*n)*x^(2*n)*sin(d*x^n + c) + 2*a^4*b^2*
e^(3*n)*x^(2*n) - (2*a^5*b - a^3*b^3)*d*e^(3*n)*x^(3*n)*cos(d*x^n + c))*sin
(2*d*x^n + 2*c) + ((2*a^5*b - 3*a^3*b^3 + a*b^5)*d*e^(3*n)*x^(3*n) + (2*a*b
^5*e^(3*n)*x^(2*n)*sin(2*c) + (2*a^3*b^3 - a*b^5)*d*e^(3*n)*x^(3*n)*cos(2*c
))*cos(2*d*x^n) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(3*n)*x^(3*n)*sin(c)
 - 2*(a^2*b^4 - b^6)*e^(3*n)*x^(2*n)*cos(c))*cos(d*x^n) + (2*a*b^5*e^(3*n)*
x^(2*n)*cos(2*c) - (2*a^3*b^3 - a*b^5)*d*e^(3*n)*x^(3*n)*sin(2*c))*sin(2*d*
x^n) + 2*((2*a^4*b^2 - 3*a^2*b^4 + b^6)*d*e^(3*n)*x^(3*n)*cos(c) + 2*(a^2*b
^4 - b^6)*e^(3*n)*x^(2*n)*sin(c))*sin(d*x^n))*sin(d*x^n + c))/(a^8*d*e*x*co
s(2*d*x^n + 2*c)^2 + a^8*d*e*x*sin(2*d*x^n + 2*c)^2 + (a^4*b^4*cos(2*c)^2 +
 a^4*b^4*sin(2*c)^2)*d*e*x*cos(2*d*x^n)^2 + 4*((a^6*b^2 - 2*a^4*b^4 + a^2*b
^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*e*x*cos(d*x^n)^2
 + (a^4*b^4*cos(2*c)^2 + a^4*b^4*sin(2*c)^2)*d*e*x*sin(2*d*x^n)^2 + 4*(a^7*
b - 2*a^5*b^3 + a^3*b^5)*d*e*x*cos(c)*sin(d*x^n) + 4*((a^6*b^2 - 2*a^4*b^4
+ a^2*b^6)*cos(c)^2 + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*sin(c)^2)*d*e*x*sin(d
*x^n)^2 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*e*x*cos(d*x^n)*sin(c) + (a^8 -
2*a^6*b^2 + a^4*b^4)*d*e*x - 2*(2*((a^5*b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a
^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*e*x*cos(d*x^n) - (a^6*b^2 - a^4*b^4)*d
*e*x*cos(2*c) - 2*((a^5*b^3 - a^3*b^5)*cos(2*c)*cos(c) + (a^5*b^3 - a^3*b^5
)*sin(2*c)*sin(c))*d*e*x*sin(d*x^n))*cos(2*d*x^n) - 2*(a^6*b^2*d*e*x*cos(2*
d*x^n)*cos(2*c) - a^6*b^2*d*e*x*sin(2*d*x^n)*sin(2*c) + 2*(a^7*b - a^5*b^3)
*d*e*x*cos(c)*sin(d*x^n) + 2*(a^7*b - a^5*b^3)*d*e*x*cos(d*x^n)*sin(c) + (a
^8 - a^6*b^2)*d*e*x)*cos(2*d*x^n + 2*c) - 2*(2*((a^5*b^3 - a^3*b^5)*cos(2*c
)*cos(c) + (a^5*b^3 - a^3*b^5)*sin(2*c)*sin(c))*d*e*x*cos(d*x^n) + 2*((a^5*
b^3 - a^3*b^5)*cos(c)*sin(2*c) - (a^5*b^3 - a^3*b^5)*cos(2*c)*sin(c))*d*e*x
*sin(d*x^n) + (a^6*b^2 - a^4*b^4)*d*e*x*sin(2*c))*sin(2*d*x^n) - 2*(a^6*b^2
*d*e*x*cos(2*c)*sin(2*d*x^n) + a^6*b^2*d*e*x*cos(2*d*x^n)*sin(2*c) - 2*(a^7
*b - a^5*b^3)*d*e*x*cos(d*x^n)*cos(c) + 2*(a^7*b - a^5*b^3)*d*e*x*sin(d*x^n
)*sin(c))*sin(2*d*x^n + 2*c)), x) + 2*(3*a*b^3*e^(3*n)*x^(2*n)*sin(d*x^n +
c) + 3*a^2*b^2*e^(3*n)*x^(2*n) - 2*(a^3*b - a*b^3)*d*e^(3*n)*x^(3*n)*cos(d*
x^n + c))*sin(2*d*x^n + 2*c))/((a^6 - a^4*b^2)*d*e*n*cos(2*d*x^n + 2*c)^2 +
 4*(a^4*b^2 - a^2*b^4)*d*e*n*cos(d*x^n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e*n*c
os(d*x^n + c)*sin(2*d*x^n + 2*c) + (a^6 - a^4*b^2)*d*e*n*sin(2*d*x^n + 2*c)
^2 + 4*(a^4*b^2 - a^2*b^4)*d*e*n*sin(d*x^n + c)^2 + 4*(a^5*b - a^3*b^3)*d*e
*n*sin(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n - 2*(2*(a^5*b - a^3*b^3)*d*e*n*si
n(d*x^n + c) + (a^6 - a^4*b^2)*d*e*n)*cos(2*d*x^n + 2*c))
```

Giac [F]

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(ex)^{3n-1}}{(b\csc(dx^n+c)+a)^2}\,dx$$

```
[In] integrate((e*x)^(-1+3*n)/(a+b*csc(c+d*x^n))^2,x, algorithm="giac")
[Out] integrate((e*x)^(3*n - 1)/(b*csc(d*x^n + c) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{-1+3n}}{(a+b\csc(c+dx^n))^2}\,dx = \int \frac{(e\,x)^{3\,n-1}}{\left(a+\frac{b}{\sin(c+d\,x^n)}\right)^2}\,dx$$

```
[In] int((e*x)^(3*n - 1)/(a + b/sin(c + d*x^n))^2,x)
[Out] int((e*x)^(3*n - 1)/(a + b/sin(c + d*x^n))^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```